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Picture Fuzzy Linear Programming Problems

Chiranjibe Jana[1*], Zdravko Nunić[2]

[1] Saveetha School of Engineering, Saveetha Institute of Medical and Technical Sciences (SIMATS), 602105 Chennai, India
[2] Faculty of Transport and Traffic Engineering, University of East Sarajevo, 74000 Doboj, Bosnia and Herzegovina

[*] Correspondence: Chiranjibe Jana (jana.chiranjibe7@gmail.com)

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Abstract: This study introduces an advanced framework for picture fuzzy linear programming problems (PFLPP), deploying picture fuzzy numbers (PFNs) to articulate diverse parameters. Integral to this approach are the three cardinal membership functions: membership, neutral, and non-membership, each contributing distinctly to the formation of the PFLPP. Emphasis is placed on employing these degrees to formulate the PFLPP in its most unadulterated form. Furthermore, the research delineates a novel optimization model, tailored specifically for the resolution of the PFLPP. A meticulous case study, accompanied by a numerical example, is presented, demonstrating the efficacy and robustness of the proposed methodology. The study culminates in a comprehensive discussion of the findings, highlighting pivotal insights and delineating potential avenues for future inquiry. This exploration not only advances the theoretical underpinnings of picture fuzzy sets but also offers practical implications for the application of linear programming in complex decision-making scenarios.

Keywords: Picture fuzzy numbers (PFNs); Linear programming optimization; Membership functions; Decision making analysis

1 Introduction

The linear programming problem (LPP) is the most well-known, straightforward, and widely applied paradigm in mathematical programming. The LPP model's simplicity makes it easily adaptable to a wide range of practical applications, including supply chain management, supplier selection, assignment issues, and challenges with transportation. Over several decades, the extension and development of standard LPP have been discussed. Examples of expanded LPP include goal LPP, multi-objective LPP, bi-level or multi-layer LPP, etc.

Many studies also use the widely used LPP, which accounts for uncertainty. Initially, Bellman and Zadeh [1] invented the fuzzy set (FS) theory, which served as the foundation for the development of fuzzy LPP (FLPP). Zimmermann [2] initially introduced the fuzzy programming technique to address the multiobjective LPP in a fuzzy setting. The goal of the fuzzy optimization technique is to maximize the marginal satisfaction of each fuzzy decision set element (belongingness degree, membership functions, etc.).

It was later demonstrated that only taking membership degrees (MD) into consideration was insufficient to adequately convey the marginal attainment of an element within the fuzzy decision set. Atanassov [3] initially presented the intuitionistic fuzzy set (IFS) theory as a means of extending or examining the fuzzy set. The degree of MD and NMD of an element in the IFS decision set are addressed by the IFS theory. The first person to suggest IF optimization strategies based on IF decision sets in decision-making issues was Angelov [4].

The literature on fuzzy and IF environments has greatly benefited from the work of numerous authors. Furthermore, when applying IFS theory to the multi-criteria decision-making (MCDM) problem, it is occasionally discovered that its performance is limited in a variety of situations. For example, suppose the decision-maker(s) gives alternatives that satisfy the requirements a score of 0.6 and those that don't a score of 0.7. Given that the values of the degrees for MD and NMD are both between 0 and 1, they are unrelated to each other. The case is not covered by the IF decision set since it requires the sum of MD and NMD to be less than or equal to 1.

To get around some of the issues in real-world situations with FSs [5] and IFSs, Cuong presented the picture fuzzy set (PFS) as a new idea for issues with computational intelligence to handle this circumstance [6, 7]. Due to its significance, several researchers have added to the idea of PFS information in actual decision-making. Wei [8] defined weighted AOs in this manner to aggregate PFS data. Wei [9] analyzed the entropy of PFS information and applied this idea to create a MADM model. Later, Wei employed the Hamacher AOs and built picture-fuzzy Hamacher AOs using this concept [10]. Additionally, Peng and Dai [11] created an approach for creating distance-based MADM issues in a PFS. Singh [12] studied the coefficient of the correlation measure using PFS arguments. New inference systems for PFS were proposed by Son et al. [13, 14]. When developing decisions, Son [15] used distribution-based picture fuzzy clustering. Thong et al. [16–20] focused on the clustering approach to analyzing complicated dates and weather forecasting in PFS environments. In order to create customer requirements, Ping et al. [21] used a novel quality function deployment (QFD) environment. Projections-based MADM issues in a PFS are discussed by Wei et al. [22]. The dice similarity measure approach was employed in PFSs by Wei and Gao to create MADM models [23]. Wei proposed new similarity measures in a picture-fuzzy environment in the study [24]. In the same setting, weighted Dombi AOs were proposed by Jana et al. [25] and utilized to build a MADM technique. Readers are referred to the following sources for more details on the decision-making process connected to the PFS [26–30]. Several study approaches have been employed to model MADM difficulties, from which we have accurately discovered a viable solution. The TODIM technique [31], the TOPOSIS approach [32, 33], the CODAS approach [34], the VIKOR approach [35], the ELECTRE approach [36], the PROMETHEE approach [37], and other MADM approaches were examined in the earlier, well-known studies. First presented by Pamučar and Ćirović [38], the multi-attributive border approximation area comparison (MABAC) is a unique MADM approach that may consider competing attributes while making decisions. In a model of MABAC structures, the fuzzy decision-theoretic environment and the intangibility of decision-makers (DMs) are taken into account so that more concrete and useful aggregate information can be gathered.

In that direction, we have seen various innovative MADM techniques. From this angle, Wu and Liao [39] investigated a novel outranking strategy that measured gains and losses in dominance scores (GLDS). The GLDS approach measured the "group utility" and the individual regret score at the same time. Subsequently, Peng and Yang [40] investigated the enlarged MABAC technique in a Pythagorean scenario. After that, the original MABAC approach was improved by Pamuvčar et al. [41].Pamuvcar et al. examined the hybrid IR-AHP-MABAC model once more. Sun et al. [42] presented the Extended MABAC approach with HFLNs, which is used to establish patient priority levels. A likelihood-based MABAC technique employing intuitionistic trapezoidal linguistics (ITLNs) was developed by Yu et al. [43]. Wang et al. [44] examined the MABAC method in an orthopair context with fuzzy Q-rungs. The MABAC approach was investigated in a visual 2-tuple linguistic context by Zhang et al. [45]. The goal of this work is to close the information gap regarding PFN decision-making difficulties and the LPP approach. This work set up a fuzzy linear programming problem procedure and constructed a PFLPP model that was based on a conventional LPP model that included PFS information. We wrap up by presenting some numerical examples to talk about a few PFLLP models in a PFN setting.

The article's remaining sections are arranged as follows: A review of some previous research is done in Section 2. In Section 3, talk about the fuzzy linear programming problem and its evolution. Update the fuzzy optimization model for the picture in Section 4. To examine the proposed method, some PFLPP instances are solved in Section 5. In the concluding Section 6, include the document that contains the future working directions.

2 Preliminaries

In this section, we annotate some essential ideas of PFSs.

Definition 2.1 [6, 7] A PFS $\mathcal{R}_f$ over the fixed set L is written as

$$\mathcal{R}_f = \{\langle \mathcal{Y}_f(l), \mathcal{A}_U(l), \mathcal{N}_f(l)\rangle \mid l \in L\},$$

$\mathcal{Y}_f(l) : L \to [0,1]$, $\mathcal{A}_f(l) : L \to [0,1]$ and $\mathcal{N}_f(l) : L \to [0,1]$ in a picture fuzzy set, are displayed in the following order: positive membership degree (PMD), neutral membership degree (NMD), and negative membership degree of $\mathcal{R}_f$, where, $0 \leq \mathcal{Y}_f(l) + \mathcal{A}_f(l) + \mathcal{N}_f(l) \leq 1$ for $l \in L$. Also, for refusal membership degree is denoted for l as $\pi_{\mathcal{R}_f}(l) = 1 - \mathcal{Y}_f(l) - \mathcal{A}_f(l) - \mathcal{N}_f(l)$. The pair $(\mathcal{Y}_f, \mathcal{A}_f, \mathcal{N}_f)$ is named as picture fuzzy numbers (PFNs) or picture fuzzy values (PFVs).

Cuong et al. [6] introduced some basic operations on PFSs given as:

Definition 2.2 [6] Let $\mathcal{R}_1 = (\mathcal{Y}_1(l), \mathcal{A}_1(l), \mathcal{N}_1(l))$ and $\mathcal{R}_2 = (\mathcal{Y}_2(l), \mathcal{A}_2(l), \mathcal{N}_2(l))$ be any two PFNs over the set L. The operations between two PFNs are below:

(i) $\mathcal{R}_1 \subseteq \mathcal{R}_2$, if $\mathcal{Y}_1(l) \leq \mathcal{Y}_2(l), \mathcal{A}_1(l) \leq \mathcal{A}_2(l)$ and $\mathcal{N}_1(l) \geq \mathcal{N}_2(l)$ for all $l \in L$,
(ii) $\mathcal{R}_1 = \mathcal{R}_2$ if $\mathcal{R}_1 \subseteq \mathcal{R}_2$ and $\mathcal{R}_2 \subseteq \mathcal{R}_1$,
(iii) $\mathcal{R}_1 \cup \mathcal{R}_2 = \{\langle l, \max\{\mathcal{Y}_1(l), \mathcal{R}_2(l)\}, \min\{\mathcal{A}_1(l), \mathcal{A}_2(l)\}, \min\{\mathcal{N}_1(l), \mathcal{N}_2\}\rangle \mid l \in L\}$,
(iv) $\mathcal{R}_1 \cap \mathcal{R}_2 = \{\langle l, \min\{\mathcal{Y}_1(l), \mathcal{Y}_2(l)\}, \max\{\mathcal{A}_1(l), \mathcal{A}_2(x)\}, \max\{\mathcal{N}_1(l), \mathcal{N}_2\}\rangle \mid l \in L\}$,

(v) $\overline{\mathcal{R}_1} = \{\langle l, \mathcal{N}_1(l), \mathcal{A}_1(l), \mathcal{Y}_1(l)\rangle \mid l \in L\}$ for all $l \in L$.

Wei [8] offered various operations on PFNs in the definition below, depending on the operations on IFSs.

Definition 2.3 [8] Let $\mathcal{R}_1 = (\mathcal{Y}_1(l), \mathcal{A}_1(l), \mathcal{N}_1(l))$ and $\mathcal{R}_2 = (\mathcal{Y}_2(l), \mathcal{A}_2(l), \mathcal{N}_2(l))$ be any two PFNs over the set L, then more operations are as follows:

(i) $\overline{\mathcal{R}_1} = \{\langle l, \mathcal{N}_1(l), \mathcal{A}_1(l), \mathcal{Y}_1(l)\rangle \mid l \in L\}$,

(ii) $\mathcal{R}_1 \wedge \mathcal{R}_2 = \{\langle l, \min\{\mathcal{Y}_1(l), \mathcal{Y}_2(l)\}, \max\{\mathcal{A}_1(l), \mathcal{A}_2(l)\}, \max\{\mathcal{N}_1(l), \mathcal{N}_2\}\rangle \mid l \in L\}$,

(iii) $\mathcal{R}_1 \vee \mathcal{R}_2 = \{\langle l, \max\{\mathcal{Y}_1(l), \mathcal{Y}_2(l)\}, \min\{\mathcal{A}_1(l), \mathcal{A}_2(l)\}, \min\{\mathcal{N}_1(l), \mathcal{N}_2\}\rangle \mid l \in L\}$,

(iv) $\mathcal{R}_1 \oplus \mathcal{R}_2 = (\langle \mathcal{Y}_1 + \mathcal{Y}_2 - \mathcal{Y}_1\mathcal{Y}_2, \mathcal{A}_1\mathcal{A}_2, \mathcal{N}_1\mathcal{N}_2\rangle)$,

(v) $\mathcal{R}_1 \otimes \mathcal{R}_2 = (\langle \mathcal{Y}_1\mathcal{Y}_2, \mathcal{A}_1 + \mathcal{A}_2 - \mathcal{A}_1\mathcal{A}_2, \mathcal{N}_1 + \mathcal{N}_2 - \mathcal{N}_1\mathcal{N}_2\rangle)$,

(vi) $\lambda\mathcal{R}_1 = \left(1 - (1 - \mathcal{Y}_1)^{\lambda}, \mathcal{A}_1^{\lambda}, \mathcal{N}_1^{\lambda}\right)$,

(vii) $\mathcal{R}_1^{\lambda} = \left(\mathcal{Y}_1^{\lambda}, 1 - (1 - \mathcal{A}_1)^{\lambda}, 1 - (1 - \mathcal{N}_1)^{\lambda}\right)$.

Definition 2.4 [25] Let $\mathcal{R}_1 = (\mathcal{Y}_1, \mathcal{A}_1, \mathcal{N}_1)$ be PFNs, then score $\Lambda(\mathcal{R}_1)$ and accuracy $\Phi(\mathcal{R}_1)$ for PFN are defined as follows:

$$\begin{aligned} \Lambda(\mathcal{R}_1) &= \frac{1 + \mathcal{Y}_1 - \mathcal{N}_1}{2}, \quad \Lambda(\mathcal{R}_1) \in [0, 1], \\ \Phi(\mathcal{R}_1) &= \mathcal{Y}_1 - \mathcal{N}_1, \quad \Phi(\mathcal{R}_1) \in [-1, 1]. \end{aligned} \tag{1}$$

3 Some Models on Picture Fuzzy LPP

The LPP is a well-liked and often utilised kind of mathematical programming. Numerous researchers have thoroughly examined the various extensions of the linear programming problem, including fuzzy LPP, IFs-LPP, and neutrosophic LPP [46–50].

The picture fuzzy linear programming problem (PFLPP), which introduces the image fuzzy concept, is used to investigate the further extension of the LPP. First, the PFLPP is presented in the Model I. The picture fuzzy number only represents the co-efficient of the goal function; all other parameters are taken to be real numbers.

Model I

Optimize $M = \sum_{f=1}^{\varpi} C_f l_f$.

Subject to

$$\sum_{f=1}^{\varpi} a_{gf} l_f \leq, =, \geq b_g, \quad \forall g = 1, 2, \ldots, \xi$$
$$l_f \geq 0, \quad \forall f = 1, 2, \ldots, \varpi$$

where, the parameters a_{gf}, b_g are real integers and C_f is a PFN. In the second PFLPP model, a real number represents the co-efficient of the objective function, whereas picture fuzzy numbers represent the co-efficient of constraints variables and right sides. Consequently, an analogous model is as follows:

Model II

Optimize $M = \sum_{f=1}^{\varpi} C_f l_f$.

Subject to

$$\sum_{f=1}^{\varpi} a_{gf} l_f \leq, =, \geq b_g, \quad \forall g = 1, 2, \ldots, \xi$$
$$l_f \geq 0, \quad \forall f = 1, 2, \ldots, \varpi$$

where, a_{gf}, b_g is a PFN, and C_f is a real number.

In the third model, which is a fully PFLPP, each parameter is represented by a PFN.

Model III

Optimize $M = \sum_{f=1}^{\varpi} C_f l_f$.

Subject to

$$\sum_{f=1}^{\varpi} a_{gf} l_f \leq, =, \geq b_g, \quad \forall g = 1, 2, \ldots, \xi$$

$$l_f \geq 0, \quad \forall f = 1, 2, \ldots, \varpi$$

where, C_f, a_{gf}, and b_g are PFNs.

The membership function $\mathcal{Y}_f$ for PFN $\mathcal{R}_f$ can be defined as follows:

$$\mathcal{Y}_f(l) = \begin{cases} \frac{l-\mathcal{Y}_1}{\mathcal{Y}_2-\mathcal{Y}_1}, & \mathcal{Y}_1 \leq l \leq \mathcal{Y}_2 \\ \frac{\mathcal{Y}_2-l}{\mathcal{Y}_3-\mathcal{Y}_2}, & \mathcal{Y}_2 \leq l \leq \mathcal{Y}_3 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The NuMF $\mathcal{A}_f$ for PFN $\mathcal{R}_f$ can be defined as follows:

$$\mathcal{A}_f(l) = \begin{cases} \frac{l-\mathcal{A}_1}{\mathcal{A}_2-\mathcal{A}_1}, & \mathcal{A}_1 \leq l \leq \mathcal{A}_2 \\ \frac{\mathcal{A}_2-l}{\mathcal{A}_3-\mathcal{A}_2}, & \mathcal{A}_2 \leq l \leq \mathcal{A}_3 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

The NMF $\mathcal{N}_f$ for PFN $\mathcal{R}_f$ can be defined as follows:

$$\mathcal{N}_f(l) = \begin{cases} \frac{l-\mathcal{N}_1}{\mathcal{N}_2-\mathcal{N}_1}, & \mathcal{N}_1 \leq l \leq \mathcal{N}_2 \\ \frac{\mathcal{N}_2-l}{\mathcal{N}_3-\mathcal{N}_2}, & \mathcal{N}_2 \leq l \leq \mathcal{N}_3 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

First, we establish the upper and lower bounds for each MF using the following formulae in order to define the PMF, NMF, and negative non-MF for the objective function under the PFN concept:

$$M_U^{\mathcal{Y}} = \max\{M(l_f)\} \text{ and } M_\theta^{\mathcal{Y}} = \min\{M(l_f)\} \quad (PMF) \tag{5}$$

$$M_U^{\mathcal{A}} = M_\theta^{\mathcal{A}} + P\left(M_U^{\mathcal{Y}} - M_\theta^{\mathcal{Y}}\right)\} \text{ and } M_\theta^{\mathcal{A}} = M_\theta^{\mathcal{Y}} \quad (NuMF) \tag{6}$$

$$M_U^{\mathcal{N}} = M_U^{\mathcal{Y}} \text{ and } M_L^{\mathcal{N}} = M_\theta^{\mathcal{Y}} + Q\left(M_U^{\mathcal{Y}} - M_L^{\mathcal{Y}}\right)\} \quad (NMF) \tag{7}$$

where, $P, Q \in [0, 1]$ are predetermined real numbers.

The PMF, NuMF, and NMF for the objective function can be obtained using Eqs. (5) to (7) as follows:

$$\mathcal{Y}(M(l)) = \begin{cases} 1, & \text{if } m \geq m_U^{\mathcal{Y}} \\ \frac{m-m_\theta^{\mathcal{Y}}}{m_U^{\mathcal{Y}}-m_\theta^{\mathcal{Y}}}, & \text{if } m_\theta^{\mathcal{Y}} \leq m \leq m_U^{\mathcal{Y}} \\ 0, & \text{if } m \leq m_\theta^{\mathcal{Y}} \end{cases}$$

$$\mathcal{A}(M(l)) = \begin{cases} 1, & \text{if } m \geq m_U^{\mathcal{A}} \\ \frac{m-m_\theta^{\mathcal{A}}}{m_\theta^{\mathcal{A}}-m_\theta^{\mathcal{A}}}, & \text{if } m_\theta^{\mathcal{A}} \leq m \leq m_U^{\mathcal{A}} \\ 0, & \text{if } m \leq m_\theta^{\mathcal{A}} \end{cases}$$

$$\mathcal{N}(M(x)) = \begin{cases} 1, & \text{if } m \geq m_U^{\mathcal{N}} \\ \frac{m-m_\theta^{\mathcal{N}}}{m_U^{\mathcal{N}}-m_\theta^{\mathcal{N}}}, & \text{if } m_\theta^{\mathcal{N}} \leq m \leq m_U^{\mathcal{N}} \\ 0, & \text{if } m \leq m_\theta^{\mathcal{N}} \end{cases}$$

where, $\mathcal{Y}(M(l))$, $\mathcal{A}(M(l))$, and $\mathcal{N}(M(l))$ are PMF, NuMF and NMF of objective function.

Following is a description of the PMF for C_f^{th} constraints in a PFS:

$$Y(C_f(l)) = \begin{cases} 1, & \text{if } b_g \geq \sum_{f=1}^{\varpi} (a_{gf} + d_{gf}) l_f \\ \frac{b_g - \sum_{f=1}^{\varpi} a_{gf} l_f}{\sum_{f=1}^{\varpi} d_{gf} l_f}, & \text{if } \sum_{f=1}^{\varpi} a_{gf} l_f \leq b_g \leq \sum_{f=1}^{\varpi} (a_{gf} + d_{gf}) l_f \\ 0, & \text{if } b_g \leq \sum_{f=1}^{\varpi} a_{gf} l_f \end{cases}$$

$$A(C_f(l)) = \begin{cases} 1, & \text{if } b_g \geq \sum\limits_{f=1}^{\varpi}(a_{gf}+d_{gf})l_f \\ \frac{b_g - \sum\limits_{f=1}^{\varpi} d_{gf}l_f}{\sum\limits_{f=1}^{\varpi} a_{gf}l_f}, & \text{if } \sum\limits_{f=1}^{\varpi} a_{gf}l_f \leq b_g \leq \sum\limits_{f=1}^{\varpi}(a_{gf}+d_{gf})l_f \\ 0, & \text{if } b_g \leq \sum\limits_{f=1}^{\varpi} a_{gf}l_f \end{cases}$$

$$N(C_f(x)) = \begin{cases} 0, & \text{if } b_g \geq \sum\limits_{f=1}^{\varpi}(a_{gf}+d_{gf})l_f \\ \frac{\sum\limits_{f=1}^{\varpi}(a_{gf}+d_{gf})l_f - b_g}{\sum\limits_{f=1}^{\varpi} d_{gf}l_f}, & \text{if } \sum\limits_{f=1}^{\varpi} a_{gf}l_f \leq b_g \leq \sum\limits_{f=1}^{\varpi}(a_{gf}+d_{gf})l_f \\ 1, & \text{if } b_g \leq \sum\limits_{f=1}^{\varpi} a_{gf}l_f \end{cases}$$

where, $d_{gf} \in [0,1]$ is the jth constraint's pre-set tolerance limit.

4 Picture Fuzzy Optimization Model

PF optimisation techniques can be used to solve the PFLP model (Section 3) that was previously discussed. We have taken into account the maximisation of PMF and the minimization of NuMF and NMF under the PF decision set in the suggested PF optimisation model. Consequently, the following statement represents the PF optimisation model:

$$\max Y(M(l)), \min A(M(l)), \min N(M(l))$$

Subject to

$$Y(M(l)) \geq A(M(l)), Y(M(l)) \geq N(M(l)),$$
$$0 \leq Y(M(l)) + A(M(l)) + N(M(l)) \leq 1,$$
$$Y(M(l)), A(M(l)), N(M(l)) \geq 0, l \geq 0.$$

where, $Y(M(l)), A(M(l))$, and $N(M(l))$ represent the PMF, NuMF, and NMF of the picture fuzzy objective optimization function and constraint.

The aforementioned problem can be converted into the following optimisation model:

$$\max \phi, \min \varphi, \min \psi$$

Subject to

$$Y(M(l)) \geq \phi, \;\; A(M(l)) \leq \varphi, \;\; N(M(l)) \leq \psi,$$
$$\phi \geq \varphi, \;\; \varphi \geq \psi,$$
$$0 \leq \phi + \varphi + \psi \leq 1, \;\; x \geq 0.$$

where, ϕ, φ and ψ respectively represents the minimal degree of acceptance of PMF, maximal degree of acceptance of NuMF, and maximal degree of acceptance of NMF.

The PF optimisation model can be converted into the following model equivalently:

$$\max(\phi - \varphi - \psi)$$

Subject to

$$Y(M(l)) \geq \phi, \;\; A(M(l)) \leq \varphi, \;\; N(M(l)) \leq \psi,$$
$$\phi \leq \varphi, \;\; \varphi \geq \psi, \;\; \phi, \varphi, \psi \geq 0,$$
$$0 \leq \phi + \varphi + \psi \leq 1, \;\; x \geq 0.$$

The above-discussed PF optimisation model covers the more realistic aspects of parameter uncertainty and can solve the PFLPP with a variety of picture fuzzy parameters.

5 Numerical Examples

The usefulness and viability of the picture fuzzy optimisation model are demonstrated by the forthcoming numerical examples. The different picture fuzzy parameters are converted into the crisp version for each decided-upon membership level using Eqs. (2) to (4).

Example (Model I)

Maximize $M = C_1 l_1 + C_2 l_2 + C_3 l_3$

Subject to

$$0.3l_1 + 0.5l_2 + 0.7l_3 \leq 12$$
$$0.4l_1 + 0.4l_2 + 0.6l_3 \leq 15$$
$$0.5l_1 + 0.8l_2 + 0.2l_3 \leq 16$$
$$l_1,\ l_2,\ l_3 \geq 0$$

where,

$$C_1 = (0.4, 0.1, 0.3), \quad C_2 = (0.6, 0.0, 0.3), \quad C_3 = (0.5, 0.1, 0.2)$$

Here C_1, C_2 and C_3 are PFNs. The values of PFNs C_1, C_2 and C_3 are transferred into crisp by using Eq. (1). The following steps can be taken to obtain the PFLPP (Model I) in crisp form:

Maximize $M = 0.5l_1 + 0.6l_2 + 0.6l_3$

Subject to

$$0.3l_1 + 0.5l_2 + 0.7l_3 \leq 12$$
$$0.4l_1 + 0.4l_2 + 0.6l_3 \leq 15$$
$$0.5l_1 + 0.8l_2 + 0.2l_3 \leq 16$$
$$l_1,\ l_2,\ l_3 \geq 0$$

The following are the solution's outcomes: $l_1 = 30.34$, $l_2 = 0$, $l_3 = 4.13$, and Max $M = 17.65$.

Example (Model II)

Maximize $M = 50l_1 + 60l_2 + 20l_3$

Subject to

$$a_{11}\ l_1 + a_{12}l_2 + a_{13}l_3 \leq b_1$$
$$a_{21}\ l_1 + a_{22}l_2 + a_{23}l_3 \leq b_2$$
$$a_{31}\ l_1 + a_{32}l_2 + a_{33}l_3 \leq b_3$$
$$l_1,\ l_2,\ l_3 \geq 0$$

where,

$$a_{11} = (0.4, 0.1, 0.3), \quad a_{12} = (0.5, 0.1, 0.3), \quad a_{13} = (0.5, 0.2, 0.2)$$
$$a_{21} = (0.6, 0.2, 0.1), \quad a_{22} = (0.8, 0.0, 0.1), \quad a_{23} = (0.4, 0.2, 0.3)$$
$$a_{31} = (0.7, 0.1, 0.1), \quad a_{32} = (0.3, 0.2, 0.2), \quad a_{33} = (0.2, 0.1, 0.4)$$
$$b_1 = (0.6, 0.0, 0.2), \quad b_2 = (0.5, 0.3, 0.1), \quad b_3 = (0.7, 0.1, 0.1)$$

Here, $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, b_1, b_2$, and b_3 are all PFNs. They are transferred into crisp by using score function defined in Eq. (1). The following steps can be taken to obtain the PFLPP (Model II) in crisp form:

Maximize $M = 50l_1 + 60l_2 + 20l_3$

Subject to

$$0.55\ l_1 + 0.60l_2 + 0.65l_3 \leq 0.70$$
$$0.75\ l_1 + 0.85l_2 + 0.55l_3 \leq 0.70$$
$$0.80\ l_1 + 0.55l_2 + 0.40l_3 \leq 0.80$$
$$l_1,\ l_2,\ l_3 \geq 0$$

The following are the solution's outcomes: $l_1 = 0.00$, $l_2 = 0.82$, $l_3 = 0.00$, and Max $M = 49.41$.

Example (Model III)

Maximize $M = C_1 l_1 + C_2 l_2 + C_3 l_3$

Subject to

$$a_{11}\ l_1 + a_{12}l_2 + a_{13}l_3 \leq b_1$$

$$a_{21}\, l_1 + a_{22}l_2 + a_{23}l_3 \le b_2$$
$$a_{31}\, l_1 + a_{32}l_2 + a_{33}l_3 \le b_3$$
$$l_1,\ l_2,\ l_3 \ge 0$$

where,

$$C_1 = (0.7, 0.0, 0.3), \quad C_2 = (0.6, 0.1, 0.1), \quad C_3 = (0.8, 0.1, 0.1)$$
$$a_{11} = (0.4, 0.1, 0.1), \quad a_{12} = (0.3, 0.1, 0.2), \quad a_{13} = (0.7, 0.1, 0.1)$$
$$a_{21} = (0.6, 0.1, 0.3), \quad a_{22} = (0.5, 0.2, 0.2), \quad a_{23} = (0.5, 0.2, 0.1)$$
$$a_{31} = (0.6, 0.1, 0.2), \quad a_{32} = (0.4, 0.2, 0.2), \quad a_{33} = (0.6, 0.1, 0.2)$$
$$b_1 = (0.6, 0.0, 0.1), \quad b_2 = (0.8, 0.1, 0.1), \quad b_3 = (0.4, 0.1, 0.1).$$

Here, $C_1, C_2, C_3, a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}, b_1, b_2$, and b_3 are all PFNs. They are transferred into crisp by using score function defined in Eq. (1). The following steps can be taken to obtain the PFLPP (Model III) in crisp form:

Maximize $M = 0.70l_1 + 0.75l_2 + 0.85l_3$

Subject to

$$0.65l_1 + 0.55l_2 + 0.80l_3 \le 0.75$$
$$0.65l_1 + 0.65l_2 + 0.70l_3 \le 0.85$$
$$0.70l_1 + 0.60l_2 + 0.70l_3 \le 0.65$$
$$l_1, l_2, l_3 \ge 0$$

The following are the solution's outcomes: $l_1 = 0.00$, $l_2 = 1.08$, $l_3 = 0.00$, and Max $M = 0.81$.

6 Conclusions

This is the first discussion of LPP that takes into consideration the recently announced picture fuzzy set. The PFS makes it easy to collect inaccurate and inconsistent data, which makes decision-making even more difficult. In this chapter, we therefore introduce PFLPP in a picture fuzzy context, which consists of decreasing neutral and non-membership functions inside the picture fuzzy decision set and maximizing membership. By employing membership, neutral, and non-membership degrees, respectively, the deterministic form of PFLPP is generated. Additionally, several numerical examples are provided to show how appropriate the suggested SFLPP solution methodology is. The PFLPP also solves two distinct real-world problems: production planning and purchasing strategy.

The LPP duality theory might be presented in this context to analyze future research attempts in the fuzzy picture domain. Moreover, the suggested approach can be expanded to address issues related to fractional programming, nonlinear programming, etc. Researchers have a variety of possibilities when it comes to applying PFLPP to real-world issues such as supply chain management, inventory control, portfolio management, and transportation-related issues.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] R. E. Bellman and L. A. Zadeh, "Decision-making in a fuzzy environment," *Manage. Sci.*, vol. 17, no. 4, pp. 140–164, 1970. https://doi.org/10.1287/mnsc.17.4.b141

[2] H. J. Zimmermann, "Fuzzy programming and linear programming with several objective functions," *Fuzzy Sets Syst.*, vol. 1, no. 1, pp. 45–55, 1978. https://doi.org/10.1016/0165-0114(78)90031-3

[3] K. T. Atanassov, *Intuitionistic Fuzzy Sets*. Physica Heidelberg, Germany, 1999.

[4] P. P. Angelov, "Optimization in an intuitionistic fuzzy environment," *Fuzzy Sets Syst.*, vol. 86, no. 3, pp. 299–306, 1997. https://doi.org/10.1016/s0165-0114(96)00009-7

[5] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965. https://doi.org/10.1016/s0019-9958(65)90241-x

[6] B. C. Cuong, "Picture fuzzy sets -first results. Part 1, seminar neuro-fuzzy systems with applications," Institute of Mathematics, Hanoi, 2013.

[7] B. C. Cuong, "Picture fuzzy sets -first results. Part 2, seminar neuro-fuzzy systems with applications," Institute of Mathematics, Hanoi, 2013.

[8] G. W. Wei, "Picture fuzzy aggregation operators and their application to multiple attribute decision making," *J. Intell. Fuzzy Syst.*, vol. 33, no. 2, pp. 713–724, 2017. https://doi.org/10.3233/jifs-161798

[9] G. W. Wei, "Picture fuzzy cross-entropy for multiple attribute decision making problems," *J. Bus. Econ. Manage.*, vol. 17, no. 4, pp. 491–502, 2016. https://doi.org/10.3846/16111699.2016.1197147

[10] G. W. Wei, "Picture fuzzy Hamacher aggregation operators and their application to multiple attribute decision making," *Fund. Inf.*, vol. 157, no. 3, pp. 271–320, 2018. https://doi.org/10.3233/fi-2018-1628

[11] X. Peng and J. Dai, "Algorithm for picture fuzzy multiple attribute decision making based on new distance measure," *Int. J. Uncertain. Quant.*, vol. 7, no. 2, pp. 177–187, 2017. https://doi.org/10.1615/int.j.uncertaintyquantification.2017020096

[12] P. Singh, "Correlation coefficients for picture fuzzy sets," *J. Intell. Fuzzy Syst.*, vol. 28, no. 2, pp. 591–604, 2015. https://doi.org/10.3233/ifs-141338

[13] L. H. Son, "Generalized picture distance measure and applications to picture fuzzy clustering," *Appl. Soft Comput.*, vol. 46, pp. 284–295, 2016. https://doi.org/10.1016/j.asoc.2016.05.009

[14] L. H. Son, P. Van Viet, and P. Van Hai, "Picture inference system: A new fuzzy inference system on picture fuzzy set," *Appl. Intell.*, vol. 46, no. 3, pp. 652–669, 2016. https://doi.org/10.1007/s10489-016-0856-1

[15] L. H. Son, "DPFCM: A novel distributed picture fuzzy clustering method on picture fuzzy sets," *Expert Syst. Appl.*, vol. 42, no. 1, pp. 51–66, 2015. https://doi.org/10.1016/j.eswa.2014.07.026

[16] P. H. Thong and L. H. Son, "Picture fuzzy clustering for complex data," *Eng. Appl. Artif. Intell.*, vol. 56, pp. 121–130, 2016. https://doi.org/10.1016/j.engappai.2016.08.009

[17] P. H. Thong and L. H. Son, "A novel automatic picture fuzzy clustering method based on particle swarm optimization and picture composite cardinality," *Knowl. Based Syst.*, vol. 109, pp. 48–60, 2016. https://doi.org/10.1016/j.knosys.2016.06.023

[18] P. H. Thong and L. H. Son, "A new approach to multi-variables fuzzy forecasting using picture fuzzy clustering and picture fuzzy rules interpolation method," in *Advances in Intelligent Systems and Computing*. Springer, Cham, 2015, pp. 679–690. https://doi.org/10.1007/978-3-319-11680-8_54

[19] P. H. Thong, L. H. Son, and H. Fujita, "Interpolative picture fuzzy rules: A novel forecast method for weather nowcasting," in *2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Vancouver, Canada*, 2016, pp. 86–93. https://doi.org/10.1109/fuzz-ieee.2016.7737672

[20] N. T. Thong and L. H. Son, "HIFCF: An effective hybrid model between picture fuzzy clustering and intuitionistic fuzzy recommender systems for medical diagnosis," *Expert Syst. Appl.*, vol. 42, no. 7, pp. 3682–3701, 2015. https://doi.org/10.1016/j.eswa.2014.12.042

[21] Y. J. Ping, R. Liu, W. Lin, and H. C. Liu, "A new integrated approach for engineering characteristic prioritization in quality function deployment," *Adv. Eng. Inf.*, vol. 45, pp. 1474–0346, 2020. https://doi.org/10.1016/j.aei.2020.101099

[22] G. W. Wei, F. E. Alsaadi, T. Hayat, and A. Alsaedi, "Projection models for multiple attribute decision making with picture fuzzy information," *Int. J. Mach. Learn. Cybern.*, vol. 9, no. 4, pp. 713–719, 2016. https://doi.org/10.1007/s13042-016-0604-1

[23] G. W. Wei and H. Gao, "The generalized Dice similarity measures for picture fuzzy sets and their applications," *Informatica*, vol. 29, no. 1, pp. 107–124, 2018. https://doi.org/10.15388/informatica.2018.160

[24] G. W. Wei, "Some similarity measures for picture fuzzy sets and their applications," *Iran. J. Fuzzy Syst.*, vol. 15, no. 1, pp. 77–89, 2018. https://doi.org/10.22111/ijfs.2018.3579

[25] C. Jana, T. Senapati, M. Pal, and R. R. Yager, "Picture fuzzy Dombi aggregation operators: Application to MADM process," *Appl. Soft Comput.*, vol. 74, no. 4, pp. 99–109, 2019. https://doi.org/10.1016/j.asoc.2018.10.021

[26] P. H. Phong, D. T. Hieu, R. T. H. Ngan, and P. T. Them, "Some compositions of picture fuzzy relations," in *Proceedings of the 7th National Conference on Fundamental and Applied Information Technology Research (FAIR7), Thai Nguyen, Vietnam*, vol. 2014, 2014, pp. 19–20.

[27] P. T. M. Phuong, P. H. Thong, and L. H. Son, "Theoretical analysis of picture fuzzy clustering," *J. Comput. Sci. Cybern.*, vol. 34, no. 1, pp. 17–32, 2018. https://doi.org/10.15625/1813-9663/34/1/12725

[28] L. H. Son, "Measuring analogousness in picture fuzzy sets: From picture distance measures to picture association measures," *Fuzzy Optim. Decis. Making*, vol. 16, no. 3, pp. 359–378, 2016. https://doi.org/10.1007/s10700-016-9249-5

[29] V. P. Van, M. C. H. Thi, L. H. Son, and V. H. Pham, "Some extensions of membership graphs for picture inference systems," in *2015 Seventh International Conference on Knowledge and Systems Engineering (KSE),*

Ho Chi Minh city, Vietnam, 2015, pp. 192–197. https://doi.org/10.1109/kse.2015.33

[30] Y. Yang, C. Liang, S. Ji, and T. Liu, "Adjustable soft discernibility matrix based on picture fuzzy soft sets and its applications in decision making," *J. Intell. Fuzzy Syst.*, vol. 29, no. 4, pp. 1711–1722, 2015. https://doi.org/10.3233/ifs-151648

[31] R. Lourenzutti and R. A. Krohling, "A study of TODIM in a intuitionistic fuzzy and random environment," *Expert Syst. Appl.*, vol. 40, no. 16, pp. 6459–6468, 2013. https://doi.org/10.1016/j.eswa.2013.05.070

[32] P. Biswas, S. Pramanik, and B. C. Giri, "Neutrosophic TOPSIS with group decision making," in *Fuzzy Multi-Criteria Decision-Making Using Neutrosophic Sets*. Springer, Cham, 2018, pp. 543–585. https://doi.org/10.1007/978-3-030-00045-5_21

[33] T. Gebrehiwet and H. B. Luo, "Risk level evaluation on construction project lifecycle using fuzzy comprehensive evaluation and TOPSIS," *Symmetry*, vol. 11, no. 1, p. 12, 2018. https://doi.org/10.3390/sym11010012

[34] J. Roy, S. Das, S. Kar, and D. Pamučar, "An extension of the CODAS approach using interval-valued intuitionistic fuzzy set for sustainable material selection in construction projects with incomplete weight information," *Symmetry*, vol. 11, no. 3, p. 393, 2019. https://doi.org/10.3390/sym11030393

[35] K. W. Shen and J. Q. Wang, "Z-VIKOR method based on a new comprehensive weighted distance measure of Z-number and its application," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3232–3245, 2018. https://doi.org/10.1109/tfuzz.2018.2816581

[36] P. Liu, H. Li, P. Wang, and J. Liu, "ELECTRE method and its application in multiple attribute decision making based on INS," *J. Shandong Univ. Financ. Econ.*, vol. 28, pp. 80–87, 2016.

[37] P. Ziemba, "NEATF-PROMETHEE – A new fuzzy multiple criteria decision making method based on the adjustment of mapping trapezoidal fuzzy numbers," *Expert Syst. Appl.*, vol. 110, pp. 363–380, 2018. https://doi.org/10.1016/j.eswa.2018.06.008

[38] D. Pamučar and G. Ćirović, "The selection of transport and handling resources in logistics centers using Multi-Attributive Border Approximation area Comparison (MABAC)," *Expert Syst. Appl.*, vol. 42, no. 6, pp. 3016–3028, 2015. https://doi.org/10.1016/j.eswa.2014.11.057

[39] X. L. Wu and H. C. Liao, "A consensus-based probabilistic linguistic gained and lost dominance score method," *Eur. J. Oper. Res.*, vol. 272, no. 3, pp. 1017–1027, 2019. https://doi.org/10.1016/j.ejor.2018.07.044

[40] X. D. Peng and Y. Yang, "Pythagorean fuzzy choquet integral based MABAC method for multiple attribute group decision making," *Int. J. Intell. Syst.*, vol. 31, no. 10, pp. 989–1020, 2016. https://doi.org/10.1002/int.21814

[41] D. Pamučar, I. Petrović, and G. Ćirović, "Modification of the Best–Worst and MABAC methods: A novel approach based on interval-valued fuzzy-rough numbers," *Expert Syst. Appl.*, vol. 91, pp. 89–106, 2018. https://doi.org/10.1016/j.eswa.2017.08.042

[42] R. Sun, J. Hu, J. Zhou, and X. Chen, "A Hesitant fuzzy linguistic projection-based MABAC method for patients's prioritization," *Int. J. Fuzzy Syst.*, vol. 20, no. 7, pp. 2144–2160, 2017. https://doi.org/10.1007/s40815-017-0345-7

[43] S. M. Yu, J. Wang, and J. Q. Wang, "An interval type-2 fuzzy likelihood-based MABAC approach and its application in selecting hotels on a tourism website," *Int. J. Fuzzy Syst.*, vol. 19, no. 1, pp. 47–61, 2016. https://doi.org/10.1007/s40815-016-0217-6

[44] J. Wang, G. W. Wei, C. Wei, and Y. Wei, "MABAC method for multiple attribute group decision making under q-rung orthopair fuzzy environment," *Defence Technol.*, vol. 16, no. 1, pp. 208–216, 2020. https://doi.org/10.1016/j.dt.2019.06.019

[45] S. Zhang, G. W. Wei, F. E. Alsaadi, T. Hayat, C. Wei, and Z. Zhang, "MABAC method for multiple attribute group decision making under picture 2-tuple linguistic environment," *Soft Comput.*, vol. 24, no. 8, pp. 5819–5829, 2019. https://doi.org/10.1007/s00500-019-04364-x

[46] F. Ahmad and A. Y. Adhami, "Neutrosophic programming approach to multiobjective nonlinear transportation problem with fuzzy parameters," *Int. J. Manage. Sci. Eng. Manage.*, vol. 14, no. 3, pp. 218–229, 2018. https://doi.org/10.1080/17509653.2018.1545608

[47] F. Ahmad, A. Y. Adhami, and F. Smarandache, "Single valued neutrosophic hesitant fuzzy computational algorithm for multiobjective nonlinear optimization problem," *Neutrosophic Sets Syst.*, vol. 22, pp. 76–86, 2018.

[48] F. Ahmad, A. Y. Adhami, and F. Smarandache, "Neutrosophic optimizationmodel and computational algorithm for optimal shale gas water management under uncertainty," *Symmetry*, vol. 11, no. 4, p. 544, 2019. https://doi.org/10.3390/sym11040544

[49] F. Ahmad, A. Adhami, and F. Smarandache, "Modified neutrosophic fuzzy optimization model for optimal closed-loop supply chain management under uncertainty," in *Optimization Theory Based on Neutrosophic and Plithogenic Sets*. Academic Press, 2020, pp. 343–403. https://doi.org/10.1016/B978-0-12-819670-0.00015-9

[50] R. M. Rizk-Allah, A. E. Hassanien, and M. Elhoseny, “A multi-objective transportation model under neutrosophic environment,” *Comput. Electr. Eng.*, vol. 69, pp. 705–719, 2018. https://doi.org/10.1016/j.compeleceng.2018.02.024