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Collapsing and Expanding Observation of Anisotropic Charged Source in $f(R,T)$ Gravity

Jan Muhammad[1*], Rifaqat Ali[2]

[1] Department of Mathematics, Shanghai University, 200444 Shanghai, China
[2] Department of Mathematics, College of Science and Arts, Muhayil, King Khalid University, 61413 Abha, Saudi Arabia

[*] Correspondence: Jan Muhammad (janmath@shu.edu.cn)

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Abstract: In the present work we investigate the collapsing and expanding solutions of the Einstein's field equation of anisotropic fluid in spherically symmetric space-time and with charge within the framework of *f(R, T)* theory, where R denotes the Ricci scalar and T denotes the trace of the energy−momentum tensor. We also evaluate the expansion scalar, whose negative values result in collapse and positive values yield expansion. We analyzed the impacts of charge in *f(R, T)* theory on the density and pressure distribution of the collapsing and expanding fluid and noticed the involvement of anisotropic fluid in the process of collapsing and expanding with charge in *f(R, T)*. Furthermore, the definition of mass function has been used to analyse the condition for the trapped surface, and it has been found that in this case there is only one horizon. In all scenarios, the effects of coupling parameters λ and q have been thoroughly examined. Additionally, we have created graphs representing pressures, anisotropy, and energy density in *f(R, T)* theory and check the effect of charge on these quantities.

Keywords: Gravitational collapse; Expending; Trapped regions; Anisotropic source; Charge

1 Introduction

The core ideas of the general theory of relativity are the Einstein field equations (EFES), which offer a wealth of knowledge about the close relationship between spacetime and matter. Gravitational physics can be explained in its entirety by this hypothesis. The universe's expansion is currently the most essential problem in gravitational physics. A number of modified theories of gravity explained the swift expansion of the universe. A modified gravity model based on the $\frac{1}{R}$ scalar where R is the Ricci scalar in the standard Einstein action was proposed by Carroll et al. [1]. They found that the acceleration of the expansion of the universe is caused by the amount of deviations at tiny curvature and cosmic scale. Dark energy's repellent gravity is the primary source of acceleration. General relativity must give way to a modified theory of gravity in order to handle the problem of universe expansion.

The *f(R)* theory is one of the most prominent modified theories of gravity, where Langragian is a universal function of Ricci scalar. In 1970, Buchdahl [2] was the first to put this idea forth. The general theory of relativity is restored by adjusting the value of the Ricci scalar function R to scalar. In such a developing universe, the transition from deceleration to acceleration is pretty naturally described by the *f(R)* theory of gravity. This concept has several applications in high-energy physics. Because it is so simple to adjust, this hypothesis is quite well-liked. Recently, there has been a lot of interest from authors to explore several facets of the *f(R)* theory of gravity. Bento et al. [3] combined langrangian matter with a Ricci scalar arbitrary function to study a unique connection. Spherically symmetric spacetimes were employed by Hollenstein and Lobo [4] to study the solutions in *f(R)* theory.

The most fundamental and important concept in general relativity is gravitational collapse, which is acknowledged as the universe's centre of structure generation. Something happens when internal pressure is not sufficient to counteract the gravitational attraction. In 1939, Oppenheimer and Synder [5] conducted the first investigation of gravity collapse using junction circumstances. Misner and Sharp [6] used junction situations to study gravitational collapse under pressure. da Rocha et al. [7] examined a self-similar collapse using matching conditions and ideal fluid. Pereira and Wang [8] used matching conditions to study the collapse of cylindrical shells. Markovic and

Shapiro [9] studied gravitational collapse with a positive cosmic constant. Sharif and Ahmed [10] studied planar symmetric gravitational collapse using junction circumstances.

The anisotropic fluid model was developed by Bowers and Liang [11], which made it possible for scientists to study how anisotropy affects a star's physical behaviour during gravitational collapse. The perturbation approach was employed by Herrera and Santos [12] to study the properties of anisotropic self-gravitating spheres. In several research investigations, a number of authors examined cylindrical collapse using an isotropic fluid. Using the junction condition, Herrera and Santos [13, 14] addressed the cylindrical collapse with fluid bearing isotropic pressure. This was developed to charged cylindrical collapse by Sharif and Fatima [15]. Ahmed et al. [16] investigated spherical collapse using a fast-moving, anisotropic fluid.The plane symmetric model was investigated by Glass [17], and later on, charge anisotropic sources were included [18]. A number of articles [19–21] proposed the use of anisotropic solutions in the study of stellar collapse.

The problem of gravitational collapse is becoming more and more popular in modified theories of gravity. Cognola et al. [22] analysed the *f(R)* theory by looking at the de-sitter world. Farasat et al. [23] investigated gravitational collapse with dust within the framework of the *f(R)* theory of gravity. Feinstein [24] investigated black string creation in a vacuum gravitational collapse using higher dimensions. In self-similar higher dimensional spacetime, gravitational collapse was studied using perfect fluid by Ghosh and Deshkar [25].

It has been found that the universal expansion and collapse on cosmic rates cannot be explained by the General Theory of Relativity. A modified theory of gravity was proposed to be presented by researchers in an attempt to resolve the universal expansion problem. Among the most notable modified theories are Chern-Simons theory of gravity, Lovelock gravity, *f(R)* gravity, *f(G)* theory of gravity, and *f(R, T)* gravity. The most widely accepted of these theories was the *f(R, T)* theory of gravity.

In *f(R,T)* theory of gravity, Zubair et al. [26] conducted a dynamical analysis of a locally anisotropic fluid with a cylindrical symmetry. In a review of *f(R, T)* theory of gravity in a dark matter form, Zaregonbadi et al. [27]. Amir and Sattar [28] studied the spherically-symmetric gravitational collapse in the *f(R,T)* gravity. Many investigators have discovered anisotropic fluid collapse as a result of studying junction conditions both in an electromagnetic field and out of one. Dissipative cylindrical collapse of charge anisotropic fluid under Darmois junction circumstances was studied by Guha and Banerji [29]. Ahmad et al. [30] conducted research on the gravitational anisotropic fluid collapse with heat flux. Prisco et al. investigated shear free cylindrical anisotropic fluid collapse [31].

The anisotropic fluid model is a theoretical framework that is used to explain fluids behavior in cases where pressure is not uniform in all directions. It is very important to comprehend the dynamics of astronomical objects such as stars, black holes and the universe itself during gravitational collapse and expansion. For instance; $P_\perp \neq P_\parallel$ and others are the pressures components that are perpendicular and parallel to the direction of motion given by this mathematically expressed inequality [32–35]. This additional approximation helps provide a more realistic description of how fluid acts when subjected to extreme environments like those experienced under intense gravity fields.Relevance in gravitational collapse and expansion are, during collapsing processes, pressure components of a fluid become anisotropic because of increased gravity. The resulting dynamics including the formation of singularities (for example, black holes) can be described by this model.It also sheds light on the role of pressure anisotropy in determining the final fate of collapsing objects. In other words if we think about an expanding universe, then we should reveal that according to this model there are different radial and tangential pressure components [36, 37]. This anisotropy affects the evolution of the universe's density, velocity, and curvature. The model helps investigate the impact of pressure anisotropy on the large-scale structure formation and the universe's overall expansion dynamics. This means that such appearance changes in density, speed or curvature as influenced by anisotropy for universe development process.

λ and q are parameters in the *f(R, T)* theory that are important from a mathematical and physical standpoint. The coupling constant between geometry and matter is represented by λ. It calculates the magnitude of the interaction between matter and spacetime curvature [38]. A dimensionless parameter called λ can be found in the normal Einstein-Hilbert term by altering it with the gravitational Lagrangian. It modifies the behavior of the gravitational field and alters the dynamics of gravity. Whereas q stands for the electromagnetic field's associated charge. It expresses the strength of the electromagnetic interaction. The conventional Maxwell term in the gravitational Lagrangian [39] is modified by a dimensionless parameter called q. It modifies the behavior of the electromagnetic field and alters electromagnetic dynamics. λ and q are crucial in the *f(R, T)* theory because they alter the gravitational and electromagnetic dynamics, describe the interplay of matter, geometry, and electromagnetic forces, and provide an explanation for phenomena such as dark energy [40], dark matter, and electromagnetic effects in gravitational systems. The strength of these interactions and the behavior of the theory depend on the values of λ and q. When generating predictions and comparing the theory to observations, they are crucial [41]. All things considered, λ and q are significant factors that influence how the f(R,T) theory behaves, and their values have a significant impact on how we comprehend the cosmos and the rules of physics.

Under the framework of the *f(R, T)* theory of gravity, a study into a spherically charged gravitational anisotropic source will be conducted. In *f(R, T)* gravity, for models of charged, spherical, anisotropic gravitational sources that collapse and expand. The research conducted by will supplement those of Abbas and Ahmad [42]. We study field equations with charge in *f(R, T)* gravity with anisotropic source in the current paper. The structure of the article is as follows. In Section 2, we derived the field equations. Collapsing solutions and expanding solutions for the model are given in Section 3. We provide a summary of the work done on this paper in Section 4.

2 Field Equations in *f(R, T)* Gravity

The mathematical description of the $f(R,T)$ gravity action with electromagnetic field contribution is

$$S=\int(d^4x\sqrt{-g}f(R,T)+\mathcal{L}_m+\mathcal{L}_e) \tag{1}$$

The Lagrangian density of electromagnetic, the form of $\mathcal{L}_e$ is $\mathcal{L}_e=mF_{\mu\nu}F^{\mu\nu}$, where m is an arbitrary constant, the electromagnetic field tensor is represented by $F_{\mu\nu}=\Phi_{\nu,\mu}-\Phi_{\mu,\nu}$, the four potentials are represented by Φ_μ, the matter Lagrangian is $\mathcal{L}_m$, and g is the determinant of metric g_{ab}. We select $\mathcal{L}_m=\rho$ in this case. For the action mentioned above, the field equations are,

$$G_{\delta\eta}=\frac{1}{f_R}[(f_T+1)T^{(m)}_{\delta\eta}-\rho g_{\delta\eta}f_T+\frac{f-Rf_R}{2}g_{\delta\eta}+(\nabla_\delta\nabla_\eta-g_{\delta\eta}\nabla^\kappa\nabla_\kappa)f_R]+8\pi E_{\delta\eta} \tag{2}$$

where the derivatives of *f(R,T)* with respect to T and R, respectively, are shown by the symbols f_R and f_T. The *f(R,T)* model in our current study can be chosen in the manner described below:

$$f(R,\,T)=f_R+f_T,\ \text{Here we take}\ f_R=R,\ \text{and}\ f_T=2\lambda T \tag{3}$$

where the electromagnetic energy-momentum tensor $E_{\delta\eta}$ is defined by and λ is a positive constant.

$$E_{\delta\eta}=\frac{1}{4\pi}\Big(-F^{\varpi}_{\delta}F_{\eta\varpi}+\frac{1}{4}F^{\varpi\beta}F_{\varpi\beta}g_{\delta\eta}\Big) \tag{4}$$

A spherically symmetric spacetime is a spacetime that remains unchanged under rotations in three-dimensional space. In other words, it looks the same in all directions, like a sphere.In general relativity, a spherically symmetric spacetime is described by the Schwarzschild metric (for a black hole) or the Friedmann-Lematre-Robertson-Walker (FLRW) metric [43] (for an expanding universe). The key features of a spherically symmetric spacetime are. Isotropy, The spacetime looks the same in all directions [44]. Homogeneity, The spacetime has the same properties at every point. Spherical coordinates, The spacetime can be described using spherical coordinates (r,θ,Φ). Metric, The spacetime metric is diagonal and has a specific form, like the Schwarzschild or FLRW metric. Spherical symmetry is a simplifying assumption that makes it easier to solve Einstein's equations and study the behavior of spacetime [45]. However, it's important to note that real-world astrophysical objects often deviate from spherical symmetry due to various factors like rotation, magnetic fields, or complex internal structures. We assume 4-dimensional spherically symmetric spacetime in the interior geometry delineated by

$$ds^2=A^2_{(t,r)}dt^2-Q^2_{(t,r)}dr^2-S^2_{(t,r)}[d\theta^2+\sin^2\theta d\phi^2] \tag{5}$$

A fluid in which the stress (or pressure) varies with direction is called an anisotropic fluid [46–48]. This implies that the fluid's characteristics change depending on direction. An anisotropic fluid in general relativity has a more complex stress-energy tensor, It is possible to write it as:

$$T^{(m)}_{\delta\eta}=(\rho+P_\perp)\mathfrak{V}_\delta\mathfrak{V}_\eta-P_\perp g_{\delta\eta}+(P_r-P_\perp)\mathfrak{X}_\delta\mathfrak{X}_\eta \tag{6}$$

where the radical four vector, the energy density of matter, the co$-$moving four$-$velocities of the source fluid, the tangential and radical pressure, and ρ are shown, respectively. Additionally, the vectors $\mathfrak{V}_\delta$, $\mathfrak{V}^\delta$, $\mathfrak{X}_\delta$, and $\mathfrak{X}^\delta$ fulfil the following relations:

$$\mathfrak{V}^\delta=A^{-1}\delta^\delta_0,\ \mathfrak{V}^\delta\mathfrak{V}_\delta=1,\ \mathfrak{X}^\delta=A^{-1}\delta^\delta_1,\ \mathfrak{X}^\delta\mathfrak{X}_\delta=-1, \tag{7}$$

The equation for the Electromagnetic field is

$$\mathfrak{F}_{\delta\eta}=\Phi_{\eta,\delta}-\Phi_{\delta,\eta},\quad \mathfrak{F}^{\delta\eta}_{;\eta}=4\pi J^\delta \tag{8}$$

where the 4 dimensional current and potential are denoted by J_δ and Φ_δ. There is a null magnetic field as the charge is thought to be at rest in relation to the co-moving reference frame. Here, the potential and current turn into

$$\Phi_\delta=\Phi(t,r)\delta^0_\delta,\ \ J^\delta=\sigma(t,r)u^\delta \tag{9}$$

In this case, the scalar potential and charge density are represented by $\Phi(t,r)$ and $\sigma(t,r)$, respectively. The equations for the Maxwell field are Eqs. (4) and (5).

$$\dot{\Phi}' + (2\frac{\dot{S}}{S} - \frac{\dot{Q}}{Q} - \frac{\dot{A}}{A})\Phi^{'} = 0 \tag{10}$$

$$\Phi^{''} + (2\frac{S'}{S} - \frac{Q'}{Q} - \frac{A'}{A})\Phi^{'} = 4\pi\varrho Q^2 A \tag{11}$$

so

$$\Phi' = \frac{qAQ}{S^2} \quad \text{where} \quad q = 4\pi\int_0^r 4\pi\varrho Q S^2 dr \tag{12}$$

For above spacetime (Eq. (2)), the expansion scalar Θ is

$$\Theta = \frac{1}{X}\left(\frac{\dot{Q}}{Q} + 2\frac{\dot{S}}{S}\right) \tag{13}$$

Here, t is represented by the dot as the derivative. The study [49] defines the dimensionless metric of anisotropy.

$$\Delta a = \frac{P_r - P_\perp}{P_r} \tag{14}$$

The non-vanishing components of EFEs in *f(R, T)* theory of gravity for spacetime (Eq. (2)) are given by

$$\begin{aligned}
G_{00} &= \frac{A^2}{f_R}\left[\rho + \frac{f-Rf_R}{2} + \frac{f''_R}{Y^2} - \dot{f}_R\left(\frac{\dot{Y}}{Y} - 2\frac{\dot{S}}{S}\right)\frac{1}{A^2} - \frac{f'_R}{Y^2}\left(\frac{Y'}{Y} - \frac{2S'}{S}\right)\right] - \frac{q^2A^2}{S^4}\\
G_{01} &= \frac{\dot{f'}_R}{f_R}\left[1 - \frac{A'\dot{f}_R}{\dot{f'}_R A} - \frac{\dot{Y}f'_R}{Y\dot{f'}_R}\right]\\
G_{11} &= \frac{Y^2}{f_R}\left[P_r + (\rho+P_\perp)f_T - \frac{f-f_R}{2} - \frac{\dot{f}_R}{SAY^2}\left(S\dot{A} - 2A\dot{S}\right) - \frac{f'_R}{SAY_2}\left(SA' + 2AS'\right)\right] + \frac{q^2Y^2}{S^4}\\
G_{22} &= \frac{S^2}{f_R}\Bigg[P_\perp + (\rho+P_\perp)f_T + \frac{Rf_R - f}{2} + \frac{\ddot{f}_R}{A^2} - \frac{f''_R}{Y^2} - \frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A} - \frac{\dot{Y}}{Y} - \frac{\dot{S}}{S}\right)\\
&\quad - \frac{f''_R}{Y^2}\left(\frac{A'}{A} - \frac{Y'}{Y} - \frac{S'}{S}\right)\Bigg] - \frac{q^2S^2}{S^4}
\end{aligned} \tag{15}$$

these calculations suggest that

$$\begin{aligned}
(1+\lambda)\rho - \lambda P_r - 2\lambda P_\perp + \frac{q^2}{S^4} &= \frac{1}{A^2}\left(\frac{\dot{S}^2}{S^2} + 2\frac{\dot{Y}\dot{S}}{YS}\right) - \frac{1}{Y^2}\left(\frac{2S''}{S} + \frac{S'^2}{S^2} - \frac{2Y'S'}{YS}\right) + \frac{1}{S^2}\\
\lambda\rho + (1+3\lambda)P_r + 2\lambda P_\perp - \frac{q^2}{S^4} &= \frac{1}{Y^2}\left(\frac{S'^2}{S^2} + \frac{2A'S'}{AS}\right) - \frac{1}{A^2}\left(\frac{2\ddot{S}}{S} - \frac{2\dot{A}\dot{S}}{AS} + \frac{\dot{S}^2}{S^2}\right) - \frac{1}{S^2}\\
\lambda\rho + \lambda P_r + (1+4\lambda)P\perp + \frac{q^2}{S^4} &= \frac{1}{A^2}\left(-\frac{\ddot{Y}}{Y} - \frac{\ddot{S}}{S} + \frac{\dot{A}\dot{Y}}{AY} - \frac{\dot{Y}\dot{S}}{SY} + \frac{\dot{A}\dot{S}}{AS}\right)\\
&\quad + \frac{1}{Y^2}\left(\frac{A''}{A} + \frac{S''}{S} - \frac{A'Y'}{AY} + \frac{A'S'}{AS} - \frac{Y'S'}{YS}\right)\\
G_{01} &= \frac{\dot{S}'}{S} - \frac{A'\dot{S}}{AS} - \frac{\dot{Y}S'}{SY} = 0
\end{aligned} \tag{16}$$

where the intensity of the electric field is $E = \frac{q}{S^2}$.

In general relativity, the mass of a spacetime region is described by a scalar quantity called the Misner-Sharp mass. It has the following definition as $m = \frac{R}{2G}(1 - (1 - (\frac{4GM}{Rc^2}))$. The variables in this equation are c, the speed of light, G, the gravitational constant, m, the Misner-Sharp mass, and M, the total mass-energy of the spacetime region. Trapped surfaces are places in spacetime where the gravitational attraction is so great that not even light can escape. The Misner-Sharp mass is a key component in describing these locations. The trapped surface border, black hole mass, and trapped surface creation are all determined using the Misner-Sharp mass [50–52]. A trapped surface forms, signifying the existence of a black hole, if the Misner-Sharp mass beyond a threshold. The mass of the black hole, which controls its gravitational attraction and other characteristics, is measured using the Misner-Sharp mass. The trapped surface boundary, which divides the area from which nothing can escape into the rest of spacetime, is defined in part by the Misner-Sharp mass. For 4-dimensions, the Misner-Sharp mass is defined as [7].

$$m(t,r)=\frac{S}{2}\left(1+\frac{\dot{S}^2}{A^2}+\frac{S'^2}{Y^2}+\frac{q^2}{S^2}\right) \tag{17}$$

An auxiliary solution exists for the G_{01} component of the field equations that we have discovered.

$$X=\frac{\dot{S}}{S^{\varpi}},\quad Q=S^{\varpi} \tag{18}$$

where the arbitrary constant ϖ is used. Now, when Eq. (18) is inserted into Eq. (13), the scalar expansion becomes

$$\Theta=(2+\varpi)S^{(\varpi-1)} \tag{19}$$

When Eq. (18) is substituted into Eq. (17), the mass function is

$$\frac{2m(t,r)}{S}-1-\frac{q^2}{S^2}=S^{2\varpi}-\frac{S'^2}{S^{2\varpi}} \tag{20}$$

There is a trapped surface at $S=2m$ when $S'=S^{2\varpi}$, hence $S'=S^{2\varpi}$ is a trapped condition. Below [25], we obtain two scalars for trapping, denoted by

$$\mathfrak{k}_1=\frac{S^{\varpi}}{S}+\frac{S'}{S^{\varpi+1}},\qquad \mathfrak{k}_2=\frac{S^{\varpi}}{S}-\frac{S'}{S^{\varpi+1}} \tag{21}$$

If the signs of $\mathfrak{k}_1$ and $\mathfrak{k}_2$ are the same, a trapped surface will move out. If the trapped surface condition is employed, the trapping scalars have the following form.

$$\mathfrak{k}_1=2S^{\varpi-1},\qquad \mathfrak{k}_2=0 \tag{22}$$

Since both $\mathfrak{k}_1$ and $\mathfrak{k}_2$ are non-negative scalars, a trapped surface arises at $S=2m$ during the gravitational collapse. The confined surface's state has the integral

$$S_{trap}^{1-2\varpi}=r(1-2\varpi)+\mathcal{H}(\sqcup) \tag{23}$$

where the integration function is $\mathcal{H}(\sqcup)$. Eqs. (18) and (23) from Eq. (17) yield the source variables in the following explicit form.

$$\begin{aligned}
\rho=&-\frac{1}{8\lambda^2+6\lambda+1}\Big[(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{2}{2\varpi-1}}((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{-2\varpi}\times((6\lambda+1)q^2(\mathcal{H}(\sqcup)-\\
&2\varpi r+r)^{\frac{2}{2\varpi-1}}((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{2\varpi}-4\varpi^2\lambda((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}-\\
&((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi})+2\varpi(3\lambda+1)((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi})\\
&+(4\lambda+1)(-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{2\varpi}-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi}+(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}))\Big]\\
p_r=&\frac{1}{8\lambda^2+6\lambda+1}\Big[(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{2}{2\varpi-1}}((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{-2\varpi}\\
&\times((6\lambda+1)q^2(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{2}{2\varpi-1}}((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{2\varpi}\\
&\times 4\varpi^2\lambda((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi})\\
&+2\varpi(3\lambda+1)((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi})\\
&+(4\lambda+1)(-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{2\varpi}\\
&-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi}+(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}))\Big]\\
p_t=&-\frac{1}{4\lambda+1}\Big[(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{2}{2\varpi-1}}((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{-2\varpi}(q^2\times(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{2}{2\varpi-1}}\\
&((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{2\varpi}-2\varpi^2((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}-((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi})\\
&+\varpi(((\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{1}{1-2\varpi}})^{4\varpi}-(\mathcal{H}(\sqcup)-2\varpi r+r)^{\frac{4\varpi}{1-2\varpi}}))\Big]
\end{aligned} \tag{24}$$

3 Generating Solutions

We describe the nature of the solutions $\varpi = -\frac{5}{2}$ in the collapsing solution and $\varpi = \frac{3}{2}$: in the expanding solution, for various values of ϖ.

3.1 Collapse solution $\varpi = -\frac{5}{2}$

Since the collapsing system requires a negative rate of expansion, ϖ must be less than -2. As a result, $\varpi = -\frac{5}{2}$. The trapping requirement $S' = S^{2\varpi}$ becomes $S' = S^{-5}$ for $\varpi = -\frac{5}{2}$. This further yields

$$S_{trap} = [6r + \mathcal{H}(\sqcup)_\infty]^{\frac{1}{6}} \tag{25}$$

$\mathcal{H}(\sqcup)_\infty$ is an arbitrary function of integration in this case. $\varpi = -\frac{5}{2}$ density and pressure equations become

$$\begin{aligned}
\rho &= \frac{(4\lambda+1)\sqrt[3]{\mathcal{H}(\sqcup)_\infty + 6r} - (6\lambda+1)q^2}{(8\lambda^2+6\lambda+1)\,(\mathcal{H}(\sqcup)_\infty + 6r)^{2/3}} \\
P_r &= \frac{(6\lambda+1)q^2 - (4\lambda+1)\sqrt[3]{\mathcal{H}(\sqcup)_\infty + 6r}}{(8\lambda^2+6\lambda+1)\,(\mathcal{H}(\sqcup)_\infty + 6r)^{2/3}} \\
P_t &= -\frac{q^2}{(4\lambda+1)(h+6r)^{2/3}}
\end{aligned} \tag{26}$$

For $\varpi = -\frac{5}{2}$, the anisotropy and mass function specified in Eqs. (14) and (17) become

$$\begin{aligned}
\Delta a &= \frac{\left(8\lambda^2+6\lambda+1\right)q^2}{(4\lambda+1)\left((6\lambda+1)q^2 - (4\lambda+1)\sqrt[3]{\mathcal{H}(\sqcup)_\infty + 6r}\right)} + 1 \\
m(t,r) &= \frac{q^2}{2\sqrt[6]{\mathcal{H}(\sqcup)_\infty + 6r}} + \frac{1}{2}\sqrt[6]{\mathcal{H}(\sqcup)_\infty + 6r}
\end{aligned} \tag{27}$$

Understanding the observable behaviors of density, pressure, and anisotropy in relation to gravitational expansion is crucial for understanding how the universe has evolved [53, 54]. Concerning density, the expanding universe's falling matter and energy density leads to a diluted cosmos. Density fluctuations spawn structures, and their evolution shapes the large-scale structure of the universe. As the cosmos expands, pressure, a gauge of a fluid's resistance to expansion, drops, enabling the universe to keep expanding.Depending on the equation of state, pressure gradients might make the expansion go faster or slower.Because anisotropy, a measure of a fluid's directional dependence, reduces with expansion, a more isotropic world is the outcome of the universe's expansion.Early universe anisotropies left behind traces in the cosmic microwave background (CMB) that provide crucial information about the early universe.The behaviors found indicate a strong dark energy component and an acceleration of the expansion of the cosmos, with far-reaching implications.The large-scale structure of the universe is shaped by density changes and gravitational evolution. A window into the early conditions of the universe is provided by isotropies seen in the cosmic microwave background [55–58]. To understand the fate of the universe and its expansion history,one must grasp the link between pressure and density in the equation of state.

Table 1 shows the collapsing case, in which density, tangential pressure, anisotropic and minser sharp mass are increasing functions of r as, we increase the value of q and always positive. Radial pressure is also increasing function of r but if we increase the value of q it becomes negative. As seen in Figure 1, the density for the collapsing solution is positive and has an increasing. As seen in Figure 2, the P_r pressure is negative of r and increases continuously wrt "r" with different values of $q^2 = 0.2, \daleth$ and $\mathcal{H}(\sqcup)_\infty = 5$. As indicated in Figure 3, P_t pressure is also rising and positive in relation to "r" at different values of $\daleth = 0.2,\ q^2$. Because of this, we may state that pressure has the opposite consequences from those seen in previous cases. Figure 4 illustrates the growing anisotropy function with respect to r with different values of $\daleth, q^2 = 0.2$. Figure 5 is for misner sharp mass with respect to r with different values of $\daleth, q^2 = 0.2$.

Table 1. The collapsing case where density, tangential pressure, anisotropic and minser sharp mass are increasing functions of r

Quntities	ρ	P_r	P_t	Δa	$m(t,r)$
Increasing ↑/Decreasing ↓	↑	↑	↑	↑	↑
Positive/ Negative	Positive	Negative	Positive	Positive	Positive

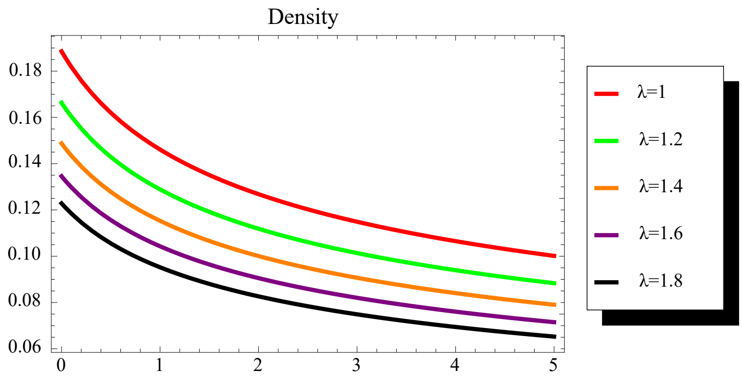

Figure 1. Variation of ρ wrt r when λ, $q = 0.2$ and $\mathcal{H}(\sqcup)_\infty = 5$

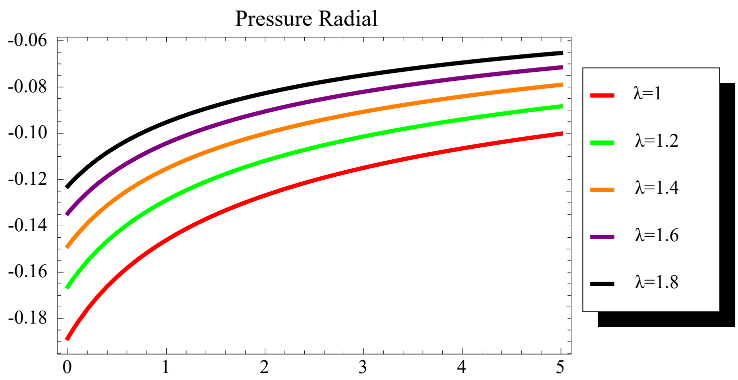

Figure 2. Variation of P_r wrt r when λ, $q = 0.2$ and $\mathcal{H}(\sqcup)_\infty = 5$

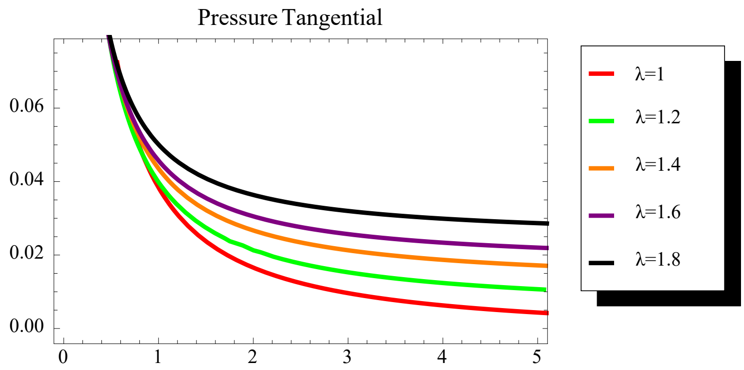

Figure 3. Variation of P_t wrt r when λ, $q = 0.2$ and $\mathcal{H}(\sqcup)_\infty = 5$

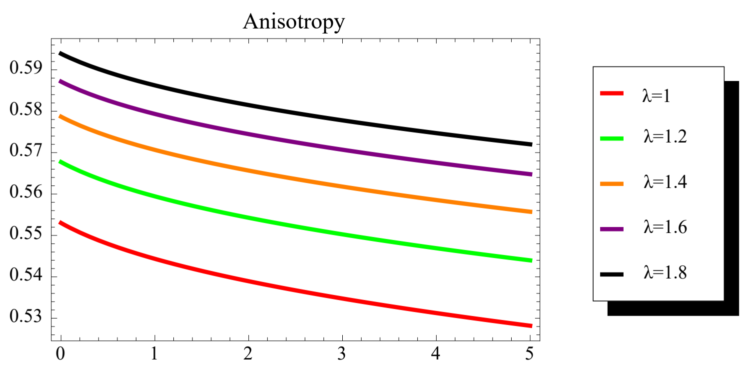

Figure 4. Variation of Δ_a wrt r when λ, $q = 0.2$ and $\mathcal{H}(\sqcup)_\infty = 5$

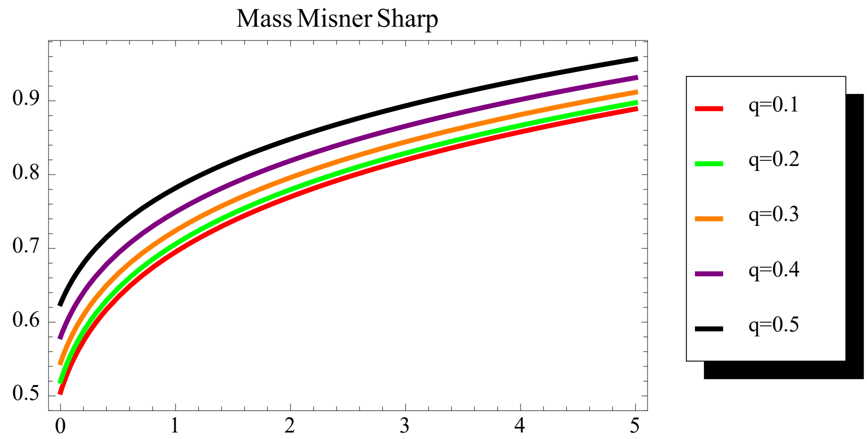

Figure 5. Variation of $m_1(r,t)$ wrt r when $q = 0.2$, and $\mathcal{H}(\sqcup)_\infty = 5$

3.2 Expansion with $\varpi = \frac{3}{2}$

Since we have an expanding solution when the expansion scalar reaches positive values, Eq. (13) suggests that $\Theta > 0$, if $\varpi > -2$. For convenience, we will choose $\varpi = \frac{3}{2}$ and suppose that

$$S = (r^2 + l_\circ^2) + \mathcal{H}(\sqcup)_\in \tag{28}$$

where $l_\circ > 0$ and $\mathcal{H}(\sqcup)_\in$ denote the integration function. Simplifying, we consider $F(t,r) = 1 + \mathfrak{H}(\mathfrak{t})_2(r^2 + l_\circ^2)$ and $S = \frac{F}{(r^2+l_\circ^2)}$. By simplifying the aforementioned field equations, we may derive the explicit form of matter variables as follows.

$$\begin{aligned}
\rho = &-\frac{1}{(1+2\lambda)(1+4\lambda)}\Bigg(-\frac{4F}{r^2+l_\circ^2} - \frac{8r^2(r^2+l_\circ^2)}{F^5} - \frac{(r^2+l_\circ^2)^2}{F^2} + \frac{4(3r^2-l_\circ^2)(r^2+l_\circ^2)^2}{F^4}\\
&+\frac{q^2(r^2+l_\circ^2)^4}{F^4} - 2\Bigg(-\frac{6F}{r^2+l_\circ^2} + \frac{12r^2(r^2+l_\circ^2)}{F^5} + \frac{(-3r^2+l_\circ^2)(r^2+l_\circ^2)}{F^4} - \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\\
&-5\Bigg(\frac{4F}{r^2+l_\circ^2} + \frac{8r^2(r^2+l_\circ^2)}{F^5} + \frac{(r^2+l_\circ^2)^2}{F^2} - \frac{4(3r^2-l_\circ^2)(r^2+l_\circ^2)^2}{F^4} - \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\\
&-\Bigg(-\frac{4F}{r^2+l_\circ^2} - \frac{8r^2(r^2+l_\circ^2)}{F^5} - \frac{(r^2+l_\circ^2)^2}{F^2} + \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\Bigg)\\
P_r = &-\frac{1}{(1+2\lambda)(1+4\lambda)}\Bigg(\frac{4F}{r^2+l_\circ^2} + \frac{8r^2(r^2+l_\circ^2)}{F^5} + \frac{(r^2+l_\circ^2)^2}{F^2} - \frac{q^2(r^2+l_\circ^2)^4}{F^4}\\
&+2\Bigg(-\frac{6F}{r^2+l_\circ^2} + \frac{12r^2(r^2+l_\circ^2)}{F^5} + \frac{(-3r^2+l_\circ^2)(r^2+l_\circ^2)}{F^4} - \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\\
&+\Bigg(\frac{4F}{r^2+l_\circ^2} + \frac{8r^2(r^2+l_\circ^2)}{F^5} + \frac{(r^2+l_\circ^2)^2}{F^2} - \frac{4(3r^2-l_\circ^2)(r^2+l_\circ^2)^2}{F^4} - \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\\
&-3\Bigg(-\frac{4F}{r^2+l_\circ^2} - \frac{8r^2(r^2+l_\circ^2)}{F^5} - \frac{(r^2+l_\circ^2)^2}{F^2} + \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\Bigg)\\
P_t = &-\frac{1}{(1+2\lambda)(1+4\lambda)}\Bigg(\frac{6F}{r^2+l_\circ^2} - \frac{12r^2(r^2+l_\circ^2)}{F^5} - \frac{(-3r^2+l_\circ^2)(r^2+l_\circ^2)}{F^4} + \frac{q^2(r^2+l_\circ^2)^4}{F^4}\\
&-2\Bigg(-\frac{6F}{r^2+l_\circ^2} + \frac{12r^2(r^2+l_\circ^2)}{F^5} + \frac{(-3r^2+l_\circ^2)(r^2+l_\circ^2)}{F^4} - \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\\
&+\Bigg(\frac{4F}{r^2+l_\circ^2} + \frac{8r^2(r^2+l_\circ^2)}{F^5} + \frac{(r^2+l_\circ^2)^2}{F^2} - \frac{4(3r^2-l_\circ^2)(r^2+l_\circ^2)^2}{F^4} - \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\\
&-\Bigg(-\frac{4F}{r^2+l_\circ^2} - \frac{8r^2(r^2+l_\circ^2)}{F^5} - \frac{(r^2+l_\circ^2)^2}{F^2} + \frac{q^2(r^2+l_\circ^2)^4}{F^4}\Bigg)\lambda\Bigg)
\end{aligned} \tag{29}$$

For $\varpi = \frac{3}{2}$, the anisotropy defined in Eq. (14) becomes

$$\Delta a = \frac{\begin{pmatrix} rF^3\left(r^2+l_\circ^2\right)^3(1+4\lambda)+4r^2\left(r^2+l_\circ^2\right)^2(-1+8\lambda)+2F^6(5+8\lambda) \\ -F\left(r^2+l_\circ^2\right)^2\left(l_\circ^2+4q^2r^6\lambda-8r_\circ^4\lambda+4q^2r_\circ^6\lambda+12r^4\left(2+q^2l_\circ^2\right)\lambda\right. \\ \left.+r^2\left(-3+16l_\circ^2\lambda+12q^2r_\circ^4\lambda\right)\right) \end{pmatrix}}{\begin{pmatrix} r4F^6(1+\lambda)+F^3\left(r^2+l_\circ^2\right)^2(1+4\lambda)+8r^2\left(r^2+l_\circ^2\right)^2(1+7\lambda) \\ -F\left(2\left(r^2+l_\circ^2\right)^2\left(6r^2-l_\circ^2\left(1+2l_\circ^2\right)+r^2\left(3+4l_0^2\right)\right)\lambda\right. \\ \left.+q^2\left(r^2+l_\circ^2\right)^5(1+6\lambda)\right) \end{pmatrix}} \tag{30}$$

The mass function for $\varpi = \frac{3}{2}$, as stated in Eq. (17), becomes

$$m_2(t,r) = \frac{f}{2\left(r^2+l_\circ^2\right)}\left(1+\frac{f^3}{\left(r^2+l_\circ^2\right)^3}-\frac{4r^2}{f^3\left(r^2+l_\circ^2\right)}+\frac{q^2\left(r^2+l_\circ^2\right)^2}{f^2}\right) \tag{31}$$

Table 2. The expansion case where density,tangential pressure, and radial pressure are decreasing functions of r

Quntities	ρ	P_r	P_t	Δa	$m(t,r)$
Increasing $\uparrow$/Decreasing $\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
Positive/ Negative	Positive	Positive	Negative	Positive	Positive

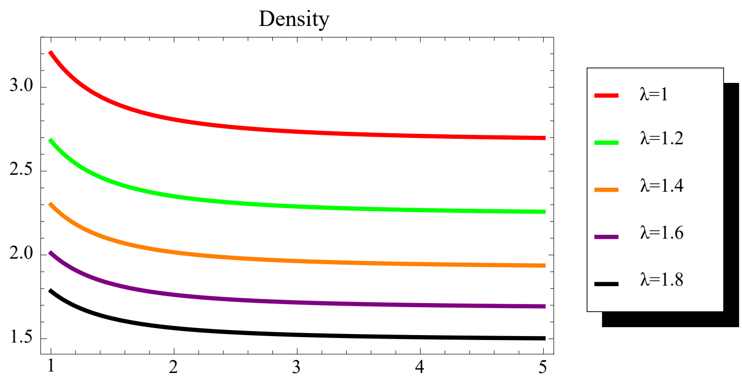

Figure 6. Variation of ρ wrt r when λ, $q = 0.2$, $l_o = 0.05$ and $\mathcal{H}(\sqcup)_\in = 5$

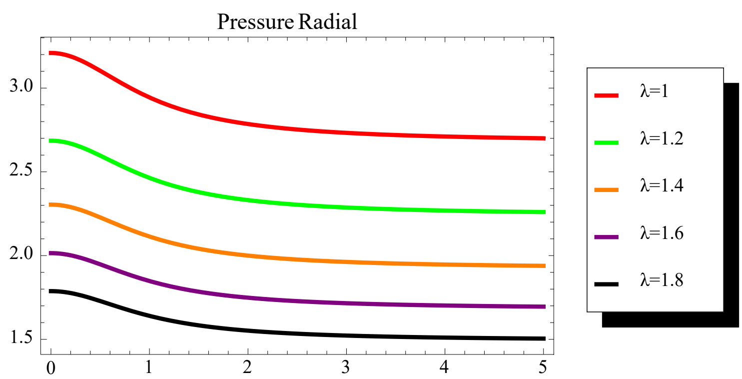

Figure 7. Variation of P_r wrt r when λ, $q = 0.2$, $l_o = 0.05$ and $\mathcal{H}(\sqcup)_\in = 5$

Table 2 shows the expansion case, in which density, tangential pressure and radial pressure are decreasing functions of r as, we increase the value of q and always negative. Miner sharp mass is increasing function of r, as we increase the value of q and it becomes positive. Figure 6 illustrates how the density is falling. The p_r continually decreases with respect to radii for various values of $\daleth$, $l_2(t) = 5$ and $q^2 = 0.2$, as shown in Figure 7. P_r declines with

“r” at various values of $\beth = 0.2$ and q^2, as seen in Figure 8. It is established that, at various values of $\beth$, $q^2 = 0.2$, the anisotropy is a decreasing with respect to r, as shown in Figure 9. The misner sharp mass is positive as we increase the value of λ with respect to r for various values of $\beth$, $l_2(t) = 5$ and $q^2 = 0.2$, as shown in Figure 10.

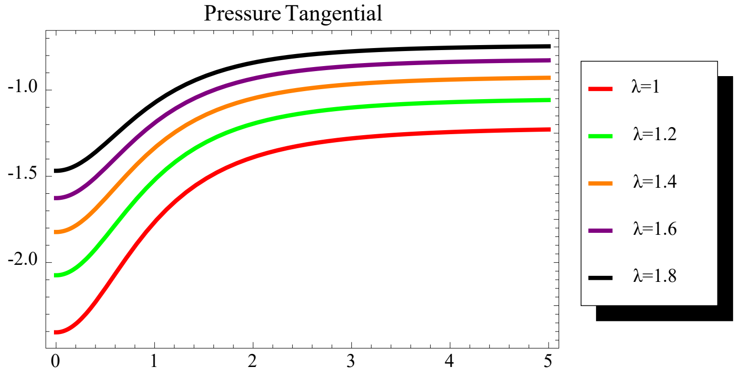

Figure 8. Variation of P_t wrt r when λ, $q = 0.2$, $l_o = 0.05$ and $\mathcal{H}(\sqcup)_\in = 5$

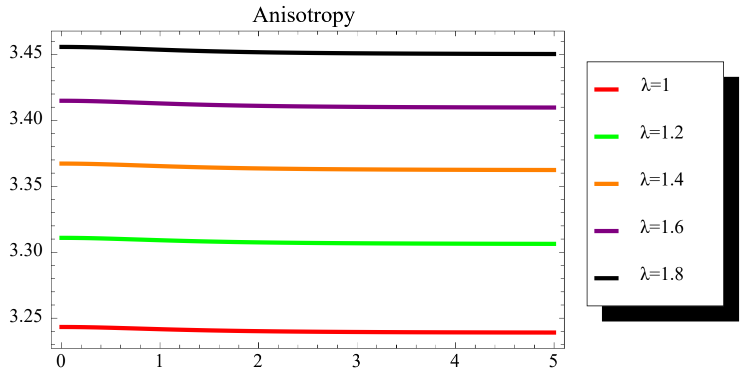

Figure 9. Variation of Δ_a wrt r when λ, $q = 0.2$, $l_o = 0.05$ and $\mathcal{H}(\sqcup)_\in = 5$

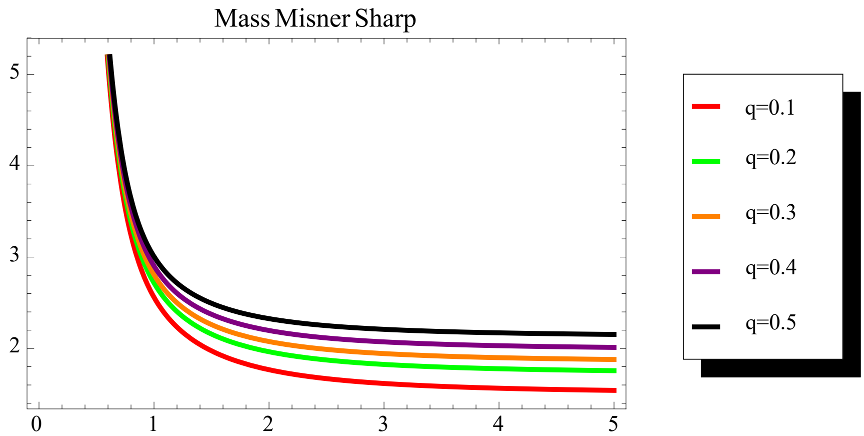

Figure 10. Variation of $m_2(r,t)$ r when q, $l_o = 0.05$ and $\mathcal{H}(\sqcup)_\in = 5$

4 Conclusion

Inspired on the formulation of *f(R,T)* gravity by Abbas and Ahmed [42], this work focuses on the gravitational collapse and expansion of anisotropic fluids. This hypothesis is predicated on a galaxy forecast that is significant to the current period of science and mathematics. The $F(R)$ and $G(R)$ theories of gravity have received a great deal of attention, whereas the *f(R,T)* theory of gravity has received less attention. Researchers’ focus in the recent age has been mostly on the *f(R,T)* theory of gravity. In *f(R,T)* theory of gravity, Zubair et al. [26] conducted a dynamical analysis of a locally anisotropic fluid with a cylindrical symmetry. Abbas and Ahmed [42] addressed the nature of solution to collapse and expansion of anisotropic fluid in the context of the *f(R,T)* theory of gravity.

The nature of the anisotropic fluid's solution to collapsing and expanding in the presence of an electromagnetic field has been developed here. A unique trapped surface for the inner matter spread in the presence of an electromagnetic field is provided by the collapse solution. A significant amount of thermal energy is released during gravitational collapse, as explained by Herrera and Santos [12]. Glass [17] obtains an anisotropic expanding and collapsing EFE solution. The work of Abbas and Ahmed [42] in the *f(R,T)* theory of gravity in the absence of an electromagnetic field is extended in this study.

This study *f(R,T)* provides a comprehensive analysis of the interior solution for anisotropic fluids used in the simulation of anisotropic galaxies in the presence of electromagnetic field within the framework of the modified theory of gravity. With the help of the auxiliary form of the metric function in the *f(R,T)* theory of gravity, we have solved the trapped conditions for fluid. In *f(R,T)* gravity, the following quantities have been calculated: For a collapsing solution where $\varpi = -\frac{5}{2}$, the ρ lowers wrt r, as Figure 1 depicts, along with the $P_\perp$, P_r, $m(t,r)$ and Δa when λ, $l_1(t) = 5$ and $q = 0.2$. In Figures 2 and 3 the values of $\lambda, q = 0.2$, and $l_1(t)$ decrease as the radius r increases, as do the tangential pressure $P_\perp$ and radical pressure P_r. The Figures 4 and 5 state that as $P_\perp$ and P_r diminish and $P_\perp < P_r$, anisotropy likewise reduces. It is clear from this that $\Delta a > 0$. From equation of density and radial pressure it is obvious that $p_r = -\rho$. This clearly shows that density and radial pressure ratio is - 1, which corresponds to dark energy. Herrera et al. [59] revealed that a singularity may exist for negative radial pressure. Further, if the ratio $\frac{p_r}{\rho} > -\frac{1}{3}$, the singularity will be covered and it will be naked if the $\frac{p_r}{\rho} \leq -\frac{1}{3}$. In our case, the ratio $\frac{p_r}{\rho} < -\frac{1}{3}$ and hence a naked singularity may exists. Regarding the expansion solution when Θ is positive and $\Delta a = \frac{3}{2}$. The behaviour of ρ, P_r, P_t, and Δa in this scenario is opposite to that of gravitational collapse.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

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