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Strategic Performance Evaluation Using the Interval Malmquist Index for Institutional Assessment

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Abstract: This study proposes an advanced framework for performance evaluation by extending the Malmquist Productivity Index (MPI) to accommodate interval data, addressing the inherent uncertainty and imprecision frequently encountered in institutional assessments. In many contexts, input-output data are often reported as intervals rather than precise values, which poses significant challenges for evaluating productivity changes. The extended MPI model allows for a more comprehensive analysis of performance by incorporating such interval data, thus providing a robust mechanism for assessing both progress and regression in the productivity of Decision-Making Units (DMUs). A case study on university departments is employed to demonstrate the practical application of this interval-based model. The results highlight notable variations in efficiency and technological advancement, offering valuable insights for institutional decision-makers. The proposed methodology enhances the accuracy of performance evaluation in dynamic and uncertain environments, making it a powerful tool for strategic planning and policy formulation. Furthermore, it is suggested that this interval-based approach offers a significant improvement over traditional models by accounting for the uncertainty present in real-world data. The study contributes to the broader field of strategic performance analytics by advancing the methodological understanding of productivity analysis, offering a more nuanced and reliable framework for institutional assessment.

Keywords: Interval data; MPI; Strategic analytics; Institutional assessment; Performance evaluation

1 Introduction

Data Envelopment Analysis (DEA) is a highly versatile and valuable methodology that provides a measure of the relative efficiency of each DMU, where efficiency encompasses various dimensions, including environmental, economic, and social impacts within the context of protected areas [1, 2]. DEA is a multivariate approach for evaluating productivity, offering insights into potential avenues for improvement when inefficiencies are identified. As a nonparametric method, DEA compares input/output data without making prior assumptions about the underlying probability distribution [3–5]. The foundation of this nonparametric programming methodology, including the assessment of relative efficiency, is based on the pioneering work of Charnes et al. [6, 7].

Numerous benchmarking studies and performance analyses of both public and private institutions, such as schools [6, 8], hospitals [9–12], and banks [13–17], have employed DEA to evaluate efficiency.

Despite its widespread utility, a key challenge of DEA lies in its reliance on precise input and output data. However, real-world scenarios frequently involve uncertainty and imprecision, where data may be represented as intervals rather than fixed values. To overcome this limitation, researchers have adapted DEA models to incorporate interval data, providing a more nuanced perspective on efficiency analysis in uncertain environments [18–24].

Building upon the core framework of DEA, the MPI serves as a complementary tool for assessing productivity changes over time [25]. Initially introduced by Malmquist in the context of consumer theory, the MPI utilizes input distance functions to construct productivity indices [26]. Subsequent research, such as that by Caves et al. [25], extended the MPI to assess efficiency changes across different time periods. The index is typically decomposed into two main components: efficiency change and technological change. While efficiency change measures the relative performance improvement or deterioration of a DMU, technological change captures shifts in the production frontier over time [27–29].

The MPI has two primary variants: the adjacent MPI and the base period MPI. The adjacent MPI, conceptualized as the geometric mean of two period-specific indices, evaluates productivity changes between consecutive time periods. In contrast, the base period MPI anchors productivity comparisons to a fixed reference point, providing a broader temporal perspective [30, 31]. Each variant has its advantages, with the adjacent MPI highlighting short-term technological advancements and the base period MPI offering insights into long-term trends.

The neighbouring and base period variants of the MPI provide equivalent assessments of efficiency change between two time periods [32, 33]. However, the measurement of technological change differs between the two. The neighbouring variant, fundamentally a two-period concept, quantifies the shift in the technology frontier as the variation between the frontier at time t and at time $t + 1$. It relies solely on contiguous time intervals to assess technological advancements, with the two shifts ultimately averaged geometrically.

Incorporating interval data into the MPI extends its applicability to situations where precise measurements are unavailable. This interval-based MPI framework enables a more comprehensive evaluation of productivity changes by accounting for the inherent variability and uncertainty present in real-world data. By leveraging interval data, the proposed model introduces a refined methodology for assessing productivity, as well as progress or regression, across DMUs. This approach addresses critical limitations in traditional productivity analysis, facilitating more accurate assessments of institutional performance in dynamic and uncertain environments.

This study contributes to the existing literature by applying the interval-based MPI to a practical case study involving university departments. The results highlight the model's ability to capture nuanced insights into efficiency changes and technological progress, demonstrating its potential as a strategic decision-support tool for institutional assessment. By bridging theoretical advancements with practical applications, the study aims to enhance the utility of DEA and MPI methodologies for both researchers and practitioners.

2 Background

2.1 Data Envelopment Analysis

In the DEA approach, each DMU can independently determine its own set of weights, which can be derived by maximizing efficiency [32]. Consider a collection of N Decision-Making Units (DMUs), each generating J outputs from I inputs. Let y and x represent the vectors of output and input quantities for the m^{th} DMU, respectively. The efficiency of the m^{th} DMU can, therefore, be computed as:

$$e_m = \frac{\sum_{j=1}^{J} u_j y_{jm}}{\sum_{i=1}^{J} v_i x_{im}} \tag{1}$$

In model (1), u_i and v_i are two weight vectors utilized by DMU m to assess the relative significance of the consumed and created components. The weights in DEA are not provided but are derived from the maximizing of the m^{th} DMU, as outlined below.

$$\begin{gathered} max\, e_m \\ s.t. \frac{\sum_{j=1}^{J} u_j y_{jn}}{\sum_{i=1}^{l} v_i x_{in}} \leq 1 \quad \forall n = 1, .., m, ., N \\ 0 \leq u_i \leq 1, 0 \leq v_i \leq 1 \end{gathered} \tag{2}$$

To facilitate calculations, one can normalize the input prices so that the cost of the inputs for DMU m equals 1. Converting the problem outlined in model (2) into the standard LP problem presented below:

$$\begin{gathered} max \sum_{j=1}^{J} u_j y_{jm} \\ s.t. \sum_{i=1}^{I} v_i x_{im} = 1 \\ \sum_{j=1}^{J} u_j y_{jn} - \sum_{i=1}^{I} v_i x_{im} \leq 0 \quad \forall n = 1, ., m, ., N, \, \varepsilon \leq u_i \leq 1, \, \varepsilon \leq v_i \leq 1 \end{gathered} \tag{3}$$

2.2 MPI

Adjacent MPI is characterized as the geometric mean of two Malmquist-based productivity indices as presented by the study [25] as follows:

$$MPI = \sqrt{\frac{\theta^t\left(X_p^{t+1}, Y_p^{t+1}\right)\theta^{t+1}\left(X_p^{t+1}, Y_p^{t+1}\right)}{\theta^t\left(X_p^t, Y_p^t\right)\theta^{t+1}\left(X_p^t, Y_p^t\right)}} \tag{4}$$

The decomposition of (4) in an efficiency change component and a technical change one is given as

$$MPI = \underbrace{\frac{\theta_0^{t+1}\left(X_0^{t+1}, Y_0^{t+1}\right)}{\theta_0^t\left(X_0^t, Y_0^t\right)}}_{\text{Efficiency Change}} \underbrace{\left[\frac{\theta_0^t\left(X_0^{t+1}, Y_0^{t+1}\right)}{\theta_0^{t+1}\left(X_0^{t+1}, Y_0^{t+1}\right)} \cdot \frac{\theta_0^t\left(X_0^t, Y_0^t\right)}{\theta_0^{t+1}\left(X_0^t, Y_0^t\right)}\right]^{\frac{1}{2}}}_{\text{Technical Change}} \tag{5}$$

The change in efficiency share in model (5) is the quotient of the Farrell technical efficiency measure at time $t+1$ and the Farrell technical efficiency measure at time t. The aspect of technological change is represented by the geometric average of the two ratios that indicate the shifts in the frontier at times t and $t+1$. $M > 1$ signifies an enhancement in overall productivity, $M < 1$ denotes a reduction, and $M = 1$ reflects stable productivity growth.

3 Interval Data in DEA

Suppose for any DMU the values of input and output are situated inside a specific interval, in which x_{ij}^L and x_{ij}^U denotes the i^{th} input lower and upper bounds related to the j^{th} DMU; Similarly, y_{ij}^L and y_{ij}^U represent the r^{th} output lower and upper bounds related to the j^{th} DMU, namely, $x_{ij}^L \le x_{ij} \le x_{ij}^U$ and $y_{ij}^L \le y_{ij} \le y_{ij}^U$. These data are referred to as interval ones, as they exist within specific intervals.

It should be considered that invariably $x_{ij}^L \le x_{ij}^U$ and $y_{ij}^L \le y_{ij}^U$. If $x_{ij}^L = x_{ij}^U$ so, then the i^{th} input of the j^{th} DMU possesses a certain value. Interval problems involve parameters whose values reside within intervals, rendering their precise values indeterminate. The CCR model for assessing DMU_P utilizing interval data is presented as follows [34]:

$$\begin{aligned} & e_p max \quad \sum_{r=1}^{s} u_r\left[y_{rp}^L, y_{rp}^U\right] \\ & s.t. \quad \sum_{i=1}^{m} v_i\left[x_{ip}^L, x_{ip}^U\right] = 1 \\ & \sum_{j=1}^{J} u_j\left[y_{rp}^L, y_{rp}^U\right] - \sum_{i=1}^{I} v_i\left[x_{ip}^L, x_{ip}^U\right] \le 0 \\ & \varepsilon \le u_i, \varepsilon \le v_i, i = 1, \ldots, m, r = 1, \ldots, s \end{aligned} \tag{6}$$

In model (6), given that all parameters of the issue are expressed as intervals and lack definitive values, the relative efficiency of DMU_P is determined by solving models (7) and (8), accordingly.

$$\begin{aligned} & max\, e_p^U = \sum_{r=1}^{s} u_r y_{ro}^u \\ & s.t. \sum_{i=1}^{m} v_i x_{ij}^L = 1 \\ & \sum_{r=1}^{s} u_r y_{ro}^u - \sum_{i=1}^{m} u_i x_{io}^L \le 0, \sum_{r=1}^{s} u_r y_{rj}^L - \sum_{i=1}^{m} v_i x_{ij}^u \le 0, \quad u_r \ge \varepsilon,\, v_i \ge \varepsilon \end{aligned} \tag{7}$$

$$\begin{aligned} & max\, e_p^L = \sum_{r=1}^{s} u_r y_{ro}^L \\ & s.t. \sum_{i=1}^{m} v_i x_{io}^u = 1 \\ & \sum_{r=1}^{s} u_r y_{ro}^L - \sum_{i=1}^{m} u_i x_{io}^u \le 0, \sum_{r=1}^{s} u_r y_{rj}^u - \sum_{i=1}^{m} v_i x_{ij}^L \le 0, \quad u_r \ge \varepsilon,\, v_i \ge \varepsilon \end{aligned} \tag{8}$$

In model (7), the evaluated DMU is in optimal condition, while the remaining DMUs are in suboptimal condition. Thus, $e_p \leq e_p^U$ is consistently valid. In model (8), the evaluated DMU is in its most unfavourable state, while the other DMUs are in their optimal states. Consequently, the derived efficiency will represent the least favourable efficiency for the evaluated DMU, so $e_p \geq e_p^L$. Based on the aforementioned information, we can assert $e_p \in \left[e_p^L, e_p^U\right]$.

4 The Proposed Models

The MPI is assessed across periods t and t+1. This article aims to derive an interval for the MPI applicable to any unit, referred to as the interval of the MPI [14]. Furthermore, it is understood that the assessment of the productivity index necessitates two single-period measures and two mixed-period measures. The two single-period metrics can be derived with the CCR DEA model as follows:

$$\begin{aligned}&min\,\theta\\&s.t.\ \textstyle\sum_{j=1}^{n}\lambda_j X_j \leq \theta X_0\\&\textstyle\sum_{j=1}^{n}\lambda_j Y_j \geq Y_0,\quad \lambda_j \geq 0, j=1,\ldots,n\end{aligned} \tag{9}$$

In model (9), x_{ip}^t stands for the i^{th} input and y_{rp}^t denotes the r^{th} output of DMU_P in time t. It can be proven that $0 < \theta^* \leq 1$. (θ^* is the optimal solution). DMU_P will be efficient if $\theta^* = 1$. Two single-period measures $\theta^t\left(X_p^t, Y_p^t\right)$ and $\theta^{t+1}\left(X_p^{t+1}, Y_p^{t+1}\right)$ can be given as below:

$$\begin{aligned}&\theta^t\left(X_p^t, Y_p^t\right) = min\theta\\&s.t.\ \textstyle\sum_{j=1}^{n}\lambda_j\left[x_{ij}^{t,L}, x_{ij}^{t,U}\right] \leq \theta\left[x_{ip}^{t,L}, x_{ip}^{t,U}\right]\quad i=1,\ldots,m\\&\textstyle\sum_{j=1}^{n}\lambda_j\left[y_{rj}^{t,L}, y_{rj}^{t,U}\right] \geq \left[y_{rp}^{t,L}, y_{rp}^{t,U}\right]\quad r=1,\ldots,s,\ \lambda_j \geq 0,\ j=1,\ldots,n\end{aligned} \tag{10}$$

By employing $t+1$ in place of t, we derive the following model.

$$\begin{aligned}&\theta^{t+1}\left(X_p^{t+1}, Y_p^{t+1}\right) = min\theta\\&s.t.\textstyle\sum_{j=1}^{n}\lambda_j\left[x_{ij}^{t+1,L}, x_{ij}^{t+1,U}\right] \leq \theta\left[x_{ip}^{t+1,L}, x_{ip}^{t+1,U}\right]\quad i=1,\ldots,m\\&\textstyle\sum_{j=1}^{n}\lambda_j\left[y_{tj}^{t+1,L}, y_{tj}^{t+1,U}\right] \geq \left[y_{rp}^{t+1,L}, y_{rp}^{t+1,U}\right]\quad r=1,\ldots,s,\ \lambda_j \geq 0,\ j=1,\ldots,n\end{aligned} \tag{11}$$

Regarding the parameters of models (10) and (11), if they are intervals, then both $\theta^t\left(X_p^t, Y_p^t\right)$ and $\theta^{t+1}\left(X_p^{t+1}, Y_p^{t+1}\right)$ are also intervals. Considering model (10), we derive:

$$\begin{aligned}&\theta^{t,L}\left(X_p^t, Y_p^t\right) = min\theta\\&s.t.\textstyle\sum_{i=1,i=p}^{n}\lambda_j x_{ij}^{t,L} + \lambda_p x_{ip}^{t,U} \leq \theta x_{ip}^{t,U},\quad i=1,\ldots,m\\&\textstyle\sum_{i=1,j=p}^{n}\lambda_j y_{rj}^{t,U} + \lambda_p y_{rp}^{t,L} \leq y_{rp}^{t,L},\quad r=1,\ldots,s,\ \lambda_i \geq 0,\ j=1,\ldots,n\end{aligned} \tag{12}$$

And

$$\begin{aligned}&\theta^{t,U}\left(X_p^t, Y_p^t\right) = min\theta\\&s.t.\textstyle\sum_{j=1,j\neq p}^{n}\lambda_j x_{ij}^{t,U} + \lambda_p x_{ip}^{t,L} \leq \theta x_{ip}^{t,L},\quad i=1,\ldots,m\\&\textstyle\sum_{\substack{j=1,j\neq p\\ \lambda_j\geq 0}}\lambda_j y_{rj}^{t,L} + \lambda_p y_{rp}^{t,U} \leq y_{rp}^{t,U},\quad r=1,\ldots,s,\ \lambda_j \geq,\ j=1,\ldots,n\end{aligned} \tag{13}$$

By using $t+1$ in place of t for the interval's lower and upper bounds, we obtain $\theta^{t+1,L}\left(X_p^{t+1}, Y_p^{t+1}\right)$ and $\theta^{t+1,U}\left(X_p^{t+1}, Y_p^{t+1}\right)$.

The subsequent stage involves calculating two mixed-period metrics, $\theta^{t+1}\left(X_p^t, Y_p^t\right)$ and $\theta^t\left(X_v^{t+1}, Y_v^{t+1}\right)$.

To compute $\theta^{t+1}\left(X_p^t, Y_p^t\right)$, we can apply model (14) as follows:

$$\begin{aligned}&\theta^{t+1}\left(X_p^t, Y_p^t\right) = min\theta\\&s.t.\textstyle\sum_{j=1}^{n}\lambda_j\left[x_{ij}^{t+1,L}, x_{ij}^{t+1,U}\right] \leq \theta\left[x_{ip}^{t,L}, x_{ip}^{t,U}\right]\quad i=1,\ldots,m\\&\textstyle\sum_{j=1}^{n}\lambda_j\left[y_{rj}^{t+1,L}, y_{1j}^{t+1,U}\right] \geq \left[y_{pp}^{t,L}, y_{rp}^{t,U}\right]\quad r=1,\ldots,s,\ \lambda_j \geq 0,\ j=1,\ldots,n\end{aligned} \tag{14}$$

By substituting $t+1$ for t, and conversely, we obtain $\theta^t\left(X_p^{t+1}, Y_p^{t+1}\right)$.

$\theta^{t+1}\left(X_p^t, Y_p^t\right)$ and $\theta^t\left(X_p^{t+1}, Y_p^{t+1}\right)$ may additionally be situated within bounded interval measurements, as $\left[\theta^{t+1,L}\left(X_p^t, Y_p^t\right), \theta^{t+1,U}\left(X_p^t, Y_p^t\right)\right]$ and $\left[\theta^{t,L}\left(X_p^{t+1}, Y_p^{t+1}\right), \theta^{t,U}\left(X_p^{t+1}, Y_p^{t+1}\right)\right]$, where the lower and upper bounds are derived from optimistic and pessimistic perspectives as follows:

$$\begin{array}{l}\theta^{t+1,L}\left(X_p^t,Y_p^t\right)=min\theta\\ s.t.\sum_{j=1}^n\lambda_j x_{ij}^{t+1,L}\leq\theta x_{ip}^{t,U}\quad i=1,\dots,m\\ \sum_{i=1}^n\lambda_j y_{rj}^{t+1,U}\geq y_{rp}^{t,L}\quad r=1,\dots,s,\ \lambda_i\geq 0,\ j=1,\dots,n\end{array}\tag{15}$$

And

$$\begin{array}{l}\theta^{t+1,U}\left(X_p^t,Y_p^t\right)=min\theta\\ s.t.\sum_{j=1}^n\lambda_j x_{ij}^{t+1,U}\leq\theta x_{ip}^{t,L}\quad i=1,\dots,m\\ \sum_{j=1}^n\lambda_j y_{rj}^{t+1,L}\geq y_{rp}^{t,U}\quad r=1,\dots,s,\ \lambda_j\geq 0,\ j=1,\dots,n\end{array}\tag{16}$$

In models (7) and (8), substituting $t+1$ for t yields $\theta^{t,L}\left(X_p^{t+1},Y_p^{t+1}\right)$ and $\theta^{t,U}\left(X_p^{t+1},Y_p^{t+1}\right)$, respectively, as the interval measure's lower and upper bounds for the unit $\left(X_p^{t+1},Y_p^{t+1}\right)$ in $t+1$ relative to t.

5 Interval Data in MPI

The MPI is computed to assess the advancement and decline of a DMU using the formula below:

$$MI_p=\sqrt{\frac{\theta^t\left(X_p^{t+1},Y_p^{t+1}\right)\theta^{t+1}\left(X_p^{t+1},Y_p^{t+1}\right)}{\theta^t\left(X_p^t,Y_p^t\right)\theta^{t+1}\left(X_p^t,Y_p^t\right)}}\tag{17}$$

It is important to recognize that we derive an interval for each element in the aforementioned formula. Consequently, the MPI will serve as an interval for any DMU. The MPI is defined by its lower and upper bounds as follows:

$$MI_p^U=\sqrt{\frac{\theta^{t,U}\left(X_p^{t+1},Y_p^{t+1}\right)\theta^{t+1,U}\left(X_p^{t+1},Y_p^{t+1}\right)}{\theta^{t,L}\left(X_p^t,Y_p^t\right)\theta^{t+1,L}\left(X_p^t,Y_p^t\right)}}\tag{18}$$

$$MI_p^L=\sqrt{\frac{\theta^{t,L}\left(X_p^{t+1},Y_p^{t+1}\right)\theta^{t+1,L}\left(X_p^{t+1},Y_p^{t+1}\right)}{\theta^{t,U}\left(X_p^t,Y_p^t\right)\theta^{t+1,U}\left(X_p^t,Y_p^t\right)}}\tag{19}$$

For every interval data denoted as DMU_P, the MPI is defined as follows, $MPI\in\left[MPI^L,MPI^U\right]$.

To evaluate units at periods t and $t+1$, we establish:

(a) If $MI_\nu^L=1$ and $MII_\nu^U=1$, for DMU_P, there is neither progress nor regress.

(b) If $MI_\nu^L\geq 1$ and $MI_\nu^U>1$, for DMU_P, there exists alone progress.

(c) If $MI_p^L<1$ and regress there exists alone, DMU_P for, $MI_p^U\leq 1$.

(d) If $MI_p^L<1$ and $MI_p^U>1$, for DMU_P, we introduce an indicator for assessing the progress or regress ratio for DMU_P in model (20):

$$\rho=\frac{MI_p^U-1}{1-MI_p^L}\tag{20}$$

where $0<\rho<\infty$. In model (20), e of progress in exhibits a greater percentag $\rho>1$ comparison to regress while $0<\rho<1$ demonstrates a higher percentage of regress relative to progress.

6 A Case Study

To demonstrate the practical application of the proposed interval MPI methodology, a case study was conducted involving 18 academic departments from a university. These departments were evaluated based on productivity changes across two time periods, utilising both interval input and output data. The dataset comprised seven input variables and thirteen output variables, as detailed in Table 1. This input-output configuration captures the diverse operational aspects of academic departments and their contributions to institutional objectives.

The result of this evaluation is shown in Table 2.

The analysis computed the MPI for each department, yielding interval productivity scores that account for the uncertainty inherent in the input-output data. The interval MPI framework facilitated the categorisation of departments into three distinct groups:

• Progressing Departments: These are departments where both the lower and upper bounds of the MPI exceeded 1, indicating consistent productivity improvement.

• Regressing Departments: Departments where both bounds of the MPI are below 1, signifying a decline in productivity.

• Ambiguous Departments: Units where the lower bound of the MPI is below 1, but the upper bound exceeds 1, reflecting mixed performance that cannot be conclusively categorised as either progress or regression.

The findings indicated that five departments exhibited significant progress, nine departments experienced a regression in productivity, and four departments displayed mixed results. This classification offers a nuanced understanding of performance trends across the university, highlighting both areas of strength and concern for management.

Table 1. Input and output sets

Inputs	Outputs
Count of full-time masters (I1)	Count of graduates (O1)
Count of semi-time masters (I2)	Count of PhD graduates (O2)
Count of masters (I3)	Average of mean graduates (O3)
Count of full-time masters (I4)	Average of mean graduates (O4)
Count of semi-time masters (I5)	Count of edited books via the masters (O51)
Count of masters (I6)	Count of edited papers via masters (O6)
Count of books in the library (I7)	Count of edited papers via the PhD students (O7)
	Count of edited papers via the students (O8)
	Count of research plans (O9)
	Count of PhD students (O10)
	Count of students (O11)
	Full-time and semi-time master ratio (O12)
	Full-time and semi-time master ratio (O13)

Table 2. The MPI upper and lower bounds

DMU	MPI_L	MPI_U	ρ
1	1.690459	3.46358	-
2	1.798158	3.358199	-
3	1.176601	1.788881	-
4	0.611566	1.00763	0.017
5	4.309994	7.361947	-
6	4.327163	5.781328	-
7	0.347285	0.670782	-
8	0.232	0.4026	-
9	0.281631	0.534205	-
10	0.299286	0.601169	-
11	0.52934	0.860132	-
12	0.287348	0.533464	-
13	0.600009	0.981197	-
14	0.502202	0.790558	-
15	0.683586	1.17938	0.564
16	0.680942	1.08376	0.259
17	0.287283	0.575906	-
18	0.716361	1.187363	0.658

7 Result Analysis

The results from the interval MPI analysis offer critical insights into the efficiency and productivity dynamics of the university's educational departments. Departments classified as progressing (MPI > 1) exhibited substantial improvements in both technical efficiency and technological advancements. This indicates that these units have effectively optimized their resource utilization and adapted to new methodologies or technologies. For instance, departments with high publication output and significant research contributions were among those showing progress. Conversely, the regressing departments (MPI < 1) highlight inefficiencies in resource utilization or a lack of adaptation to changing academic and operational demands. Common characteristics of these units included lower research outputs, reduced graduate numbers, or underutilization of available faculty and resources. This regression signals the need for targeted interventions to address resource allocation, faculty training, and strategic planning. The ambiguous departments, where the MPI intervals straddled 1, present a unique challenge for decision-makers. These departments may require a more granular analysis to identify factors contributing to the mixed results. Such ambiguity could stem from inconsistent performance across metrics or external factors influencing the departments' operations during the assessment period.

From a management perspective, the interval MPI analysis provides actionable insights that support data-driven decision-making:

Resource Allocation: Progressing departments can serve as benchmarks for best practices, and additional resources can be allocated to further enhance their performance. For regressing departments, targeted resource optimization strategies, such as investing in faculty development or infrastructure, are essential.

Strategic Interventions: Ambiguous departments require a deeper investigation to identify root causes of mixed performance, such as inconsistent resource utilization or variations in external demands. Tailored strategies can be developed to stabilize and improve their productivity.

Policy Formation: The findings emphasize the importance of adopting a dynamic approach to institutional assessment that accounts for uncertainty. Policies that encourage innovation, collaboration, and continuous improvement across departments can significantly enhance institutional productivity.

By leveraging the interval MPI framework, university administrators can move beyond simplistic assessments and gain a comprehensive understanding of productivity dynamics, enabling them to implement targeted strategies for sustained institutional growth.

8 Conclusions

This study proposed an advanced methodology for assessing institutional productivity through the MPI within the DEA framework. By extending the traditional MPI to incorporate interval data, the research addressed critical challenges in real-world productivity analysis, where uncertainty and imprecision in input-output measurements are common. The interval MPI provides a robust mechanism for evaluating performance changes, enabling the categorisation of DMUs into progressing, regressing, and ambiguous groups based on their productivity trends. The application of the proposed model to a university case study demonstrated its practical relevance and effectiveness. The analysis yielded significant insights into the operational dynamics of the educational departments, identifying areas of strength and concern. Progressing departments exemplified their ability to optimise resources and adapt to changing environments, while regressing departments highlighted inefficiencies that require targeted interventions. Ambiguous departments indicated the need for further investigation due to mixed performance results.

The findings offer actionable insights for university administrators and policymakers. By identifying performance trends and categorising departments based on their productivity dynamics, the interval MPI facilitates data-driven resource allocation, targeted interventions, and the formulation of policies aimed at fostering continuous improvement. This model empowers institutions to enhance operational efficiency and overall strategic performance in dynamic and uncertain environments. Although this study focused on a specific case, future research could extend the application of the interval MPI framework to other sectors such as healthcare, banking, and manufacturing. Furthermore, integrating advanced computational techniques and artificial intelligence could enhance the model's efficiency and scalability, enabling broader adoption across diverse fields. By bridging theoretical advancements with practical applications, this study contributes to the growing body of research on uncertainty modelling and strategic analytics, emphasising the importance of innovative tools for institutional assessment in an ever-changing world.

Data Availability

The datasets used and analyzed during the current study are available from the corresponding author upon request. The data includes interval input and output measures used for the assessment of productivity across the studied university departments.

Conflicts of Interest

The authors declare no conflict of interest.

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