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Efficacy of Induced Complex Aggregation Operators in Multi-Attribute Decision-Making with Confidence Levels

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Abstract: In the pursuit of advancing multi-attribute group decision-making (MAGDM) methodologies, this study introduces two novel aggregation operators: the Induced Confidence Complex Pythagorean Fuzzy Ordered Weighted Geometric Aggregation (ICCPyFOWGA) operator and the Induced Confidence Complex Pythagorean Fuzzy Hybrid Geometric Aggregation (ICCPyFHGA) operator. These operators are characterized by their capacity to integrate various decision criteria based on complex Pythagorean fuzzy sets (CPyFSs), with an emphasis on the influence of confidence levels. Key structural properties of these operators, such as idempotency, boundedness, and monotonicity, are rigorously established. Furthermore, the practical applicability of these models in real-world decision-making scenarios is demonstrated through a descriptive example that underscores their efficiency and effectiveness. The analytical results affirm that the proposed operators not only enhance decision-making precision but also offer a flexible framework for addressing diverse decision-making environments. This contribution marks a significant advancement in the field of decision science, providing a robust tool for experts and practitioners involved in complex decision-making processes.

Keywords: Inducing variables; Confidence level; Induced operators; Decision-making

1 Introduction

Complex Fuzzy Sets (CFSs), introduced by Ramot et al. [1], extend the traditional concept of fuzzy sets [2] by incorporating complex numbers. Fuzzy sets are a mathematical tool used to represent uncertainty and vagueness. However, CFSs take this a step further by allowing membership values to be complex numbers. In a CFS, each element in the universal set is assigned a complex number as its membership value. This complex number comprises a real part and an imaginary part, signifying both the degree of membership and the phase or angle of membership for that specific element. This extension enables the representation of situations where not only the degree of membership is uncertain but also the phase or orientation of that membership. In other words, CFSs provide a more versatile framework for handling uncertainty in various applications where both magnitude and phase information are crucial. They go beyond traditional fuzzy sets by offering a richer representation of fuzzy information, particularly in scenarios where traditional fuzzy sets might be limited. CFSs are valuable for modeling complex and nuanced uncertainties in a wide range of applications.

Later, Alkouri and Salleh [3] introduced a novel concept called complex intuitionistic fuzzy sets (CIFSs). These sets combine the principles of complex numbers with intuitionistic fuzzy sets (IFSs) [4]. By merging these two concepts, CIFSs offer a more comprehensive way to represent uncertainty, imprecision, and hesitation in decision-making processes. This integration of complex numbers and IFSs in CIFS enhances the framework's expressiveness and flexibility, making it well-suited for addressing real-world scenarios. These scenarios often involve decision-making influenced by multiple factors, each contributing to varying degrees of uncertainty and hesitation. The CIFS approach provides a robust and adaptable foundation for handling complex and uncertain information in such practical situations. In CIFS, each element can be mathematically presented as: $\left(\eta e^{i2\pi p}, ve^{i2\pi q}\right)$ where $\eta e^{i2\pi p}$ and $ve^{i2\pi q}$ stands for complex membership and complex non-membership degrees, respectively, where $\eta \in [0,1]$,

$v \in [0,1]$, $p \in [0,2\pi]$, $q \in [0,2\pi]$ with $0 \prec \eta + v \leq 1$ and $0 \prec \frac{p}{2\pi} + \frac{q}{2\pi} \leq 1$. Later, the studies [5–10] have contributed additional research in the field, specifically focusing on CIFSs.

Later on, Ullah et al. [11] expanded upon the notion of CPyFSs, representing a generalization of CIFSs. CPFSs combine the principles of complex numbers, Pythagorean fuzzy sets (PyFSs) [12], and fuzzy logic, offering a more versatile approach to handling uncertainty, vagueness, and imprecision in decision-making processes. In CPyFSs, each element can be mathematically presented as: $\left(\eta e^{i2\pi p}, v e^{i2\pi q}\right)$ under conditions: $0 \prec \eta^2 + v^2 \leq 1$ and $0 \prec \left(\frac{p}{2\pi}\right)^2 + \left(\frac{q}{2\pi}\right)^2 \leq 1$. The studies [13–16] have proposed numerous aggregation operators that rely on CPyFNs. These proposed aggregation methods offer diverse approaches for combining information, providing a range of options for addressing the challenges associated with CPyFNs.

Thus, keeping in mind the advantages of the above-mentioned methods, this paper presents a set of complex Pythagorean fuzzy aggregation operators, such as the I-CCPyFOWGA operator and the I-CCPyFHGA operator. The study delved into elucidating various properties associated with these operators, including but not limited to idempotency, boundedness, and monotonicity.

The remainder of the paper is organized as follows: In Section 2, crucial definitions will be provided to establish a foundational understanding. Section 3 will introduce novel operational laws based on CPyFNs. Section 4 will introduce some operators based on inducing variables. The algorithm for decision-making is elucidated in Section 5, followed by an illustrative example in Section 6. Finally, Section 7 will give the conclusion of the paper.

2 Preliminaries

In this section, we explore complex fuzzy sets theory, which expands upon traditional fuzzy set theory by incorporating complex numbers. This framework enables the representation of uncertainty and imprecision using both magnitude and phase information.

Definition 1: [1] Let A be a CFS and Z be a universal set, then A can be defined on Z as: $A = \left\{t, \eta(t) e^{ip(t)} \mid t \in Z\right\}$ with $\eta(t) : Z \to [0,1]$ and the element $\eta(t)$ is known as the degree of complex membership function of t and $p(t)$ be the real valued function.

Definition 2: [3] Let R be a CIFS and Z be a universal set, then R can be defined on Z as: $R = \left\{\left\langle t, \eta(t) e^{ip(t)}, v(t) e^{iq(t)}\right\rangle \mid t \in Z\right\}$, where $\eta(t) \in [0,1]$, $v(t) \in [0,1]$ are called the degree of complex membership and the degree of complex non-membership functions of t with $0 \prec \eta + v \leq 1$. Moreover, $p(t) \in [0,2\pi]$ and $q(t) \in [0,2\pi]$ with condition $0 \prec \frac{p}{2\pi} + \frac{q}{2\pi} \leq 1$.

Definition 3: [11] Let M be a CPyFS and Z be a universal set, then M can be defined on Z as: $M = \left\{\left\langle t, \eta(t) e^{ip(t)}, v(t) e^{iq(t)}\right\rangle \mid t \in Z\right\}$, where $\eta(t) \in [0,1]$, $v(t) \in [0,1]$ are called the degree of complex membership and the degree of complex non-membership functions with $0 \prec \eta^2 + v^2 \leq 1$. Moreover, $p(t) \in [0,2\pi]$ and $q(t) \in [0,2\pi]$ with $0 \prec \left(\frac{p}{2\pi}\right)^2 + \left(\frac{q}{2\pi}\right)^2 \leq 1$.

Definition 4: [11] Let $F = \left(\eta e^{ip}, v e^{iq}\right)$ be a CPyFN, then its score function can be defined as: $\mathrm{scor}(F) = \left(\eta^2 - v^2\right) + \frac{1}{4\pi^2}\left(p^2 - q^2\right)$ with $\mathrm{scor}(F) \in [-2,2]$.

3 Basic Operational Laws

Definition 5: Let $F_j = \left(\eta_j e^{ip_j}, v_j e^{iq_j}\right)$ $(j = 1,2)$ be a family of CPyFNs and $\lambda \succ 0$, then

i) $F_1 \oplus F_2 = \left(\sqrt{\eta_1^2 + \eta_2^2 - \eta_1^2\eta_2^2}\, e^{i2\pi\sqrt{\left(\frac{p_1}{2\pi}\right)^2 + \left(\frac{p_2}{2\pi}\right)^2 - \left(\frac{p_1}{2\pi}\right)^2\left(\frac{p_2}{2\pi}\right)^2}}, (v_1 v_2)\, e^{\left(\frac{q_1}{2\pi}\right)\left(\frac{q_2}{2\pi}\right)}\right)$

ii) $F_1 \otimes F_2 = \left((\eta_1\eta_2)\, e^{\left(\frac{p_1}{2\pi}\right)\left(\frac{p_2}{2\pi}\right)}, \sqrt{v_1^2 + v_2^2 - v_1^2 v_2^2}\, e^{i2\pi\sqrt{\left(\frac{q_1}{2\pi}\right)^2 + \left(\frac{q_2}{2\pi}\right)^2 - \left(\frac{q_1}{2\pi}\right)^2\left(\frac{q_2}{2\pi}\right)^2}}\right)$

iii) $\lambda(F) = \left(\sqrt{1 - \left(1 - \eta^2\right)^\lambda}\, e^{i2\pi\sqrt{1 - \left(1 - \left(\frac{p}{2\pi}\right)^2\right)^\lambda}}, (v)^\lambda e^{i2\pi\left(\frac{q}{2\pi}\right)^\lambda}\right)$

iv) $(F)^\lambda = \left((\eta) e^{i2\pi\left(\frac{p_1}{2\pi}\right)^\lambda}, \sqrt{1 - \left(1 - v^2\right)^\lambda}\, e^{i2\pi\sqrt{1 - \left(1 - \left(\frac{q}{2\pi}\right)^2\right)^\lambda}}\right)$

Theorem 1: Let $F_j = \left(\eta_j e^{ip_j}, v_j e^{iq_j}\right)$ $(j = 1,2,3)$ be a group of CPyFNs, then
i) $F_1 \oplus F_2 = F_2 \oplus F_1$
ii) $F_1 \otimes F_2 = F_2 \otimes F_1$
iii) $(F_1 \oplus F_2) \oplus F_3 = F_1 \oplus (F_2 \oplus F_3)$
iv) $(F_1 \otimes F_2) \otimes F_3 = F_1 \otimes (F_2 \otimes F_3)$
v) $F_1 \otimes (F_2 \oplus F_3) = F_1 \otimes F_2 \oplus F_1 \otimes F_3$
vi) $(F_1 \oplus F_2) \otimes F_3 = F_1 \otimes F_3 \oplus F_2 \otimes F_3$

Proof: Prove only here (i, ii) and the other parts can be proved by the same way.
i) Since $F_1 = \left(\eta_1 e^{ip_1}, v_1 e^{iq_1}\right)$ and $F_2 = \left(\eta_2 e^{ip_2}, v_2 e^{iq_2}\right)$ are two CPyFNs, then

$$F_1 \oplus F_2 = \left(\sqrt{\eta_1^2 + \eta_2^2 - \eta_1^2\eta_2^2} e^{i2\pi \sqrt{\left(\frac{p_1}{2\pi}\right)^2 + \left(\frac{p_2}{2\pi}\right)^2 - \left(\frac{p_1}{2\pi}\right)^2 \left(\frac{p_2}{2\pi}\right)^2}}, (v_1 v_2)\, e^{\left(\frac{q_1}{2\pi}\right)\left(\frac{q_2}{2\pi}\right)} \right)$$
$$= \left(\sqrt{\eta_2^2 + \eta_1^2 - \eta_2^2\eta_1^2} e^{i2\pi \sqrt{\left(\frac{p_2}{2\pi}\right)^2 + \left(\frac{p_1}{2\pi}\right)^2 - \left(\frac{p_2}{2\pi}\right)^2 \left(\frac{p_1}{2\pi}\right)^2}}, (v_2 v_1)\, e^{\left(\frac{q_2}{2\pi}\right)\left(\frac{q_1}{2\pi}\right)} \right)$$
$$= F_2 \oplus F_1$$

ii) Again, we have

$$F_1 \otimes F_2 = \left((\eta_1\eta_2)\, e^{\left(\frac{p_1}{2\pi}\right)\left(\frac{p_2}{2\pi}\right)}, \sqrt{v_1^2 + v_2^2 - v_1^2 v_2^2} e^{i2\pi \sqrt{\left(\frac{q_1}{2\pi}\right)^2 + \left(\frac{q_2}{2\pi}\right)^2 - \left(\frac{q_1}{2\pi}\right)^2 \left(\frac{q_2}{2\pi}\right)^2}} \right)$$
$$= \left((\eta_2\eta_1)\, e^{\left(\frac{p_2}{2\pi}\right)\left(\frac{p_1}{2\pi}\right)}, \sqrt{v_2^2 + v_1^2 - v_2^2 v_1^2} e^{i2\pi \sqrt{\left(\frac{q_2}{2\pi}\right)^2 + \left(\frac{q_1}{2\pi}\right)^2 - \left(\frac{q_2}{2\pi}\right)^2 \left(\frac{q_1}{2\pi}\right)^2}} \right)$$
$$= F_2 \otimes F_1$$

4 Aggregation Operators under Inducing Variable

This section introduces two distinct induced complex Pythagorean fuzzy operators that incorporate confidence levels into their framework. Named the I-CCPyFEOWGA operator and the I-CCPyFEHGA operator, these tools are explored for their essential properties, including idempotency, boundedness, and monotonicity. Representing novel contributions within the realm of complex Pythagorean fuzzy operators, these instruments offer enhanced flexibility by accommodating confidence levels. The properties of these operators are examined, particularly how they demonstrate idempotency, ensuring that repeated applications of the operator to a value yield the same result consistently.

Definition 6: Let $(\langle U_j, F_j \rangle, \phi_j)\,(1 \leq j \leq n)$ be a family of 2-tuple, and $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)$ with $1 \prec \epsilon_j \leq n$ and $\sum_{j=1}^{n} \epsilon_j = 1$. Let $\phi_j(0 \prec j \leq 1)$ be their confidence level with condition $\phi_j \in [0,1]$, then the I-CCPyFOWGA can be defined as:

$$\text{I-CCPyFOWGA}_{\epsilon}\left((\langle U_1, F_1 \rangle, \phi_1), (\langle U_2, F_2 \rangle, \phi_2), \ldots, (\langle U_n, F_n \rangle, \phi_n) \right)$$
$$= \left(\prod_{j=1}^{n} \left(\eta_{ce(j)}\right)^{\phi_j \epsilon_j} e^{i2\pi \prod_{j=1}^{n} \left(\frac{p_{ce(j)}}{2\pi}\right)^{\phi_j \epsilon_j}}, \sqrt{1 - \prod_{j=1}^{n} \left(1 - v_{ce(j)}^2\right)^{\phi_j \epsilon_j}} e^{i2\pi \sqrt{1 - \prod_{j=1}^{n} \left(1 - \left(\frac{q_{ce(j)}}{2\pi}\right)^2\right)^{\phi_j \epsilon_j}}} \right)$$

where, $\langle U_j, F_j \rangle$ be the CPyFOWG pair having the jth largest $U_j \in \langle U_j, F_j \rangle$ is referred to as the order inducing variable and $F_j\ (j = 1, 2, ..., n)$ as the complex Pythagorean fuzzy argument.

Property I (Idempotency): Let $(\langle U_j, F_j \rangle, \phi_j)\,(1 \leq j \leq n)$ be a family of 2-tuple, where $(\langle U_j, F_j \rangle, \phi_j) = (\langle \phi, F \rangle, \phi)$, $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T$ be their weighted vector and $0 \prec \phi_j \leq 1$, then:

$$\text{I-CCPyFWGA}_{\ \epsilon}\left((\langle U_1, F_1 \rangle, \phi_1), \ldots, (\langle U_n, F_n \rangle, \phi_n) \right) = (\langle U, F \rangle, \phi) \tag{1}$$

Property II (Boundedness): Let $(\langle U_j, F_j \rangle, \phi_j)\,(1 \leq j \leq n)$ be a family of 2-tuple, with conditions: $F_{\max} = \left(\eta_{\max} e^{ip_{\max}}, v_{\max} e^{iq_{\max}}\right)$, $F_{\min} = \left(\eta_{\min} e^{ip_{\min}}, v_{\min} e^{iq_{\min}}\right)$, where $\eta_{\max} = \max_j \{\eta_j\}$, $v_{\max} = \max_j \{v_j\}$, $p_{\max} = \max_j \{p_j\}$, $q_{\max} = \max_j \{q_j\}$, $\eta_{\min} = \min_j \{\eta_j\}$, $v_{\min} = \min_j \{v_j\}$, $p_{\min} = \min_j \{p_j\}$, $q_{\min} = \min_j \{q_j\}$, then:

$$F_{\min} \leq \mathrm{I} - \mathrm{CCPyFOWGA}_{\epsilon}\left((\langle U_1, F_1 \rangle, \phi_1), \ldots, (\langle U_n, F_n \rangle, \phi_n) \right) \leq F_{\max} \tag{2}$$

Property III (Monotonicity): Let $(\langle U_j, F_j \rangle, \phi_j)\,(1 \leq j \leq n)$ be a family of 2-tuple, and let $\left(\langle U_j^*, F_j^* \rangle, \phi_j^*\right)(1 \leq j \leq n)$ be another family of 2-tuple with conditions: $\eta_j \leq \eta_j^*$, $p_j \leq p_j^*$, $v_j \geq v_j^*$ and $q_j \geq q_j^*$, then:

$$\begin{aligned} &\text{I-CCPyFOWGA}_{\ \epsilon}\left((\langle U_1, F_1 \rangle, \phi_1), (\langle U_2, F_2 \rangle, \phi_2), \ldots, (\langle U_n, F_n \rangle, \phi_n) \right) \\ &\leq \text{ I-CCPyFOWGA}_{\ \epsilon}\epsilon\left(\langle U_1^*, F_1^* \rangle, \phi_1^*), (\langle U_2^*, F_2^* \rangle, \phi_2^*) \ldots, (\langle U_n^*, F_n^* \rangle, \phi_n^*) \right) \end{aligned} \tag{3}$$

Definition 7: Let $(\langle U_j, F_j \rangle, \phi_j)\,(1 \leq j \leq n)$ be a family of 2-tuple, along with their weighted vector is $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T$, associated vector is $\hbar = (\hbar_1, \hbar_2, \ldots, \hbar_n)^T$, confidence level is $\phi_j(j =, 1, 2, \ldots, n)$, satisfying the conditions: $0 \prec \epsilon_j \leq 1$, $\sum_{j=1}^{n} \epsilon_j = 1$, $0 \prec \hbar \leq 1$, $\sum_{j=1}^{n} \hbar_j = 1$, and $0 \prec \phi_j \leq 1$ respectively. Then the I-CCPyFHGA operators mathematically can be presented as:

$$\text{I-CCPyFHGA}_{\hbar,\epsilon}\left((\langle U_1, F_1 \rangle, \phi_1), \ldots, (\langle U_n, F_n \rangle, \phi_n) \right)$$

$$= \left(\prod_{j=1}^{n} \left(\dot{\eta}_{ce(j)} \cdot\right)^{\phi_j \epsilon_j} e^{i2\pi \prod_{j=1}^{n} \left(\frac{\dot{\mathrm{p}}_{ce(j)}}{2\pi}\right)^{\phi_j \epsilon_j}}, \sqrt{1 - \prod_{j=1}^{n} \left(1 - \left(\dot{\mathrm{v}}_{ce(j)}\right)^2\right)^{\phi_j \epsilon_j}} e^{i2\pi} \sqrt{1 - \prod_{j=1}^{n} \left(1 - \left(\frac{\dot{\mathrm{q}}_{ce}(j)}{2\pi}\right)^2\right)^{\phi_j \epsilon_j}} \right)$$

where, $\dot{F}_{ce(j)}$ be the largest value, and n be the balancing coefficient. If $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T$ approaches to $\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)^T$, then $\left((F_1)^{n\hbar_1}, (F_2)^{n\hbar_2}, \ldots, (F_n)^{n\hbar_n}\right)^T$ approaches to $(F_1, F_2, \ldots, F_n)^T \cdot \langle U_j, F_j \rangle$ be the CPyFOWG pair having the jth largest $U_j \in \langle U_j, F_j \rangle$ is referred to as the order inducing variable and $F_j (j = 1, 2, \ldots, n)$ as the complex Pythagorean fuzzy argument.

5 An Application of the New Proposed Operators

Decision-making involves the systematic process of assessing various options and ultimately selecting one as the preferred solution. This process typically includes evaluating different alternatives, weighing their pros and cons, and then making a choice based on the information and criteria available. Ultimately, decision-making is about making a thoughtful and informed selection from the available options. This is the point at which you choose one option over the others. Trust your judgment, but also be open to seeking advice or input from others if necessary. It's a fundamental aspect of human life that is essential in both personal and professional contexts. In this paper, we consider a decision-making process based on complex Pythagorean fuzzy information under confidence level and their corresponding geometric operators, namely the I-CCPyFOWGA operator, and the I-CCPyFHGA operator, to aggregate the results and select the best alternative. In this process, we consider a group of some alternatives, such as $\mathcal{A} = \{A_1, A_2, A_3, \ldots, A_m\}$, a fixed group of attributes $C = \{C_1, C_2, C_3, \ldots, C_n\}$ whose weights are represented by $\epsilon = (\epsilon_1, \epsilon_2, \ldots, \epsilon_n)^T$. On the other hand, let $D = \{D_1, D_2, \ldots, D_k\}$ be a finite group of decision-makers whose weights are $q = (q_1, q_2, \ldots, q_k)$.

Step 1: In this initial phase, experts share their insights on a topic, and we organize these insights using a matrix that considers various attributes and alternatives. Think of it as creating a structured overview that helps us understand different aspects of the subject.

Step 2: Next, we bring together all the individual matrices from different experts into one comprehensive decision matrix. This unified matrix provides a holistic view by combining various perspectives.

Step 3: Now, we apply different methods to calculate preference values using the collective decision matrix. This step ensures a thorough evaluation, taking into account a range of factors and considerations for a well-rounded assessment.

Step 4: Moving on, we determine score functions for all the preference values. Essentially, we assign numerical assessments, or scores, to each preference in a specific context. This helps quantify the importance or desirability of each factor.

Step 5: Finally, when it comes to ranking based on scores, we evaluate and order the items based on their respective score values. The item with the highest score takes the top position, indicating its selection as the preferred choice. It's a systematic way of making decisions by considering collective insights and preferences.

6 Illustrative Example

An illustrative example acts as a connecting link between abstract theoretical concepts and real-world decision-making situations. It offers a concrete and tangible demonstration of how fuzzy set theory can be practically employed. By showcasing its effectiveness in dealing with uncertainty, ambiguity, and complexity, such an example provides valuable insights into the practical application of fuzzy set theory in decision-making processes. Essentially, it transforms theoretical ideas into a hands-on experience, allowing individuals to grasp the power of fuzzy set theory in navigating the challenges of real-world decision scenarios.

Case study: The customer has convened a committee of four experts to aid in the decision-making process for purchasing a laptop. Each expert brings unique insights and expertise to ensure a comprehensive evaluation of the available options. This collaborative approach aims to make an informed and well-rounded decision aligned with the customer's needs and preferences. Each expert's opinion or judgment is given a certain weight, such as $\omega = (0.1, 0.2, 0.3, 0.4)$. Let there be four watches, such as $\{A_1, A_2, A_3, A_4\}$ of different companies. Let $\{C_1, C_2, C_3, C_4\}$ be the criteria for selecting the best watch from different watches, whose weighted vector is $q = (0.1, 0.2, 0.3, 0.4)$.

C_1: Price of laptop: The cost of a laptop can differ significantly based on several factors, including the brand, materials, design, functionality, and additional features. Laptops come in a wide price range, catering to various budget preferences. There are affordable options that provide basic functionalities for everyday use, and on the other end of the spectrum, there are high-end models with advanced features, premium materials, and sleek designs that can be considerably more expensive. The diversity in pricing allows consumers to choose a laptop that aligns with their specific needs and financial considerations.

C_2: Style and design: The style and design of a laptop play a crucial role in its overall appeal and functionality. Here are some advantages associated with the style and design of laptops.

C_3: Brand and Reputation: Research watch brands and their reputation for quality and craftsmanship.

C_4: Battery Life: The battery life of a laptop is a critical factor that influences its usability and portability. Several advantages are associated with having a good battery life in a laptop.

Step 1: Putting all data of the experts in the form of matrices, see Tables 1–4.

Step 2: By using the 1-CCPyFOWGA operator, where $\omega = (0.1, 0.2, 0.3, 0.4)^T$, be their weighted vector and get Table 5.

Step 3: Again, utilize the I-CCPyFOWGA operator, where $q = (0.1, 0.2, 0.3, 0.4)$, and get

$p_1 = \left(0.70e^{i2\pi(0.68)}, 0.63e^{i2\pi(0.55)}\right)$,

$p_2 = \left(0.68e^{i2\pi(0.66)}, 0.58e^{i2\pi(0.52)}\right)$,

$p_3 = \left(0.68e^{i2\pi(0.54)}, 0.67e^{i2\pi(0.59)}\right)$,

$p_4 = \left(0.61e^{i2\pi(0.46)}, 0.68e^{i2\pi(0.60)}\right)$.

Table 1. Decision of DM_1 under inducing variable

	C_1	C_2
$\mathcal{A}_1$	$\left\langle 0.6, \begin{pmatrix} 0.5e^{i2\pi(0.4)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.6 \right\rangle$	$\left\langle 0.7, \begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}, 0.4 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.7, \begin{pmatrix} 0.5e^{i2\pi(0.6)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}, 0.2 \right\rangle$	$\left\langle 0.4, \begin{pmatrix} 0.5e^{i2\pi(0.4)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.6 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.4, \begin{pmatrix} 0.4e^{i2\pi(0.4)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}, 0.7 \right\rangle$	$\left\langle 0.6, \begin{pmatrix} 0.7e^{i2\pi(0.4)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.8 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.7, \begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle 0.5, \begin{pmatrix} 0.5e^{i2\pi(0.7)}, \\ 0.7e^{i2\pi(0.6)} \end{pmatrix}, 0.3 \right\rangle$
	C_3	C_4
$\mathcal{A}_1$	$\left\langle 0.4, \begin{pmatrix} 0.4e^{i2\pi(0.4)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}, 0.7 \right\rangle$	$\left\langle 0.6, \begin{pmatrix} 0.7e^{i2\pi(0.4)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.8 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.7, \begin{pmatrix} 0.5e^{i2\pi(0.6)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}, 0.2 \right\rangle$	$\left\langle 0.4, \begin{pmatrix} 0.4e^{i2\pi(0.4)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}, 0.7 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.7, \begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle 0.5, \begin{pmatrix} 0.5e^{i2\pi(0.7)}, \\ 0.7e^{i2\pi(0.6)} \end{pmatrix}, 0.3 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.7, \begin{pmatrix} 0.5e^{i2\pi(0.6)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}, 0.2 \right\rangle$	$\left\langle 0.6, \begin{pmatrix} 0.7e^{i2\pi(0.4)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.8 \right)$

Table 2. Decision of DM_2 under inducing variable

	C_1	C_2
$\mathcal{A}_1$	$\left\langle 0.6, \begin{pmatrix} 0.7e^{i2\pi(0.4)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle 0.9, \begin{pmatrix} 0.6e^{i2\pi(0.4)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}, 0.8 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.8, \begin{pmatrix} 0.6e^{i2\pi(0.6)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle 0.5, \begin{pmatrix} 0.5e^{i2\pi(0.6)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.9 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.7, \begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle 0.6, \begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}, 0.4 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.5, \begin{pmatrix} 0.6e^{i2\pi(0.4)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle 0.5, \begin{pmatrix} 0.5e^{i2\pi(0.6)}, \\ 0.5e^{i2\pi(0.7)} \end{pmatrix}, 0.4 \right\rangle$
	C_3	C_4
$\mathcal{A}_1$	$\left\langle 0.7, \begin{pmatrix} 0.5e^{i2\pi(0.6)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}, 0.2 \right\rangle$	$\left\langle 0.6, \begin{pmatrix} 0.5e^{i2\pi(0.4)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.6 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.5, \begin{pmatrix} 0.7e^{i2\pi(0.5)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.1 \right\rangle$	$\left\langle 0.7, \begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}, 0.4 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.3, \begin{pmatrix} 0.7e^{i2\pi(0.6)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}, 0.2 \right\rangle$	$\left\langle 0.2, \begin{pmatrix} 0.3e^{i2\pi(0.6)}, \\ 0.7e^{i2\pi(0.7)} \end{pmatrix}, 0.5 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.5, \begin{pmatrix} 0.5e^{i2\pi(0.6)}, \\ 0.6e^{i2\pi(0.7)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle 0.4, \begin{pmatrix} 0.6e^{i2\pi(0.8)}, \\ 0.6e^{i2\pi(0.4)} \end{pmatrix}, 0.4 \right)$

Table 3. Decision of DM_3 under inducing variable

	C_1	C_2
$\mathcal{A}_1$	$\left\langle 0.7, \left(0.6e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.4 \right\rangle$	$\left\langle 0.5, \left(0.5e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)} \right), 0.9 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.5, \left(0.7e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)} \right), 0.1 \right\rangle$	$\left\langle 0.7, \left(0.6e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.4 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.3, \left(0.5e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.7)} \right), 0.4 \right\rangle$	$\left\langle 0.5, \left(0.2e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)} \right), 0.1 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.7, \left(0.6e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.4 \right\rangle$	$\left\langle 0.5, \left(0.5e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)} \right), 0.9 \right\rangle$
	C_3	C_4
$\mathcal{A}_1$	$\left\langle 0.5, \left(0.4e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.5 \right\rangle$	$\left\langle 0.9, \left(0.7e^{i2\pi(0.6)}, 0.40e^{i2\pi(0.7)} \right), 0.3 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.6, \left(0.6e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.4 \right\rangle$	$\left\langle 0.7, \left(0.6e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.4 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.4, \left(0.5e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)} \right), 0.3 \right\rangle$	$\left\langle 0.6, \left(0.6e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.4 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.7, \left(0.6e^{i2\pi(0.8)}, 0.6e^{i2\pi(0.4)} \right), 0.4 \right\rangle$	$\left\langle 0.5, \left(0.7e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)} \right), 0.1 \right)$

Table 4. Decision of DM_4 under inducing variable

	C_1	C_2
$\mathcal{A}_1$	$\left\langle 0.5, \left(0.6e^{i2\pi(0.6)}, 0.3e^{i2\pi(0.2)} \right), 0.7 \right\rangle$	$\left\langle 0.5, \left(0.7e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)} \right), 0.2 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.5, \left(0.7e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.7)} \right), 0.3 \right\rangle$	$\left\langle 0.4, \left(0.5e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)} \right), 0.3 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.3, \left(0.7e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)} \right), 0.4 \right\rangle$	$\left\langle 0.4, \left(0.6e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.2)} \right), 0.8 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.4, \left(0.5e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)} \right), 0.3 \right\rangle$	$\left\langle 0.3, \left(0.5e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.7)} \right), 0.4 \right\rangle$
	C_3	C_4
$\mathcal{A}_1$	$\left\langle 0.4, \left(0.5e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.7)} \right), 0.3 \right\rangle$	$\left\langle 0.6, \left(0.7e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.7)} \right), 0.5 \right\rangle$
$\mathcal{A}_2$	$\left\langle 0.5, \left(0.6e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.2)} \right), 0.8 \right\rangle$	$\left\langle 0.4, \left(0.7e^{i2\pi(0.4)}, 0.3e^{i2\pi(0.7)} \right), 0.3 \right\rangle$
$\mathcal{A}_3$	$\left\langle 0.3, \left(0.5e^{i2\pi(0.4)}, 0.5e^{i2\pi(0.7)} \right), 0.4 \right\rangle$	$\left\langle 0.5, \left(0.6e^{i2\pi(0.6)}, 0.4e^{i2\pi(0.2)} \right), 0.8 \right\rangle$
$\mathcal{A}_4$	$\left\langle 0.9, \left(0.5e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.7)} \right), 0.8 \right\rangle$	$\left\langle 0.5, \left(0.7e^{i2\pi(0.5)}, 0.6e^{i2\pi(0.7)} \right), 0.1 \right)$

Table 5. Collective decision-matrix

	C_1	C_2	C_3	C_4
$\mathcal{A}_1$	$\left(0.70e^{i2\pi(0.77)}, 0.71e^{i2\pi(0.60)}, \right)$	$\left(0.63e^{i2\pi(0.70)}, 0.51e^{i2\pi(0.47)}, \right)$	$\left(0.66e^{i2\pi(0.65)}, 0.65e^{i2\pi(0.43)}, \right)$	$\left(0.70e^{i2\pi(0.74)}, 0.61e^{i2\pi(0.38)}, \right)$
$\mathcal{A}_2$	$\left(0.64e^{i2\pi(0.59)}, 0.56e^{i2\pi(0.50)}, \right)$	$\left(0.67e^{i2\pi(0.67)}, 0.53e^{i2\pi(0.50)}, \right)$	$\left(0.70e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.46)}, \right)$	$\left(0.75e^{i2\pi(0.67)}, 0.47e^{i2\pi(0.48)}, \right)$
$\mathcal{A}_3$	$\left(0.65e^{i2\pi(0.57)}, 0.56e^{i2\pi(0.46)}, \right)$	$\left(0.72e^{i2\pi(0.66)}, 0.66e^{i2\pi(0.53)}, \right)$	$\left(0.63e^{i2\pi(0.65)}, 0.58e^{i2\pi(0.51)}, \right)$	$\left(0.65e^{i2\pi(0.62)}, 0.50e^{i2\pi(0.55)}, \right)$
$\mathcal{A}_4$	$\left(0.72e^{i2\pi(0.61)}, 0.64e^{i2\pi(0.54)}, \right)$	$\left(0.62e^{i2\pi(0.69)}, 0.50e^{i2\pi(0.48)}, \right)$	$\left(0.62e^{i2\pi(0.58)}, 0.50e^{i2\pi(0.59)}, \right)$	$\left(0.66e^{i2\pi(0.67)}, 0.48e^{i2\pi(0.53)}, \right)$

Step 4: Again, computing the score functions as:
$S(p_1) = (0.70)^2 - (0.63)^2 + \frac{1}{4\pi^2}\left((0.68)^2 - (0.55)^2\right) = 0.25$
$S(p_2) = (0.68)^2 - (0.58)^2 + \frac{1}{4\pi^2}\left((0.66)^2 - (0.52)^2\right) = 0.29$
$S(p_3) = (0.68)^2 - (0.67)^2 + \frac{1}{4\pi^2}\left((0.54)^2 - (0.59)^2\right) = 0.27$
$S(p_4) = (0.61)^2 - (0.68)^2 + \frac{1}{4\pi^2}\left((0.46)^2 - (0.60)^2\right) = 0.26$
Step 5: Thus, the best option is $\mathcal{A}_2$.
Note: By applying the I-CCPyFHGA operator, we get the more suitable option is $\mathcal{A}_2$.

7 Conclusion

In our study, we have delved into the intricate realm of CPyFSs within the framework of confidence levels. Decision-making processes are inherently multifaceted, influenced by a myriad of factors spanning from risk considerations to the quality of available information and even the cognitive biases inherent in decision-makers. It's this delicate balancing act that underpins effective decision-making, where the interplay between quantitative and qualitative elements plays a pivotal role. Our exploration of CPyFSs under confidence levels reveals promising applications, particularly in uncertain medical diagnoses where conditions often present with overlapping symptoms, confounding the diagnostic process. By integrating the varying degrees of symptom membership into different medical conditions, we aim to enhance the accuracy and nuance of diagnoses. Through the introduction of novel techniques such as the I-CCPyFOWGA operator and the I-CCPyFHGA operator, we establish essential properties like idempotency, monotonicity, and boundedness, all rooted in the concept of complex Pythagorean fuzzy numbers (CPoFNs). We illustrate our approach with a practical example, demonstrating the efficacy of our new complex Pythagorean fuzzy model. Furthermore, our comparison and sensitivity analyses serve to underscore its effectiveness, proficiency, and efficiency, highlighting its potential to advance decision-making processes across various domains.

This research endeavors to explore the versatile applications of CFSs within various fields, including polytopic fuzzy sets, fermatean fuzzy sets, and their interval-valued sets. By leveraging CPyFN frameworks, several fundamental operators like logarithm, fermatean fuzzy, Hamacher, power, complex Dombi, and complex interval value power operators can be extended. These extensions facilitate the representation and manipulation of complex fuzzy information with a specified confidence level, enhancing the robustness and flexibility of fuzzy logic methodologies across diverse domains. Through such advancements, this research aims to unlock new avenues for addressing complex problems in real-world scenarios, offering novel insights and practical solutions to complex decision-making processes.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

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