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Solution and Interpretation of Neutrosophic Fuzzy Equation with Applications

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Abstract: Neutrosophy is a special area of philosophy that explains the nature, genesis and scope of neutralities, like the interactions with diverse ideational hues. It showed the degree of indeterminacy as an independent component that was the extension of an intuitionistic set. In this paper, the interpretation of the linear equation of type $\mathcal{AX}+\mathcal{B}=\mathcal{C}$ are discussed in a neutrosophic environment. It is observed that the equations $\mathcal{AX}+\mathcal{B}=\mathcal{C}$, $\mathcal{AX}=\mathcal{C}-\mathcal{B}$ and $\mathcal{AX}-\mathcal{C}=-\mathcal{B}$ are same and their solution are also same in crisp sense. But, in the neutrosophic sense, the solutions to the above equations are different. Mathematical operations on intervals are considered for the purpose of solution and analysis. Further, an application of budgeting-financing is described with the help of neutrosophic fuzzy equation.

Keywords: Fuzzy set theory; Triangular neutrosophic fuzzy number (TNFN); Neutrosophic fuzzy equation

1 Introduction

1.1 Fuzzy Sets Theory with Its Extension and Applications

Zadeh [1] introduced a fuzzy set in his first paper in 1965. The degree of belongingness in terms of membership function is captured by the theory. Further, the extension was developed in 1986 by Prof. Atanassov, namely the intuitionistic fuzzy set theory [2]. It captured the degree of belongingness and non belongingness by non-membership and degree of membership function. Another theory introduced by Smarandache [3] is neutrosophic sets, which are extensions of fuzzy sets by incorporating three membership degrees: truth, indeterminacy and falsity idea. There are several extensions like Type-2 Fuzzy Sets [4], Interval-Valued Fuzzy Sets [5], Fuzzy Rough Sets [6], etc. All ideas are based on providing that extra flexibility in representing uncertainty.

Not only in theoretical perspective, fuzzy sets and its application are very important for modelling real world problems. Modelling real-world problems with fuzzy sets and their extensions is vital for handling the integral uncertainty, ambiguity, and vagueness that describe many complex fields, such as decision-making theory, control theory, optimization, model simulation and risk valuation, etc. Fuzzy sets allow and its extensions supplementary layers of uncertainty, such as inadequate knowledge and indistinguishability, thus making models further robust and adaptive in uncertain settings. For example, anyone can follow the recent application in the field of Women's empowerment [7], Diabetes modelling [8], Industrial engineering [9], Population dynamics [10], Transportation problem [11] and Epidemic modelling [12].

1.2 Neutrosophic Set and Its Application in Different Areas

In 1995, Smarandache [3] proposed the degree of uncertainty as an independent element representing the neutrosophy set.

There are many applications of neutrosophic numbers in real life, some of which are mentioned below. Majumder et al. [13] mainly proposed single valued neutrosophic fuzzy set (SVNFS) to avoid the difficulties of some real-life problems with the nonstandard interval of neutrosophic elements. This work also introduced the distance measures between two SVNFS in 2020. A multi-criteria decision making (MCDM) procedure is also presented to apply this approach [14]. Khalil et al. [15] have applied the neutrosophic number to decision making in their work [16] respectively. Koundal et al. [17] mainly showed that prognosis and diagnosis are the most complex and challenging work in the world of medical science due to the limited subjectivity of experts and it is generalized by neutrosophic theory. In 2020, Fei [18] have presented the application in a wireless network with the help of a neutrosophic graph. Gulistan et al. [19] demonstrated the application in industry performance evaluation by using neutrosophic cubic graphs in 2018.

1.3 Motivation and Novelties of the Study

Fuzzy equations can address different real world's problems within their mathematical representatives. These are primarily represented by the models of the ambiguity linear system. The neutrosophic fuzzy equation investigates various mathematical approaches to the models' volatility with the help of neutrosophic fuzzy set theory. Sometimes, it is used in different fields, such as artificial intelligence, decision-making, etc. To solve various real-life applications such as budgeting and financing, we have used and processed neutrosophic fuzzy equations.

1.4 Structure of the Paper

This portion provides a proper explanation of this research. The introduction of this work is highlighted in Section 1. Then, Section 2 conveys the literature survey of this work. Then, the preliminary concepts of mathematical tools are discussed in Section 3. Further, in Section 4, we consider three different types of linear equations and solve these with neutrosophic numbers. Therefore, we take the numerical examples in Section 5. Additionally, Section 6 covered the application of neutrosophic fuzzy equations. The research findings are explained in Section 7 in this paper. Finally, conclusions and future research scope are exhibited in Section 8.

2 Literature Survey of on Neutrosophic Fuzzy Set Theory

This section explains the literature review of this paper. At first, we describe the neutrosophic set and number [20, 21] and then the fuzzy equation [22] very shortly.

2.1 Background on Neutrosophic Set and Number

An extension of the fuzzy set, which is also called the classical set with a certain amount of ambiguity, was the notion of the neutrosophic set. It studied the nature, origin and scope of neutralities and solved the problems to find the solution of having vagueness, indeterminacy or uncertainty [1]. neutrosophic numbers are mainly used to solve integral equations, differential equations, difference equations, etc. [23–25]. The perfect review of the neutrosophic set are described in Table 1.

Table 1. Literature review on neutrosophic set

Author	Year	Improvement in Theoretical Structure	Contribution in Applied Field
Smarandache [26]	2005	Beutrosophic set, intuitionistic fuzzy set, intuitionistic set	Application on generalization of intuitionistic set to neutrosophic set
Broumi et al. [27]	2013	Neutrosophic soft sets, interval valued neutrosophic soft sets	Extension the concept of interval valued intuitionstic fuzzy soft relation and introduced interval valued neutrosophic soft relation with its' properties
Broumi [28]	2013	Neutrosophic set, generalized neutrosophic set, generalized neutrosophic soft set	Development the concept of generalized neutrosophic soft set and its' application on MCDM
Deli et al. [29]	2014	Soft set, neutrosophic set, neutrosophic refined set, neutrosophic soft multi-set	Application on MCDM with the help of neutrosophic soft multi-set theory
Broumia et al. [30]	2014	Neutrosophic sets, neutrosophic refined sets	This paper mainly introduced neutrosophic refined sets (multisets) with the help of neutrosophic refined relation (NRR) with it's properties, s.t., symmetry, reflexivity, transitivity, etc.
Broumi and Smarandache [31]	2015	Interval-valued neutrosophic soft rough sets	Discuss the interval-valued neutrosophic soft rough sets with its' basic properties
Das et al. [20]	2020	Fuzzy set, neutrosophic set, neutrosophic fuzzy set, SVNFS	Application for decision making

Table 2. Literature review on neutrosophic numbers

Author	Year	Improvement in Theoretical Structure	Contribution in Applied Field
Deli and Subas [32]	2014	Neutrosophic set, single valued neutrosophic numbers, trapezoidal neutrosophic numbers, triangular neutrosophic numbers	This paper discussed about two spherical forms which are single valued trapezoidal and triangular neutrosophic numbers
Chakraborty et al. [33]	2018	Neutrosophic numbers and triangular neutrosophic numbers	Several applications of TNNs and de-neutrosophication processes
Deli [34]	2018	Single valued neutrosophic sets, neutrosophic numbers, trapezoidal neutrosophic, single valued trapezoidal neutrosophic numbers	Show the operators on single valued trapezoidal neutrosophic numbers and then apply it to the decision making process of SVTN-group
Edalatpanah [35]	2020	Single valued neutrosophic number	Application to the structure of the direct model for triangular neutrosophic linear programming problem
Deveci et al. [36]	2021	Fuzzy sets, Type-2 neutrosophic number	Offshore wind farm site selection problem solved in this paper based on Type-2 neutrosophic number
Muthulakshmi et al. [21]	2022	Neutrosophic numbers	Describe all the properties of it
Bilgin et al. [37]	2022	Fermatean neutrosophic numbers	Selection of supplier in a copper production industry
Reig-Mullor and Salas-Molina [38]	2022	Fuzzy set, neutrosophic fuzzy sets, non-linear neutrosophic numbers	Discuss the notion of non-linear neutrosophic numbers and its' application on multiple criteria performance assessment
Rahaman et al. [39]	2025	Neutrosophic numbers	System of linear differential equations in neutrosophic environment

Table 3. Literature review on uncertain equation with the help of different types of fuzzy number

Author	Year	Uncertain Environment	Contribution
Buckley [40]	1992	Fuzzy equation	Solve the fuzzy equations with different techniques
Buckley and Eslami [41]	2002	Fuzzy equation	Find the solution of linear equation with fuzzy number and show it with different examples
Chadli and Melliani [42]	2003	Intuitionistic fuzzy set	Find the required solution of fuzzy equation
Allahviranloo and Salahshour [43]	2011	Fuzzy linear system	Solve the fuzzy linear system including triangular fuzzy number with non-zero spreads and matrix
Behera and Chakraverty [44]	2013	Fuzzy number, fuzzy system of linear equations, triangular fuzzy number	A new process to solve fuzzy system of linear equations with crisp co-efficient
Ye [45]	2017	Neutrosophic number, neutrosophic linear equations	Application in traffic flow problems applying neutrosophic linear equations
Mondal and Mandal [46]	2017	Pentagonal fuzzy number	Application in fuzzy equation with the help of pentagonal fuzzy number and it's properties
Razvarz and Tahmasbi [47]	2017	Fuzzy equation	How are fuzzy equations applied to represent uncertain nonlinear system models
Allahviranloo et al. [48]	2018	Fuzzy number	Discuss a new process of fuzzy equations to get its' solution
Yu and Jafari [22]	2019	Fuzzy equation	The solutions of fuzzy equations and dual fuzzy equations
Akram et al. [49]	2019	Fuzzy number, bipolar fuzzy set	Development of bipolar fuzzy arithmetic and solve the fuzzy linear equation
Edalatpanah [50]	2020	Neutrosophic set, neutrosophic number	Application of neutrosophic linear equations
Abbasi and Allahviranloo [51]	2021	Fuzzy equation	A new computational technique for solving the fuzzy equations
Saw and Hazra [52]	2021	Non-negative intuitionistic fuzzy number, parametric form of intuitionistic fuzzy number	Find the solution of intuitionistic fuzzy linear system of equations
Alhasan [53]	2021	Neutrosophic liner equation	Application of the system of the neutrosophic linear equations with the help of Cramer's rule
Jdid and Smarandache [54]	2024	Neutrosophic science	Find the proper solution of systems of neutrosophic linear equations
Sing et al. [55]	2024	Fuzzy set theory, Hukuhara difference, generalized Hukuhara difference	Application on fuzzy linear equation
Shams et al. [56]	2024	Triangular intuitionistic fuzzy number	Application on linear fuzzy equation
Alamin et al. [57]	2024	Neutrosophic fuzzy sets	Solve financial problems

A neutrosophic fuzzy number (NFN) allows for a more comprehensive representation of ambiguity, especially when addressing undetermined or unreliable information. By including truth, indeterminacy, and falsity membership values, it enriches fuzzy numbers. While fuzzy numbers basically depend only on membership functions, the NFN account for the degree to which an element belongs to a set, simply because they include membership, indeterminacy and non-membership. In 2018, Deli [34] represented many kinds of neutrosophic sets and numbers, such as single valued neutrosophic sets, trapezoidal neutrosophic numbers, single valued trapezoidal neutrosophic numbers, etc., in his research work. Muthulakshmi et al. [21] also discussed the neutrosophic numbers and described their properties in 2022.

Neutrosophic numbers are used in numerous fields, including MCDM problems [58], differential equations [59] and difference equations [25], respectively. More literature on the neutrosophic numbers are disclosed in Table 2.

2.2 Background on Uncertain Equation

In this section, we discuss different papers on fuzzy equations. We also notice here that these fuzzy equations are solved using different fuzzy numbers, such as pentagonal fuzzy numbers, intuitionistic fuzzy numbers, neutrosophic numbers, etc. To de-marking the ambiguity in different mathematical modelling, fuzzy equations are generally studied in fuzzy logic and control systems. Zadeh's major work on fuzzy sets and its improvements in uncertain arithmetic and systems play an important role in it. Applications of fuzzy equations are now spread in various fields, i.e., optimisation, decision-making and artificial intelligence. A short literature review on uncertain equations with the help of different types of fuzzy numbers is noted in Table 3.

3 Preliminary Ideology

This section talks about fuzzy sets, fuzzy numbers, different properties of fuzzy numbers, fuzzy functions and fuzzy equations in detail.

3.1 Fuzzy Set Theory

Zadeh [60] proposed the fuzzy set notion. Instead of having the single elements, the fuzzy set has ordered pair set elements. Also, rather than being completely in or out, each element can have various membership functions and belong to a fuzzy set.

Definition 1. Fuzzy Set
Let us choose $\mathscr{U}$ to be a universal set and $\tilde{\mathscr{F}}$ to be a fuzzy set [61] defined on it. Then, the fuzzy set $\tilde{\mathscr{F}}$ defined as,

$$\tilde{\mathscr{F}} = \{(b, \mu_{\tilde{\mathscr{F}}}) : b \in \mathscr{U}\}, \tag{1}$$

where, $b \in \mathscr{U}$ be arbitrary element and $\mu_{\tilde{\mathscr{F}}} : \mathscr{U} \to [0, 1]$ be a membership function.

Example 1. Now, we consider a real-life oriented problem. We want to estimate whether a day is warm or not. Let, the reference set of days is $\mathscr{U} = \{b_1, b_2, b_3, b_4, b_5\}$. Using "warm" as a fuzzy term, let $\tilde{\mathscr{F}}$ be the fuzzy set of "warm" days. Since simply saying "warm" days does not express how "warm" a day is, it can be considered a fuzzy set. Then, the fuzzy set is, $\tilde{\mathscr{F}} = \{(b_1, 0.5), (b_2, 0.6), (b_3, 1), (b_4, 0.8), (b_5, 0.7)\}$, where the b_1 day is warm of 0.5 range, b_2 day is warm of 0.6 range and so on.

3.2 Fuzzy Number

A fuzzy number [62] is a type of value used in fuzzy logic, which allows for more flexibility compared to crisp numbers in uncertain environments. It can be distinguished by a membership function. Fuzzy numbers are specially used to model on vague environments like artificial intelligence, decision-making and control processes. It can handle uncertainty, as opposed to crisp numbers, which makes it helpful in challenging various real-world problems where accuracy is difficult to get.

Definition 2. Fuzzy Number
A fuzzy set $\tilde{\mathscr{F}}$ on the set of real numbers $\mathbb{R}$ is called to be a fuzzy number [63, 64] if it fulfills the properties listed below,

(i) $\tilde{\mathscr{F}}$ is normal and here for at least one point, the membership value should be 1 of it.
(ii) The membership function of the fuzzy set needs to be pairwise continuous.
(iii) $0-$cut of a fuzzy set, the support of $\tilde{\mathscr{F}}$, ${}^{0}\tilde{\mathscr{F}} = \{b; \mu_{\tilde{\mathscr{F}}(b)} \geq 0\}$ must be bounded.
(iv) ${}^{\alpha}\tilde{\mathscr{F}} = \{b : \mu_{\tilde{\mathscr{F}}}(b) \geq \alpha\}$ should be a closed interval for every $\alpha \in (0, 1]$.
(v) The set $\tilde{\mathscr{F}}$ must be a convex fuzzy set.

Example 2. Let us choose, Sima is a "good" girl in the class. This word alone can never express how "good" Sima is or can't fully describe this word. So, "good" differs from person to person. Then, we can apply the fuzzy number concept with select this as an object. So, from 0 ("Not good" in fuzzy concept) to 1 ("good" in fuzzy concept) range be the membership function of the above stated object.

Remark 1. In fuzzy sets, the range from 0 to 1 is the degree of membership and it indicates this numeric value in fuzzy numbers.

3.3 α-Cut of the Fuzzy Number (or, Parametric Form of the Fuzzy Number)

Suppose, $\tilde{\mathscr{T}} = (x, y, z)$ be a triangular fuzzy number. We know that the $\alpha-$cut form [65] is known as the parametric form of a fuzzy number. It is presented with an ordered pair of functions, i.e.,

$$\tilde{\mathscr{T}} = \left\{\left[\tilde{\mathscr{T}}_L(\alpha), \tilde{\mathscr{T}}_R(\alpha)\right]\right\} \tag{2}$$

Eq. (2) can also be explained in the following way, s.t.,

$$\left\{\left[\tilde{\mathscr{T}}_L(\alpha), \tilde{\mathscr{T}}_R(\alpha)\right]\right\} = [(y - x)\alpha + x, -(z - y)\alpha + z] \tag{3}$$

where, $\alpha \in [0, 1]$; $\tilde{\mathscr{T}}_L(\alpha)$ and $\tilde{\mathscr{T}}_R(\alpha)$ are the left continuous non-decreasing and right continuous non-increasing function over $[0, 1]$; $\tilde{\mathscr{T}}_L(\alpha) \leq \tilde{\mathscr{T}}_R(\alpha)$ when, $0 \leq \alpha \leq 1$.

3.4 Neutrosophic Fuzzy Number (NFN)

Definition 3. Neutrosophic Fuzzy Number (NFN)
Let us choose $\mathbb{R}$ to be a universal set and $\tilde{\mathscr{A}}$ to be a single valued NFN [66] defined on it. Then, the NFN $\tilde{\mathscr{A}}$ defined as,

$$\tilde{\mathscr{A}} = \{(b, \mathcal{T}_{\tilde{\mathscr{A}}}(b), \mathcal{I}_{\tilde{\mathscr{A}}}(b), \mathcal{F}_{\tilde{\mathscr{A}}}(b)) : b \in \mathbb{R}\}, \tag{4}$$

where, $\mathcal{T}_{\tilde{\mathscr{A}}}(b)$ be the degree of membership, $\mathcal{I}_{\tilde{\mathscr{A}}}(b)$ be the degree of indeterministic and $\mathcal{F}_{\tilde{\mathscr{A}}}(b)$ be the degree of non-membership function, where, $\mathcal{T}_{\tilde{\mathscr{A}}}(b), \mathcal{I}_{\tilde{\mathscr{A}}}(b), \mathcal{F}_{\tilde{\mathscr{A}}}(b) : \mathbb{R} \to [0, 1]$ and an element $b \in \mathbb{R}$. And, the main condition is that, $0 \leq \mathcal{T}_{\tilde{\mathscr{A}}}(b) + \mathcal{I}_{\tilde{\mathscr{A}}}(b) + \mathcal{F}_{\tilde{\mathscr{A}}}(b) \leq 1$.

A SVNFS $\tilde{\mathscr{A}}$ on the set of real numbers $\mathbb{R}$ is called a NFN if it satisfies the following properties,

(i) $\tilde{\mathscr{A}}$ is normal if $\exists\, b_0 \in \mathbb{R}$, i.e., $\mathcal{T}_{\tilde{\mathscr{A}}}(b_0) = 1$, where $\mathcal{I}_{\tilde{\mathscr{A}}}(b_0) = \mathcal{F}_{\tilde{\mathscr{A}}}(b_0) = 0$.

(ii) $\tilde{\mathscr{A}}$ is convex set for membership function $\mathcal{T}_{\tilde{\mathscr{A}}}(b)$, such that, $\mathcal{T}_{\tilde{\mathscr{A}}}(\Lambda b_1 + (1-\Lambda) b_2) \geq \min(\mathcal{T}_{\tilde{\mathscr{A}}}(b_1), \mathcal{T}_{\tilde{\mathscr{A}}}(b_2)) \forall \Lambda \in [0, 1]$ and $b_1, b_2 \in \mathbb{R}$.

(iii) $\tilde{\mathscr{A}}$ is convex set for indeterministic function $\mathcal{I}_{\tilde{\mathscr{A}}}(b)$, such that, $\mathcal{I}_{\tilde{\mathscr{A}}}(\Lambda b_1 + (1 - \Lambda) b_2) \leq \min(\mathcal{I}_{\tilde{\mathscr{A}}}(b_1), \mathcal{I}_{\tilde{\mathscr{A}}}(b_2)) \forall \Lambda \in [0, 1]$ and $b_1, b_2 \in \mathbb{R}$.

(iv) $\tilde{\mathscr{A}}$ is convex set for non-membership function $\mathcal{F}_{\tilde{\mathscr{A}}}(b)$, such that, $\mathcal{F}_{\tilde{\mathscr{A}}}(\Lambda b_1 + (1 - \Lambda) b_2) \leq \min(\mathcal{F}_{\tilde{\mathscr{A}}}(b_1), \mathcal{F}_{\tilde{\mathscr{A}}}(b_2)) \forall \Lambda \in [0, 1]$ and $b_1, b_2 \in \mathbb{R}$.

Remark 2. NFN can also be concave if the degree of membership function, the degree of indeterministic function and the degree of non-membership function, i.e., $\mathcal{T}_{\tilde{\mathscr{A}}}(b)$, $\mathcal{I}_{\tilde{\mathscr{A}}}(b)$ and $\mathcal{F}_{\tilde{\mathscr{A}}}(b)$ be concave. But in this paper, we work on NFN with a convex concept.

3.5 (α, β, γ)-Cut of Two NFN (or, Parametric From of Two NFN)

Let, $\tilde{\Delta} = \{(b, \mathcal{T}_{\tilde{\Delta}}(b), \mathcal{I}_{\tilde{\Delta}}(b), \mathcal{F}_{\tilde{\Delta}}(b)) : b \in \mathbb{R}\}$ and $\tilde{\Lambda} = \{(b, \mathcal{T}_{\tilde{\Lambda}}(b), \mathcal{I}_{\tilde{\Lambda}}(b), \mathcal{F}_{\tilde{\Lambda}}(b)) : b \in \mathbb{R}\}$ be two neutrosophic numbers and the parametric form [67] of these are,

$$\tilde{\Delta} = \left\{\left[\tilde{\Delta}_L(\alpha), \tilde{\Delta}_R(\alpha)\right], \left[\tilde{\Delta}_L(\beta), \tilde{\Delta}_R(\beta)\right], \left[\tilde{\Delta}_L(\gamma), \tilde{\Delta}_R(\gamma)\right]; \alpha, \beta, \gamma \in [0, 1]\right\} \tag{5}$$

and

$$\tilde{\Lambda} = \left\{\left[\tilde{\Lambda}_L(\alpha), \tilde{\Lambda}_R(\alpha)\right], \left[\tilde{\Lambda}_L(\beta), \tilde{\Lambda}_R(\beta)\right], \left[\tilde{\Lambda}_L(\gamma), \tilde{\Lambda}_R(\gamma)\right]; \alpha, \beta, \gamma \in [0, 1]\right\} \tag{6}$$

where, the three variables α, β, γ are mainly used for membership, indeterministic and non-membership functions.

Eqs. (5) and (6) satisfies the following conditions, s.t.,

(i) $\tilde{\Delta}_L(\alpha), \tilde{\Delta}_L(\beta), \tilde{\Delta}_L(\gamma)$; $\tilde{\Lambda}_L(\alpha), \tilde{\Lambda}_L(\beta), \tilde{\Lambda}_L(\gamma)$ are left continuous non-decreasing function and they are bounded over $[0, 1]$.

(ii) $\tilde{\Delta}_R(\alpha), \tilde{\Delta}_R(\beta), \tilde{\Delta}_R(\gamma)$; $\tilde{\Lambda}_R(\alpha), \tilde{\Lambda}_R(\beta), \tilde{\Lambda}_R(\gamma)$ are right continuous non-increasing function and they are bounded over $[0, 1]$.

(iii) $\tilde{\Delta}_L(\alpha) \leq \tilde{\Delta}_R(\alpha), \tilde{\Delta}_L(\beta) \leq \tilde{\Delta}_R(\beta), \tilde{\Delta}_L(\gamma) \leq \tilde{\Delta}_R(\gamma)$ and $\tilde{\Lambda}_L(\alpha) \leq \tilde{\Lambda}_R(\alpha), \tilde{\Lambda}_L(\beta) \leq \tilde{\Lambda}_R(\beta), \tilde{\Lambda}_L(\gamma) \leq \tilde{\Lambda}_R(\gamma)$ where, $0 \leq \alpha, \beta, \gamma \leq 1$.

3.6 Distance Between Two Neutrosophic Fuzzy Numbers in Parametric Form

The distance [68] between two neutrospheric fuzzy numbers, already mentioned in Subsection 3.5, is given here in parametric form,

$$\mathscr{D}(\tilde{\Delta}, \tilde{\Lambda}) = \sup_{0 \leq \alpha, \beta, \gamma \leq 1} \left\{d\left(\tilde{\Delta}(\alpha), \tilde{\Lambda}(\alpha)\right), d\left(\tilde{\Delta}(\beta), \tilde{\Lambda}(\beta)\right), d\left(\tilde{\Delta}(\gamma), \tilde{\Lambda}(\gamma)\right)\right\} \tag{7}$$

So,

$$\mathscr{D}(\tilde{\Delta},\tilde{\Lambda}) = \sup_{0\le\alpha,\beta,\gamma;\delta\le1} \Big[d\Big\{ \Big(\tilde{\Delta}_R(\alpha)-\tilde{\Delta}_L(\alpha)\Big)\delta+\tilde{\Delta}_L(\alpha), \Big(\tilde{\Lambda}_R(\alpha)-\tilde{\Lambda}_L(\alpha)\Big)\delta+\tilde{\Lambda}_L(\alpha)\Big\}, \\ d\Big\{ \Big(\tilde{\Delta}_R(\beta)-\tilde{\Delta}_L(\beta)\Big)\delta+\tilde{\Delta}_L(\beta), \Big(\tilde{\Lambda}_R(\beta)-\tilde{\Lambda}_L(\beta)\Big)\delta+\tilde{\Lambda}_L(\beta)\Big\}, \\ d\Big\{ \Big(\tilde{\Delta}_R(\gamma)-\tilde{\Delta}_L(\gamma)\Big)\delta+\tilde{\Delta}_L(\gamma), \Big(\tilde{\Lambda}_R(\gamma)-\tilde{\Lambda}_L(\gamma)\Big)\delta+\tilde{\Lambda}_L(\gamma)\Big\}\Big] \tag{8}$$

and by the Eq. (8), we get,

$$d\left(\tilde{\Delta}(\iota),\tilde{\Lambda}(\iota)\right) = \max\left(\mid \tilde{\Delta}_L(\iota),\tilde{\Lambda}_L(\iota)\mid, \mid \tilde{\Delta}_R(\iota),\tilde{\Lambda}_R(\iota)\mid\right) \tag{9}$$

where, $\iota = \alpha,\beta,\gamma;\delta$.

3.7 Triangular Neutrosophic Fuzzy Number (TNFN)

A triangular neutrosophic fuzzy number (TNFN) [69] $\tilde{\Delta} = \langle (b, \mathcal{T}_{\tilde{\Delta}}(b), \mathcal{I}_{\tilde{\Delta}}(b), \mathcal{F}_{\tilde{\Delta}}(b))$ $(p_1,p_2,p_3;q_1,q_2,q_3;r_1,r_2,r_3)\rangle$ is a subset of NFN in $\mathbb{R}$ with the base of convex membership, indeterministic and non-membership function which is denoted by,

$$\mathcal{T}_{\tilde{\Delta}}(b) = \begin{cases} \frac{b-p_1}{p_2-p_1} & ;\text{ when } p_1 \le b < p_2 \\ 1 & ;\text{ when } b = p_2 \\ \frac{p_3-b}{p_3-p_2} & ;\text{ when } p_2 < b \le p_3 \\ 0 & ;\text{ otherwise} \end{cases} \tag{10}$$

$$\mathcal{I}_{\tilde{\Delta}}(b) = \begin{cases} \frac{q_2-b}{q_2-q_1} & ;\text{ when } q_1 \le b < q_2 \\ 0 & ;\text{ when } b = q_2 \\ \frac{b-q_2}{q_3-q_2} & ;\text{ when } q_2 < b \le q_3 \\ 1 & ;\text{ otherwise} \end{cases} \tag{11}$$

$$\mathcal{F}_{\tilde{\Delta}}(b) = \begin{cases} \frac{r_2-b}{r_2-r_1} & ;\text{ when } r_1 \le b < r_2 \\ 0 & ;\text{ when } b = r_2 \\ \frac{b-r_2}{r_3-r_2} & ;\text{ when } r_2 < b \le r_3 \\ 1 & ;\text{ otherwise} \end{cases} \tag{12}$$

where, $p_1 \le p_2 \le p_3$, $q_1 \le q_2 \le q_3$, $r_1 \le r_2 \le r_3$, $0 \le \mathcal{T}_{\tilde{\Delta}}(b) + \mathcal{I}_{\tilde{\Delta}}(b) + \mathcal{F}_{\tilde{\Delta}}(b) \le 3$ and $b \in \mathbb{R}$.

3.8 (α,β,γ)-Cut of TNFN

The parametric form of NFN is actually the (α,β,γ)- cut of NFN and this is a parametric representation of the uncertain number, which is represented in a classical way. In (α,β,γ)-cut of above Subsection 3.7 TNFN $\tilde{\Delta}_{\alpha\beta\gamma} = [\tilde{\Delta}_L(\alpha),\tilde{\Delta}_R(\alpha);\tilde{\Delta}_L(\beta),\tilde{\Delta}_R(\beta);\tilde{\Delta}_L(\gamma),\tilde{\Delta}_R(\gamma)]$ is described as follows.

For the membership function, the left continuous non-decreasing bounded function obtained from Eq. (10) is,

$$\alpha = \frac{b-p_1}{p_2-p_1} \\ \text{or, } b = p_1 + \alpha(p_2-p_1) \tag{13}$$

and the right continuous non-increasing bounded function is,

$$\alpha = \frac{p_3-b}{p_3-p_2} \\ \text{or, } b = p_3 - \alpha(p_3-p_2) \tag{14}$$

For the indeterministic function, the left continuous non-decreasing bounded function obtained from Eq. (11) is,

$$\beta = \frac{q_2-b}{q_2-q_1} \\ \text{or, } b = q_2 - \beta(q_2-q_1) \tag{15}$$

and the right continuous non-increasing bounded function is,

$$\begin{aligned}\beta =&\frac{b-q_2}{q_3-q_2}\\ \text{or, } b =&q_2+\beta(q_3-q_2)\end{aligned} \tag{16}$$

Lastly, for the non-membership function, the left continuous non-decreasing bounded function obtained from Eq. (12) is,

$$\begin{aligned}\gamma =&\frac{r_2-b}{r_2-r_1}\\ \text{or, } b =&r_2-\gamma(r_2-r_1)\end{aligned} \tag{17}$$

and the right continuous non-increasing bounded function is,

$$\begin{aligned}\gamma =&\frac{b-r_2}{r_3-r_2}\\ \text{or, } b =&r_2+\gamma(r_3-r_2)\end{aligned} \tag{18}$$

Here, $b \in \mathbb{R}$ and the three variables α, β, γ are used for membership, indeterministic and non-membership functions for the TNFN. So, we can define the above equations are,

$$\begin{aligned}\tilde{\Delta}_L(\alpha) =&p_1+\alpha(p_2-p_1)\\ \tilde{\Delta}_R(\alpha) =&p_3-\alpha(p_3-p_2)\\ \tilde{\Delta}_L(\beta) =&q_2-\beta(q_2-q_1)\\ \tilde{\Delta}_R(\beta) =&q_2+\beta(q_3-q_2)\\ \tilde{\Delta}_L(\gamma) =&r_2-\gamma(r_2-r_1)\\ \tilde{\Delta}_R(\gamma) =&r_2+\gamma(r_3-r_2)\end{aligned} \tag{19}$$

where, $0 < \alpha, \beta, \gamma \leq 1$ and $0 < \alpha + \beta + \gamma \leq 3$.

3.9 Interval Arithmetic Concept

This section discussed interval arithmetic in detail. Guerra and Stefanini [70] and Quevedo [71] discuss the arithmetic operations on intervals [64]. Basic operations on intervals are slightly changed compared with crisp arithmetic.

Consider an interval $I = [a_L, a_R]$, where $a_L \leq a_R$, then we calculate the midpoint representation value as,

$$\hat{a} = \frac{a_R + a_L}{2} \text{ and } \bar{a} = \frac{a_R - a_L}{2} \tag{20}$$

then the interval I can be represented as follows:

$$a_L = \hat{a} - \bar{a} \text{ and } a_R = \hat{a} + \bar{a} \tag{21}$$

and

$$I = [a_L, a_R] = [\hat{a} - \bar{a}, \hat{a} + \bar{a}] \tag{22}$$

Further, the interval can be written as $I = [a_L, a_R] = (\hat{a}; \bar{a})$. The set of all real intervals can be represented as $\mathbb{IR}$ and in short, $\mathbb{I}$.

Assume, $P = [c_L, c_R] = (\hat{c}; \bar{c})$ and $Q = [d_L, d_R] = (\hat{d}; \bar{d})$, where $\hat{c} = \frac{c_R+c_L}{2}$, $\bar{c} = \frac{c_R-c_L}{2}$, $\hat{d} = \frac{d_R+d_R}{2}$ and $\bar{d} = \frac{d_R-d_L}{2}$, respectively. Then, the basic arithmetic operations on intervals are determined as,

1. Addition of two intervals P and Q:

$$P + Q = [c_L, c_R] + [d_L, d_R] = [c_L + d_L, c_R + d_R] = (\hat{c} + \hat{d}; \bar{c} + \bar{d}) \tag{23}$$

2. Subtraction of two intervals P and Q:

$$P - Q = [c_L, c_R] - [d_L, d_R] = [c_L - d_R, c_R - d_L] = (\hat{c} - \hat{d}; \bar{c} - \bar{d}) \tag{24}$$

3. Scalar multiplication of interval P:

$$\lambda P = \lambda \times P = \lambda \times [c_L, c_R] = \begin{cases} [\lambda c_L, \lambda c_R] & ;\text{ if } \lambda \geq 0 \\ [\lambda c_R, \lambda c_L] & ;\text{ if } \lambda < 0 \end{cases} \\ = (\lambda \hat{c}; |\lambda|\bar{c}) \tag{25}$$

where, λ is a scalar number.

4. Multiplication of two intervals P and Q:

$$P \times Q = [c_L, c_R] \times [d_L, d_R] = [e_L, e_R] = (\hat{e}; \bar{e}) \tag{26}$$

where, $e_L = \min\{c_L d_L, c_L d_R, c_R d_L, c_R d_R\}$ and $e_R = \max\{c_L d_L, c_L d_R, c_R d_L, c_R d_R\}$, $\hat{e} = \frac{e_R + e_L}{2}$ and $\bar{e} = \frac{e_R - e_L}{2}$, respectively.

5. Division of two intervals P and Q:

$$P \div Q = [c_L, c_R] \div [d_L, d_R] = [e_L, e_R] = (\hat{e}; \bar{e}) \tag{27}$$

where, $e_L = \min\{\frac{c_L}{d_L}, \frac{c_L}{d_R}, \frac{c_R}{d_L}, \frac{c_R}{d_R}\}$ and $e_R = \max\{\frac{c_L}{d_L}, \frac{c_L}{d_R}, \frac{c_R}{d_L}, \frac{c_R}{d_R}\}$, $\hat{e} = \frac{e_R + e_L}{2}$ and $\bar{e} = \frac{e_R - e_L}{2}$, respectively. The decision of two intervals only possible when 0 not in Q, i.e., $0 \notin [d_L, d_R] (= Q)$.

3.10 Hukuhara Difference Between Two Neutrosophic Number

Consider that, $\tilde{\Delta}$, $\tilde{\Lambda}$ be two neutrosophic numbers. And, the Hukuhara difference [72] between these two numbers is denoted by $\tilde{\xi}$. So, the Hukuhara difference occurs when, $\tilde{\Delta} = \tilde{\Lambda} + \tilde{\xi}$. Then, it is represented as,

$$\begin{cases} \tilde{\Delta}_L(\alpha) = \tilde{\Lambda}_L(\alpha) + \tilde{\xi}_L(\alpha) \\ \tilde{\Delta}_R(\alpha) = \tilde{\Lambda}_R(\alpha) + \tilde{\xi}_R(\alpha) \\ \tilde{\Delta}_L(\beta) = \tilde{\Lambda}_L(\beta) + \tilde{\xi}_L(\beta) \\ \tilde{\Delta}_R(\beta) = \tilde{\Lambda}_R(\beta) + \tilde{\xi}_R(\beta) \\ \tilde{\Delta}_L(\gamma) = \tilde{\Lambda}_L(\gamma) + \tilde{\xi}_L(\gamma) \\ \tilde{\Delta}_R(\gamma) = \tilde{\Lambda}_R(\gamma) + \tilde{\xi}_R(\gamma) \end{cases} \tag{28}$$

Now, we get from Eq. (28),

$$\begin{cases} \tilde{\xi}_L(\alpha) = \tilde{\Delta}_L(\alpha) - \tilde{\Lambda}_L(\alpha) \\ \tilde{\xi}_R(\alpha) = \tilde{\Delta}_R(\alpha) - \tilde{\Lambda}_R(\alpha) \\ \tilde{\xi}_L(\beta) = \tilde{\Delta}_L(\beta) - \tilde{\Lambda}_L(\beta) \\ \tilde{\xi}_R(\beta) = \tilde{\Delta}_R(\beta) - \tilde{\Lambda}_R(\beta) \\ \tilde{\xi}_L(\gamma) = \tilde{\Delta}_L(\gamma) - \tilde{\Lambda}_L(\gamma) \\ \tilde{\xi}_R(\gamma) = \tilde{\Delta}_R(\gamma) - \tilde{\Lambda}_R(\gamma) \end{cases} \tag{29}$$

where, $\forall \alpha, \beta, \gamma \in [0, 1]$. Mathematically, the Hukuhara difference can be expressed as,

$$\tilde{\xi} = \tilde{\Delta} \ominus_{\mathscr{H}} \tilde{\Lambda} \tag{30}$$

3.11 Characterisation Theorem for Neutrosophic Function and It's Operation

Choose that, the neutrosophic fuzzy equation of the form [73],

$$\tilde{x} = \tilde{f}(\tilde{x}) \tag{31}$$

where, f: $\mathscr{E} \times \mathbb{Z} \to \mathscr{E}$ and $\mathscr{E}$ be the set of all fuzzy function. Now, two forms of fuzzy function using Hukuhara difference are described below, s.t.,

(a) The parametric form of the fuzzy neutrosophic function is,
$\tilde{f}(x)_{\alpha,\beta,\gamma} = \{[f_L[x_L(\alpha), x_R(\alpha), \alpha], f_R[x_L(\alpha), x_R(\alpha), \alpha]],$
$[f_L[x_L(\beta), x_R(\beta), \beta], f_R[x_L(\beta), x_R(\beta), \beta]], \ [f_L[x_L(\gamma), x_R(\gamma), \gamma], f_R[x_L(\gamma), x_R(\gamma), \gamma]]\}$.

(b) The functions $[f_L[x_L(\alpha), x_R(\alpha), \alpha], f_R[x_L(\alpha), x_R(\alpha), \alpha]]; [f_L[x_L(\beta), x_R(\beta), \beta], f_R[x_L(\beta), x_R(\beta), \beta]]$ and $[f_L[x_L(\gamma), x_R(\beta), \gamma], f_R[x_L(\gamma), x_R(\gamma), \gamma]]$ are taken as continuous functions, s.t.,
$\|f_L[x_L(\alpha), x_R(\alpha), \alpha] - f_L[x_L(n_1 + 1)(\alpha), x_R(\alpha), \alpha]\| < \eta_1$ and
$\|f_L[x_L(\beta), x_R(\beta), \beta] - f_R[x_L(\beta), x_R(\beta), \beta]\| < \eta_2$ and
$\|f_L[x_L(\gamma), x_R(\gamma), n, \gamma] - f_L[x_L(\gamma), x_R(\gamma), \gamma]\| < \eta_2$, where, $\forall \alpha, \beta, \gamma \in [0, 1]$ and $\eta_1, \eta_2, \eta_3 > 0$ with

$\|(x_L(\alpha), x_R(\alpha)) - (x_L(\alpha), x_R(\alpha))\| < \delta_1$ and $\|(x_L(\beta), x_R(\beta)) - (x_L(\beta), x_R(\beta))\| < \delta_2$ and $\|(x_L(\gamma), x_R(\gamma)) - (x_L(\gamma), x_R(\gamma))\| < \delta_3$, where, $\delta_1, \delta_2, \delta_3 > 0$.
Then, for $\forall \alpha, \beta, \gamma \in [0,1]$ and $\eta_4, \eta_5, \eta_6 > 0$. So, $\|f_R[x_L(\alpha), x_R(\alpha), n, \alpha] - f_R[x_L(\alpha), x_R(\alpha)]\| < \eta_4$ and $\|f_R[x_L(\beta), x_R(\beta), \beta] - f_R[x_L(\beta), x_R(\beta)]\| < \eta_5$ and $\|f_R[x_L(\gamma), x_R(\gamma), \gamma] - f_R[x_L(\gamma), x_R(\gamma)]\| < \eta_6$ with $\|(x_L(\alpha), x_R(\alpha)) - (x_L(\alpha), x_R(\alpha))\| < \delta_4$ and $\|(x_L(\beta), x_R(\beta)) - (x_L\beta), x_R(\beta))\| < \delta_5$ and $\|(x_L(\gamma), x_R(\gamma)) - (x_L(\gamma), x_R(\gamma))\| < \delta_6$ where $\delta_4, \delta_5, \delta_6 > 0$.

Therefore, the fuzzy Eq. (31) reduces the system of six fuzzy equations, i.e.,

$$\begin{aligned} x_L(\alpha) =& f_L[x_L(\alpha), x_R(\alpha), \alpha] \\ x_R(\alpha) =& f_R[x_L(\alpha), x_R(\alpha), \alpha] \end{aligned} \tag{32}$$

and

$$\begin{aligned} x_L(\beta) =& f_L[x_L(\beta), x_R(\beta), \beta] \\ x_R(\beta) =& f_R[x_L(\beta), x_R(\beta), \beta] \end{aligned} \tag{33}$$

and

$$\begin{aligned} x_L(\gamma) =& f_L[x_L(\gamma), x_R(\gamma), \gamma] \\ x_R(\gamma) =& f_R[x_L(\gamma), x_R(\gamma), \gamma] \end{aligned} \tag{34}$$

4 Neutrosophic Fuzzy Equation and Its Solution

In this section, we consider one linear equation in three different forms and evaluate its solutions using neutrosophic numbers. Let us consider the three forms of linear equations as follows:

$$\mathcal{A}\mathcal{X} + \mathcal{B} = \mathcal{C} \tag{35}$$

$$\mathcal{A}\mathcal{X} = \mathcal{C} - \mathcal{B} \tag{36}$$

$$\mathcal{A}\mathcal{X} - \mathcal{C} = -\mathcal{B} \tag{37}$$

Remark 3. It is obvious that all of the above three linear equations are the same in the crisp environment. So, their solution should be the same and the solution is $\mathcal{X} = \frac{\mathcal{C}-\mathcal{B}}{\mathcal{A}}$. But, if we consider the fuzzy or intuitionistic fuzzy or neutrosophic coefficient, then the solution of the above three equations may not be the same. In this section, we determine the solutions of the above three equations in the neutrosophic environment in detail.

4.1 Neutrosophic Equation (Type 1)

In this section, we determine the solution of Eq. (35) in the neutrosophic field, i.e., all the coefficients are neutrosophic numbers. Then, we replace the crisp coefficients A, B and C with neutrosophic coefficient $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$, respectively, in Eq. (35). Then the solution must be a neutrosophic number, let $\tilde{\mathcal{X}}$ instead of X and the equation becomes neutrosophic equation, as

$$\tilde{\mathcal{A}}\tilde{\mathcal{X}} + \tilde{\mathcal{B}} = \tilde{\mathcal{C}} \tag{38}$$

and corresponding $(\alpha, \beta, \gamma)-$cut of Eq. (38) is

$$\begin{aligned} &\{[\mathcal{A}_L(\alpha), \mathcal{A}_R(\alpha)], [\mathcal{A}_L(\beta), \mathcal{A}_R(\beta)], [\mathcal{A}_L(\gamma), \mathcal{A}_R(\gamma)]\}\{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{W}_L(\beta), \mathcal{W}_R(\beta)], \\ &[\mathcal{W}_L(\gamma), \mathcal{W}_R(\gamma)]\} + \{[\mathcal{B}_L(\alpha), \mathcal{B}_R(\alpha)], [\mathcal{B}_L(\beta), \mathcal{B}_R(\beta)], [\mathcal{B}_L(\gamma), \mathcal{B}_R(\gamma)]\} \\ &= \{[\mathcal{C}_L(\alpha), \mathcal{C}_R(\alpha)], [\mathcal{C}_L(\beta), \mathcal{C}_R(\beta)], [\mathcal{C}_L(\gamma), \mathcal{C}_R(\gamma)]\} \end{aligned} \tag{39}$$

Remark 4. Since the coefficient, non-coefficient and non-homogeneous parts are neutrosophic fuzzy values, then we consider the solution $\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$ to be also neutrosophic fuzzy in nature.

Here, we take the concept of Hukuhara difference and characterisation theorem with Eq. (39) and convert it as

$$\begin{cases} \min\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} + \mathcal{B}_L(\alpha) = \mathcal{C}_L(\alpha) \\ \max\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} + \mathcal{B}_R(\alpha) = \mathcal{C}_R(\alpha) \end{cases} \tag{40}$$

$$\begin{cases} \min\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} + \mathcal{B}_L(\beta) = \mathcal{C}_L(\beta) \\ \max\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\alpha), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} + \mathcal{B}_R(\beta) = \mathcal{C}_R(\beta) \end{cases} \tag{41}$$

$$\begin{cases} \min\left\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\right\} + \mathcal{B}_L(\gamma) = \mathcal{C}_L(\gamma) \\ \max\left\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\right\} + \mathcal{B}_R(\gamma) = \mathcal{C}_R(\gamma) \end{cases} \tag{42}$$

After applying the interval arithmetic described in paper [74] and we already know that $0 \leq \alpha, \beta, \gamma \leq 1$. Here, we consider $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are always positive $\forall \alpha \in [0,1]$. Now, using Interval arithmetic on Eq. (40), we get

$$\begin{cases} \mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha) + \mathcal{B}_L(\alpha) = \mathcal{C}_L(\alpha) \\ \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha) + \mathcal{B}_R(\alpha) = \mathcal{C}_R(\alpha) \end{cases} \tag{43}$$

where, $[\min\left\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\right\},$ $\max\left\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\right\}]$ is $[\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)]$.

Further, we consider $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are always positive $\forall \beta \in [0,1]$. Then, form Eq. (41) using interval arithmetic, we get

$$\begin{cases} \mathcal{A}_L(\beta)\mathcal{Y}_L(\beta) + \mathcal{B}_L(\beta) = \mathcal{C}_L(\beta) \\ \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta) + \mathcal{B}_R(\beta) = \mathcal{C}_R(\beta) \end{cases} \tag{44}$$

where, $[\min\left\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\right\},$ $\max\left\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\alpha), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\right\}]$ is $[\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)]$.

Simultaneously, considering $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are always positive $\forall \gamma \in [0,1]$ and form Eq. (42) utilizing interval arithmetic, we obtain

$$\begin{cases} \mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma) + \mathcal{B}_L(\gamma) = \mathcal{C}_L(\gamma) \\ \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma) + \mathcal{B}_R(\gamma) = \mathcal{C}_R(\gamma) \end{cases} \tag{45}$$

where, $[\min\left\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\right\},$ $\max\left\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\right\}]$ is $[\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)]$.

Now, from Eq. (43), we find

$$\begin{cases} \mathcal{W}_L(\alpha) = \frac{\mathcal{C}_L(\alpha) - \mathcal{B}_L(\alpha)}{\mathcal{A}_L(\alpha)} \\ \mathcal{W}_R(\alpha) = \frac{\mathcal{C}_R(\alpha) - \mathcal{B}_R(\alpha)}{\mathcal{A}_R(\alpha)} \end{cases} \tag{46}$$

In Eq. (46), $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) > 0$, then $\mathcal{W}_L(\alpha)$ be a monotonically increasing function of α and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) < 0$ then $\mathcal{W}_R(\alpha)$ be a monotonically decreasing function of α when $0 \leq \alpha \leq 1$. Then the membership part ($\alpha-$cut) of the solution of Eq. (39) is Eq. (46). If $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) < 0$ and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) > 0$ for all $\alpha \in [0,1]$ simultaneously, then $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are monotonically decreasing and increasing functions, respectively and $\mathcal{W}_L(\alpha) > \mathcal{W}_R(\alpha)$ for all $\alpha \in [0,1]$. Then the corrected solution of Eq. (46) is $[\mathcal{W}_L(\alpha)^*, \mathcal{W}_R(\alpha)^*] = [\min\left\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\right\}, \max\left\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\right\}] = [\mathcal{W}_R(\alpha), \mathcal{W}_L(\alpha)]$.

Similarly, from Eq. (44), we find

$$\begin{cases} \mathcal{Y}_L(\beta) = \frac{\mathcal{C}_L(\beta) - \mathcal{B}_L(\beta)}{\mathcal{A}_L(\beta)} \\ \mathcal{Y}_R(\beta) = \frac{\mathcal{C}_R(\beta) - \mathcal{B}_R(\beta)}{\mathcal{A}_R(\beta)} \end{cases} \tag{47}$$

In Eq. (47), $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) < 0$, then $\mathcal{Y}_L(\beta)$ be a monotonically decreasing function of β and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) > 0$ then $\mathcal{Y}_R(\beta)$ be a monotonically increasing function of β when $0 \leq \beta \leq 1$. Then the indeterminacy part ($\beta-$cut) of the solution of Eq. (39) is Eq. (47). If $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) > 0$ and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) < 0$ for all $\beta \in [0,1]$ simultaneously, then $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are monotonically increasing and decreasing functions, respectively and $\mathcal{Y}_L(\beta) > \mathcal{Y}_R(\beta)$ for all $\beta \in [0,1]$. Then the corrected solution of Eq. (47) is $[\mathcal{Y}_L(\beta)^*, \mathcal{Y}_R(\beta)^*] = [\min\left\{\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)\right\}, \max\left\{\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)\right\}] = [\mathcal{Y}_R(\beta), \mathcal{Y}_L(\beta)]$.

Further more, from Eq. (45), we find

$$\begin{cases} \mathcal{Z}_L(\gamma) = \frac{\mathcal{C}_L(\gamma) - \mathcal{B}_L(\gamma)}{\mathcal{A}_L(\gamma)} \\ \mathcal{Z}_R(\gamma) = \frac{\mathcal{C}_R(\gamma) - \mathcal{B}_R(\gamma)}{\mathcal{A}_R(\gamma)} \end{cases} \tag{48}$$

In Eq. (48), $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) < 0$, then $\mathcal{Z}_L(\gamma)$ be a monotonically decreasing function of γ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) > 0$ then $\mathcal{Z}_R(\gamma)$ be a monotonically increasing function of γ when $0 \leq \gamma \leq 1$. Then the non-membership part ($\gamma-$cut) of the solution of Eq. (39) is Eq. (48). If $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) > 0$ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) < 0$ for all $\gamma \in [0,1]$ simultaneously, then $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are monotonically increasing and decreasing functions, respectively and $\mathcal{Z}_L(\gamma) > \mathcal{Z}_R(\gamma)$ for all $\gamma \in [0,1]$. Then the corrected solution of Eq. (48) is $[\mathcal{Z}_L(\gamma)^*, \mathcal{Z}_R(\gamma)^*] = [\min\left\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\right\}, \max\left\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\right\}] = [\mathcal{Z}_R(\gamma), \mathcal{Z}_L(\gamma)]$.

The above solution exists when Eqs. (46)-(48) are satisfied the following conditions as follows:

(a) $\mathcal{W}_L(\alpha), \mathcal{Y}_R(\beta), \mathcal{Z}_R(\gamma)$ be a monotonically increasing functions of α, β, γ, respectively, when $0 \leq \alpha, \beta, \gamma \leq 1$.

(b) $\mathcal{W}_R(\alpha), \mathcal{Y}_L(\beta), \mathcal{Z}_L(\gamma)$ be a monotonically decreasing functions of α, β, γ, respectively, when $0 \leq \alpha, \beta, \gamma \leq 1$.
(c) $\mathcal{W}_L(\alpha) \leq \mathcal{W}_R(\alpha), \mathcal{Y}_L(\beta) \leq \mathcal{Y}_R(\beta)$ and $\mathcal{Z}_L(\gamma) \leq \mathcal{Z}_R(\gamma)$, for all α, β and γ, respectively.

Then the solution $\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$ of Eq. (38) is evaluated by Eqs. (46)-(48), respectively.

4.2 Neutrosophic Equation (Type 2)

In this section, we determine the solution of Eq. (36) in the neutrosophic field, i.e., all the coefficients are neutrosophic numbers. Then, we replace the crisp coefficients A, B and C with neutrosophic coefficient $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{C}}$, respectively, in Eq. (36). Then the solution must be a neutrosophic number, let $\tilde{\mathcal{X}}$ instead of X and the equation becomes neutrosophic equation, as

$$\tilde{\mathcal{A}}\tilde{\mathcal{X}} = \tilde{\mathcal{C}} - \tilde{\mathcal{B}} \tag{49}$$

and corresponding $(\alpha, \beta, \gamma)-$cut of Eq. (49) is

$$\begin{aligned} &\{[\mathcal{A}_L(\alpha), \mathcal{A}_R(\alpha)], [\mathcal{A}_L(\beta), \mathcal{A}_R(\beta)], [\mathcal{A}_L(\gamma), \mathcal{A}_R(\gamma)]\}\{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{W}_L(\beta), \mathcal{W}_R(\beta)], \\ &[\mathcal{W}_L(\gamma), \mathcal{W}_R(\gamma)]\} = \{[\mathcal{C}_L(\alpha), \mathcal{C}_R(\alpha)], [\mathcal{C}_L(\beta), \mathcal{C}_R(\beta)], [\mathcal{C}_L(\gamma), \mathcal{C}_R(\gamma)]\} \\ &- \{[\mathcal{B}_L(\alpha), \mathcal{B}_R(\alpha)], [\mathcal{B}_L(\beta), \mathcal{B}_R(\beta)], [\mathcal{B}_L(\gamma), \mathcal{B}_R(\gamma)]\} \end{aligned} \tag{50}$$

Remark 5. Since the coefficient, non-coefficient and non-homogeneous parts are neutrosophic fuzzy values, then we consider the solution $\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$ to be also neutrosophic fuzzy in nature.

Here, we consider the Hukuhara difference and characterisation theorem with Eq. (50) and convert it as

$$\begin{cases} \min\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} = \mathcal{C}_L(\alpha) - \mathcal{B}_R(\alpha) \\ \max\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} = \mathcal{C}_R(\alpha) - \mathcal{B}_L(\alpha) \end{cases} \tag{51}$$

and

$$\begin{cases} \min\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} = \mathcal{C}_L(\beta) - \mathcal{B}_R(\beta) \\ \max\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} = \mathcal{C}_R(\beta) - \mathcal{B}_L(\beta) \end{cases} \tag{52}$$

and

$$\begin{cases} \min\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\} = \mathcal{C}_L(\gamma) - \mathcal{B}_R(\gamma) \\ \max\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\} = \mathcal{C}_R(\gamma) - \mathcal{B}_L(\gamma) \end{cases} \tag{53}$$

After applying the interval arithmetic described in paper [74] and we already know that $0 \leq \alpha, \beta, \gamma \leq 1$. Here, we consider $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are always positive $\forall \alpha \in [0, 1]$. Now, using Interval arithmetic on Eq. (51), we get

$$\begin{cases} \mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha) = \mathcal{C}_L(\alpha) - \mathcal{B}_R(\alpha) \\ \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha) = \mathcal{C}_R(\alpha) - \mathcal{B}_L(\alpha) \end{cases} \tag{54}$$

where, $[\min\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\},$
$\max\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\}]$ is $[\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)]$.

Further, we consider $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are always positive $\forall \beta \in [0, 1]$. Then, form Eq. (52) using interval arithmetic, we get

$$\begin{cases} \mathcal{A}_L(\beta)\mathcal{Y}_L(\beta) = \mathcal{C}_L(\beta) - \mathcal{B}_R(\beta) \\ \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta) = \mathcal{C}_R(\beta) - \mathcal{B}_L(\beta) \end{cases} \tag{55}$$

where, $[\min\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\},$
$\max\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\alpha), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\}]$ is $[\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)]$.

Simultaneously, considering $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are always positive $\forall \gamma \in [0, 1]$ and form Eq. (53) utilizing interval arithmetic, we obtain

$$\begin{cases} \mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma) = \mathcal{C}_L(\gamma) - \mathcal{B}_R(\gamma) \\ \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma) = \mathcal{C}_R(\gamma) - \mathcal{B}_L(\gamma) \end{cases} \tag{56}$$

where, $[\min\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\},$
$\max\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\}]$ is $[\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)]$.

Now, from Eq. (54), we find

$$\begin{cases} \mathcal{W}_L(\alpha) = \frac{\mathcal{C}_L(\alpha) - \mathcal{B}_R(\alpha)}{\mathcal{A}_L(\alpha)} \\ \mathcal{W}_R(\alpha) = \frac{\mathcal{C}_R(\alpha) - \mathcal{B}_L(\alpha)}{\mathcal{A}_R(\alpha)} \end{cases} \tag{57}$$

In Eq. (57), $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) > 0$, then $\mathcal{W}_L(\alpha)$ be a monotonically increasing function of α and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) < 0$ then $\mathcal{W}_R(\alpha)$ be a monotonically decreasing function of α when $0 \leq \alpha \leq 1$. Then the membership part ($\alpha-$cut) of the solution of Eq. (50) is Eq. (57). If $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) < 0$ and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) > 0$ for all $\alpha \in [0,1]$ simultaneously, then $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are monotonically decreasing and increasing functions, respectively and $\mathcal{W}_L(\alpha) > \mathcal{W}_R(\alpha)$ for all $\alpha \in [0,1]$. Then the corrected solution of Eq. (57) is $[\mathcal{W}_L(\alpha)^*, \mathcal{W}_R(\alpha)^*] = [\min\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\}, \max\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\}] = [\mathcal{W}_R(\alpha), \mathcal{W}_L(\alpha)]$.

Similarly, from Eq. (55), we find

$$\begin{cases} \mathcal{Y}_L(\beta) = \frac{\mathcal{C}_L(\beta)-\mathcal{B}_R(\beta)}{\mathcal{A}_L(\beta)} \\ \mathcal{Y}_R(\beta) = \frac{\mathcal{C}_R(\beta)-\mathcal{B}_L(\beta)}{\mathcal{A}_R(\beta)} \end{cases} \tag{58}$$

In Eq. (58), $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) < 0$, then $\mathcal{Y}_L(\beta)$ be a monotonically decreasing function of β and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) > 0$ then $\mathcal{Y}_R(\beta)$ be a monotonically increasing function of β when $0 \leq \beta \leq 1$. Then the indeterminacy part ($\beta-$cut) of the solution of Eq. (50) is Eq. (58). If $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) > 0$ and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) < 0$ for all $\beta \in [0,1]$ simultaneously, then $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are monotonically increasing and decreasing functions, respectively and $\mathcal{Y}_L(\beta) > \mathcal{Y}_R(\beta)$ for all $\beta \in [0,1]$. Then the corrected solution of Eq. (58) is $[\mathcal{Y}_L(\beta)^*, \mathcal{Y}_R(\beta)^*] = [\min\{\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)\}, \max\{\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)\}] = [\mathcal{Y}_R(\beta), \mathcal{Y}_L(\beta)]$.

Further more, from Eq. (56), we find

$$\begin{cases} \mathcal{Z}_L(\gamma) = \frac{\mathcal{C}_L(\gamma)-\mathcal{B}_R(\gamma)}{\mathcal{A}_L(\gamma)} \\ \mathcal{Z}_R(\gamma) = \frac{\mathcal{C}_R(\gamma)-\mathcal{B}_L(\gamma)}{\mathcal{A}_R(\gamma)} \end{cases} \tag{59}$$

In Eq. (59), $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) < 0$, then $\mathcal{Z}_L(\gamma)$ be a monotonically decreasing function of γ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) > 0$ then $\mathcal{Z}_R(\gamma)$ be a monotonically increasing function of γ when $0 \leq \gamma \leq 1$. Then the non-membership part ($\gamma-$cut) of the solution of Eq. (50) is Eq. (59). If $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) > 0$ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) < 0$ for all $\gamma \in [0,1]$ simultaneously, then $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are monotonically increasing and decreasing functions, respectively and $\mathcal{Z}_L(\gamma) > \mathcal{Z}_R(\gamma)$ for all $\gamma \in [0,1]$. Then the corrected solution of Eq. (59) is $[\mathcal{Z}_L(\gamma)^*, \mathcal{Z}_R(\gamma)^*] = [\min\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\}, \max\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\}] = [\mathcal{Z}_R(\gamma), \mathcal{Z}_L(\gamma)]$.

The above solution exists when Eqs. (57)-(59) are satisfied the following conditions as follows:

(a) $\mathcal{W}_L(\alpha), \mathcal{Y}_R(\beta), \mathcal{Z}_R(\gamma)$ be a monotonically increasing functions of α, β, γ, respectively, when $0 \leq \alpha, \beta, \gamma \leq 1$.

(b) $\mathcal{W}_R(\alpha), \mathcal{Y}_L(\beta), \mathcal{Z}_L(\gamma)$ be a monotonically decreasing functions of α, β, γ, respectively, when $0 \leq \alpha, \beta, \gamma \leq 1$.

(c) $\mathcal{W}_L(\alpha) \leq \mathcal{W}_R(\alpha)$, $\mathcal{Y}_L(\beta) \leq \mathcal{Y}_R(\beta)$ and $\mathcal{Z}_L(\gamma) \leq \mathcal{Z}_R(\gamma)$, for all α, β and γ, respectively.

Then the solution $\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$ of Eq. (49) is evaluated by Eqs. (57)-(59), respectively.

4.3 Neutrosophic Equation (Type 3)

In this section, we determine the solution of Eq. (37) in the neutrosophic field, i.e., all the coefficients are neutrosophic numbers. Then, we replace the crisp coefficients A, B and C with neutrosophic coefficient $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{C}}$, respectively, in Eq. (37). Then the solution must be a neutrosophic number, let $\tilde{\mathcal{X}}$ instead of X and the equation becomes neutrosophic equation, as

$$\tilde{\mathcal{A}}\tilde{\mathcal{X}} - \tilde{\mathcal{C}} = -\tilde{\mathcal{B}} \tag{60}$$

and corresponding $(\alpha, \beta, \gamma)-$cut of Eq. (60) is

$$\begin{aligned} \{[\mathcal{A}_L(\alpha), \mathcal{A}_R(\alpha)], [\mathcal{A}_L(\beta), \mathcal{A}_R(\beta)], [\mathcal{A}_L(\gamma), \mathcal{A}_R(\gamma)]\} \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{W}_L(\beta), \mathcal{W}_R(\beta)], \\ [\mathcal{W}_L(\gamma), \mathcal{W}_R(\gamma)]\} - \{[\mathcal{C}_L(\alpha), \mathcal{C}_R(\alpha)], [\mathcal{C}_L(\beta), \mathcal{C}_R(\beta)], [\mathcal{C}_L(\gamma), \mathcal{C}_R(\gamma)]\} \\ = -\{[\mathcal{B}_L(\alpha), \mathcal{B}_R(\alpha)], [\mathcal{B}_L(\beta), \mathcal{B}_R(\beta)], [\mathcal{B}_L(\gamma), \mathcal{B}_R(\gamma)]\} \end{aligned} \tag{61}$$

Remark 6. Since the coefficient, non-coefficient and non-homogeneous parts are neutrosophic fuzzy values, then we consider the solution $\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$ to be also neutrosophic fuzzy in nature.

Here, we consider the Hukuhara difference and characterisation theorem with Eq. (61) and convert it as

$$\begin{cases} \min\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} - \mathcal{C}_R(\alpha) = -\mathcal{B}_R(\alpha) \\ \max\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} - \mathcal{C}_L(\alpha) = -\mathcal{B}_L(\alpha) \end{cases} \tag{62}$$

$$\begin{cases} \min\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} - \mathcal{C}_R(\beta) = -\mathcal{B}_R(\beta) \\ \max\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\alpha), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} - \mathcal{C}_L(\beta) = -\mathcal{B}_L(\beta) \end{cases} \tag{63}$$

$$\begin{cases}\min\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\}-\mathcal{C}_R(\gamma)=-\mathcal{B}_R(\gamma)\\ \max\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\}-\mathcal{C}_L(\gamma)=-\mathcal{B}_L(\gamma)\end{cases} \tag{64}$$

After applying the interval arithmetic described in paper [74] and we already know that $0 \leq \alpha, \beta, \gamma \leq 1$. Here, we consider $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are always positive $\forall\alpha \in [0, 1]$. Now, using Interval arithmetic, we get from Eq. (62),

$$\begin{cases}\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha)=-\mathcal{B}_R(\alpha)+\mathcal{C}_R(\alpha)\\ \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)=-\mathcal{B}_L(\alpha)+\mathcal{C}_L(\alpha)\end{cases} \tag{65}$$

where, $[\min\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha),\mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha),\mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha),\mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\},$
$\max\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha),\mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha),\mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha),\mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\}]$ is $[\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha),\mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)]$.

Further, we consider $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are always positive $\forall\beta \in [0, 1]$. Then, form Eq. (63) using interval arithmetic, we get

$$\begin{cases}\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta)=-\mathcal{B}_R(\beta)+\mathcal{C}_R(\beta)\\ \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)=-\mathcal{B}_L(\beta)+\mathcal{C}_L(\beta)\end{cases} \tag{66}$$

where, $[\min\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta),\mathcal{A}_L(\beta)\mathcal{Y}_R(\beta),\mathcal{A}_R(\beta)\mathcal{Y}_L(\beta),\mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\},$
$\max\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta),\mathcal{A}_L(\beta)\mathcal{Y}_R(\beta),\mathcal{A}_R(\beta)\mathcal{Y}_L(\alpha),\mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\}]$ is $[\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta),\mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)]$.

Simultaneously, considering $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are always positive $\forall\gamma \in [0, 1]$ and form Eq. (64) utilizing interval arithmetic, we obtain

$$\begin{cases}\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma)=-\mathcal{B}_R(\gamma)+\mathcal{C}_R(\gamma)\\ \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)=-\mathcal{B}_L(\gamma)+\mathcal{C}_L(\gamma)\end{cases} \tag{67}$$

where, $[\min\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\},$
$\max\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\}]$ is $[\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma),\mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)]$.

Now, from Eq. (65), we find

$$\begin{cases}\mathcal{W}_L(\alpha)=\frac{-\mathcal{B}_R(\alpha)+\mathcal{C}_R(\alpha)}{\mathcal{A}_L(\alpha)}\\ \mathcal{W}_R(\alpha)=\frac{-\mathcal{B}_L(\alpha)+\mathcal{C}_L(\alpha)}{\mathcal{A}_R(\alpha)}\end{cases} \tag{68}$$

In Eq. (68), $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) > 0$, then $\mathcal{W}_L(\alpha)$ be a monotonically increasing function of α and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) < 0$ then $\mathcal{W}_R(\alpha)$ be a monotonically decreasing function of α when $0 \leq \alpha \leq 1$. Then the membership part ($\alpha-$cut) of the solution of Eq. (61) is Eq. (68). If $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) < 0$ and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) > 0$ for all $\alpha \in [0, 1]$ simultaneously, then $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are monotonically decreasing and increasing functions, respectively and $\mathcal{W}_L(\alpha) > \mathcal{W}_R(\alpha)$ for all $\alpha \in [0, 1]$. Then the corrected solution of Eq. (68) is $[\mathcal{W}_L(\alpha)^*, \mathcal{W}_R(\alpha)^*] = [\min\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\}, \max\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\}] = [\mathcal{W}_R(\alpha), \mathcal{W}_L(\alpha)]$.

Similarly, from Eq. (66), we find

$$\begin{cases}\mathcal{Y}_L(\beta)=\frac{-\mathcal{B}_R(\beta)+\mathcal{C}_R(\beta)}{\mathcal{A}_L(\beta)}\\ \mathcal{Y}_R(\beta)=\frac{-\mathcal{B}_L(\beta)+\mathcal{C}_L(\beta)}{\mathcal{A}_R(\beta)}\end{cases} \tag{69}$$

In Eq. (69), $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) < 0$, then $\mathcal{Y}_L(\beta)$ be a monotonically decreasing function of β and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) > 0$ then $\mathcal{Y}_R(\beta)$ be a monotonically increasing function of β when $0 \leq \beta \leq 1$. Then the indeterminacy part ($\beta-$cut) of the solution of Eq. (61) is Eq. (69). If $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) > 0$ and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) < 0$ for all $\beta \in [0, 1]$ simultaneously, then $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are monotonically increasing and decreasing functions, respectively and $\mathcal{Y}_L(\beta) > \mathcal{Y}_R(\beta)$ for all $\beta \in [0, 1]$. Then the corrected solution of Eq. (69) is $[\mathcal{Y}_L(\beta)^*, \mathcal{Y}_R(\beta)^*] = [\min\{\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)\}, \max\{\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)\}] = [\mathcal{Y}_R(\beta), \mathcal{Y}_L(\beta)]$.

Further more, from Eq. (67), we find

$$\begin{cases}\mathcal{Z}_L(\gamma)=\frac{-\mathcal{B}_R(\gamma)+\mathcal{C}_R(\gamma)}{\mathcal{A}_L(\gamma)}\\ \mathcal{Z}_R(\gamma)=\frac{-\mathcal{B}_L(\gamma)+\mathcal{C}_L(\gamma)}{\mathcal{A}_R(\gamma)}\end{cases} \tag{70}$$

In Eq. (70), $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) < 0$, then $\mathcal{Z}_L(\gamma)$ be a monotonically decreasing function of γ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) > 0$ then $\mathcal{Z}_R(\gamma)$ be a monotonically increasing function of γ when $0 \leq \gamma \leq 1$. Then the non-membership part ($\gamma-$cut) of the solution of Eq. (61) is Eq. (70). If $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) > 0$ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) < 0$ for all $\gamma \in [0, 1]$ simultaneously, then $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are monotonically increasing and decreasing functions, respectively and $\mathcal{Z}_L(\gamma) > \mathcal{Z}_R(\gamma)$ for all $\gamma \in [0, 1]$. Then the corrected solution of Eq. (70) is $[\mathcal{Z}_L(\gamma)^*, \mathcal{Z}_R(\gamma)^*] = [\min\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\}, \max\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\}] = [\mathcal{Z}_R(\gamma), \mathcal{Z}_L(\gamma)]$.

The above solution exists when Eqs. (60)-(70) are satisfied the following conditions as follows:

(a) $\mathcal{W}_L(\alpha), \mathcal{Y}_R(\beta), \mathcal{Z}_R(\gamma)$ be a monotonically increasing functions of α, β, γ, respectively, when $0 \leq \alpha, \beta, \gamma \leq 1$.

(b) $\mathcal{W}_R(\alpha), \mathcal{Y}_L(\beta), \mathcal{Z}_L(\gamma)$ be a monotonically decreasing functions of α, β, γ, respectively, when $0 \leq \alpha, \beta, \gamma \leq 1$.
(c) $\mathcal{W}_L(\alpha) \leq \mathcal{W}_R(\alpha)$, $\mathcal{Y}_L(\beta) \leq \mathcal{Y}_R(\beta)$ and $\mathcal{Z}_L(\gamma) \leq \mathcal{Z}_R(\gamma)$, for all α, β and γ, respectively.

Then the solution $\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$ of Eq. (60) is evaluated by Eqs. (60)-(70), respectively.

Remark 7. Here, we consider the three different types of neutrosophic equations. $\tilde{A}, \tilde{B}$ and $\tilde{C}$ are consider as NFN. After applying the (α, β, γ)-cut, we convert the Eq. (38), Eq. (49) and Eq. (60), respectively with Hukuhara difference and Characterization theorem. Then, we find the six equations and finally, we get the required value of $\tilde{\mathcal{X}}$ using the classical method.

Remark 8. To solve the neutrosophic equations, we considered
$\min\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} = \mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha)$,
$\max\{\mathcal{A}_L(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_L(\alpha)\mathcal{W}_R(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_L(\alpha), \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)\} = \mathcal{A}_R(\alpha)\mathcal{W}_R(\alpha)$ for all $\alpha \in [0, 1]$;
$\min\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} = \mathcal{A}_L(\beta)\mathcal{Y}_L(\beta)$,
$\max\{\mathcal{A}_L(\beta)\mathcal{Y}_L(\beta), \mathcal{A}_L(\beta)\mathcal{Y}_R(\beta), \mathcal{A}_R(\beta)\mathcal{Y}_L(\alpha), \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)\} = \mathcal{A}_R(\beta)\mathcal{Y}_R(\beta)$ for all $\beta \in [0, 1]$ and
$\min\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\} = \mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma)$,
$\max\{\mathcal{A}_L(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_L(\gamma)\mathcal{Z}_R(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_L(\gamma), \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)\} = \mathcal{A}_R(\gamma)\mathcal{Z}_R(\gamma)$ for all $\gamma \in [0, 1]$ in all three cases. If anyone considered other maximum and minimum values, then the solutions may differ. Then, it is one of the solutions, not the only solution.

Remark 9. We notice that Eq. (38), Eq. (49) and Eq. (60) give three different solutions when treated as neutrosophic equation. However, it gives the same solution when we consider all equations as crisp.

5 Numerical Example

In this section, we process the neutrosophic fuzzy equations numerically. Here, we consider three neutrosophic equations and solve them using the methods mentioned above.

5.1 Numerical Example of Neutrosophic Equation (Type 1)

Consider the neutrosophic equation Type 1 shown in Eq. (38) as follows:

$$\tilde{\mathcal{A}}\tilde{\mathcal{X}} + \tilde{\mathcal{B}} = \tilde{\mathcal{C}} \tag{71}$$

and we consider the constants
$\tilde{\mathcal{A}} = \{1, 3, 5; 2, 3, 4; 1.5, 3, 4.5\}$,
$\tilde{\mathcal{B}} = \{2, 4, 6; 3, 4, 5; 2.5, 4, 5.5\}$,
$\tilde{\mathcal{C}} = \{4, 7, 10; 3, 7, 11; 5, 7, 9\}$ are three with neutrosophic numbers, and the variable,
$\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$.

In this study, we only consider triangular neutrosophic numbers as coefficients and variables. Now, we take the $(\alpha, \beta, \gamma)-$cut of $\tilde{\mathcal{A}}, \tilde{\mathcal{B}}, \tilde{\mathcal{C}}$ and $\tilde{\mathcal{X}}$ with the help of Subsection 3.8, as follows

$$\begin{aligned} \mathcal{A}(\alpha) &= [1 + 2\alpha, 5 - 2\alpha] \\ \mathcal{B}(\alpha) &= [2 + 2\alpha, 6 - 2\alpha] \\ \mathcal{C}(\alpha) &= [4 + 3\alpha, 10 - 3\alpha] \\ \mathcal{X}(\alpha) &= [\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)] \end{aligned}$$

where, $0 \leq \alpha \leq 1$ and

$$\begin{aligned} \mathcal{A}(\beta) &= [3 - \beta, 3 + \beta] \\ \mathcal{B}(\beta) &= [4 - \beta, 4 + \beta] \\ \mathcal{C}(\beta) &= [7 - 4\beta, 7 + 4\beta] \\ \mathcal{X}(\beta) &= [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)] \end{aligned}$$

where, $0 \leq \beta \leq 1$ and

$$\begin{aligned} \mathcal{A}(\gamma) &= [3 - 1.5\gamma, 3 + 1.5\gamma] \\ \mathcal{B}(\gamma) &= [4 - 1.5\gamma, 4 + 1.5\gamma] \\ \mathcal{C}(\gamma) &= [7 - 2\gamma, 7 + 2\gamma] \\ \mathcal{X}(\gamma) &= [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)] \end{aligned}$$

where, $0 \leq \gamma \leq 1$ and $0 \leq \alpha + \beta + \gamma \leq 3$. We consider here the basic concept of Hukuhara difference and characterisation theorem with Eq. (71) and it is converted as,

$$\begin{cases} \min\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{X}_R(\alpha)\} + (2+2\alpha) = (4+3\alpha) \\ \max\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\} + (6-2\alpha) = (10-3\alpha) \end{cases} \tag{72}$$

and

$$\begin{cases} \min\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\} + (4-\beta) = (7-4\beta) \\ \max\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\} + (4+\beta) = (7+4\beta) \end{cases} \tag{73}$$

and

$$\begin{cases} \min\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\} + (4-1.5\gamma) = (7-2\gamma) \\ \max\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\} + (4+1.5\gamma) = (7+2\gamma) \end{cases} \tag{74}$$

After applying the interval arithmetic described in paper [74] and we already know that $0 \leq \alpha, \beta, \gamma \leq 1$. Here, we consider $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are always positive $\forall \alpha \in [0,1]$. Now, using Interval arithmetic, we get from Eq. (72), as

$$\begin{cases} (1+2\alpha)\mathcal{W}_L(\alpha) + (2+2\alpha) = (4+3\alpha) \\ (5-2\alpha)\mathcal{W}_R(\alpha) + (6-2\alpha) = (10-3\alpha) \end{cases} \tag{75}$$

where, $[\min\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\},$ $\max\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\}]$ is $[(1+2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)]$.

Further, we consider $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are always positive $\forall \beta \in [0,1]$. Then, form Eq. (73) using interval arithmetic, we get

$$\begin{cases} (3-\beta)\mathcal{Y}_L(\beta) + (4-\beta) = (7-4\beta) \\ (3+\beta)\mathcal{Y}_R(\beta) + (4+\beta) = (7+4\beta) \end{cases} \tag{76}$$

where, $[\min\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\},$ $\max\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\}]$ is $[(3-\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)]$.

Simultaneously, considering $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are always positive $\forall \gamma \in [0,1]$ and form Eq. (74) utilizing interval arithmetic, we obtain

$$\begin{cases} (3-1.5\gamma)\mathcal{Z}_L(\gamma) + (4-1.5\gamma) = (7-2\gamma) \\ (3+1.5\gamma)\mathcal{Z}_R(\gamma) + (4+1.5\gamma) = (7+2\gamma) \end{cases} \tag{77}$$

where, $[\min\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\},$ $\max\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\}]$ is $[(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)]$.

Now, from Eq. (75),

$$\begin{cases} \mathcal{W}_L(\alpha) = \dfrac{(4+3\alpha)-(2+2\alpha)}{(1+2\alpha)} \\ \qquad\quad = \dfrac{2+\alpha}{(1+2\alpha)} \\ \mathcal{W}_R(\alpha) = \dfrac{(10-3\alpha)-(6-2\alpha)}{(5-2\alpha)} \\ \qquad\quad = \dfrac{4-\alpha}{(5-2\alpha)} \end{cases} \tag{78}$$

Since, in Eq. (78), $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) < 0$, then $\mathcal{W}_L(\alpha)$ be a monotonically decreasing function of α when $0 \leq \alpha \leq 1$ and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) > 0$ then $\mathcal{W}_R(\alpha)$ be a monotonically increasing function of α when $0 \leq \alpha \leq 1$. This implies $\mathcal{W}_L(\alpha) > \mathcal{W}_R(\alpha)$, then the correct solution is, $\mathcal{W}_L^*(\alpha) = \min\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\} = \mathcal{W}_R(\alpha)$ and $\mathcal{W}_R * (\alpha) = \max\{\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)\} = \mathcal{W}_L(\alpha)$. Now, the proper solution is,

$$\begin{cases} \mathcal{W}_L^*(\alpha) = \dfrac{(4-\alpha)}{(5-2\alpha)} \\ \mathcal{W}_R^*(\alpha) = \dfrac{(2+\alpha)}{(1+2\alpha)} \end{cases} \tag{79}$$

Then, from Eq. (76),

$$\begin{cases} \mathcal{Y}_L(\beta) = \dfrac{(7-4\beta)-(4-\beta)}{(3-\beta)} \\ \qquad\quad = \dfrac{3-3\beta}{(3-\beta)} \\ \mathcal{Y}_R(\beta) = \dfrac{(7+4\beta)-(4+\beta)}{(3+\beta)} \\ \qquad\quad = \dfrac{3+3\beta}{(3+\beta)} \end{cases} \tag{80}$$

Since, in Eq. (80), $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) < 0$, then $\mathcal{Y}_L(\beta)$ be a monotonically decreasing function of β when $0 \leq \beta \leq 1$ and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) > 0$, then $\mathcal{Y}_R(\beta)$ be a monotonically increasing function of β when $0 \leq \beta \leq 1$. Then, from Eq. (77),

$$\begin{cases} \mathcal{Z}_L(\gamma) = \dfrac{(7-2\gamma)-(4-1.5\gamma)}{(3-1.5\gamma)} \\ \qquad\quad = \dfrac{3-0.5\gamma}{(3-1.5\gamma)} \\ \mathcal{Z}_R(\gamma) = \dfrac{(7+2\gamma)-(4+1.5\gamma)}{(3+1.5\gamma)} \\ \qquad\quad = \dfrac{3+0.5\gamma}{(3+1.5\gamma)} \end{cases} \tag{81}$$

Since, in Eq. (81), $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) > 0$, then $\mathcal{Z}_L(\gamma)$ be a monotonically increasing function of γ when $0 \leq \gamma \leq 1$ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) < 0$, then $\mathcal{Z}_R(\gamma)$ be a monotonically decreasing function of γ when $0 \leq \gamma \leq 1$. This implies $\mathcal{Z}_L(\gamma) > \mathcal{Z}_R(\gamma)$, then the correct solution is, $\mathcal{Z}^*_L(\gamma) = \min\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\} = \mathcal{Z}_R(\gamma)$ and $\mathcal{Z}_R * (\gamma) = \max\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\} = \mathcal{Z}_L(\gamma)$. Now, the modified solution is,

$$\begin{cases} \mathcal{Z}^*_L(\gamma) = \dfrac{(3+0.5\gamma)}{(3+1.5\gamma)} \\ \mathcal{Z}^*_R(\gamma) = \dfrac{(3-0.5\gamma)}{(3-1.5\gamma)} \end{cases} \tag{82}$$

So, the solution of $\mathcal{W}^*_L(\alpha)$ and $\mathcal{W}^*_R(\alpha)$; $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$; $\mathcal{Z}^*_L(\gamma)$ and $\mathcal{Z}^*_R(\gamma)$ are confirmed using $(\alpha, \beta, \gamma)-$cut. Here, Figure 1 shows the membership, indeterminacy and non-membership curves of the solutions of the Type 1 example (Eq. (71)) of the neutrosophic equation.

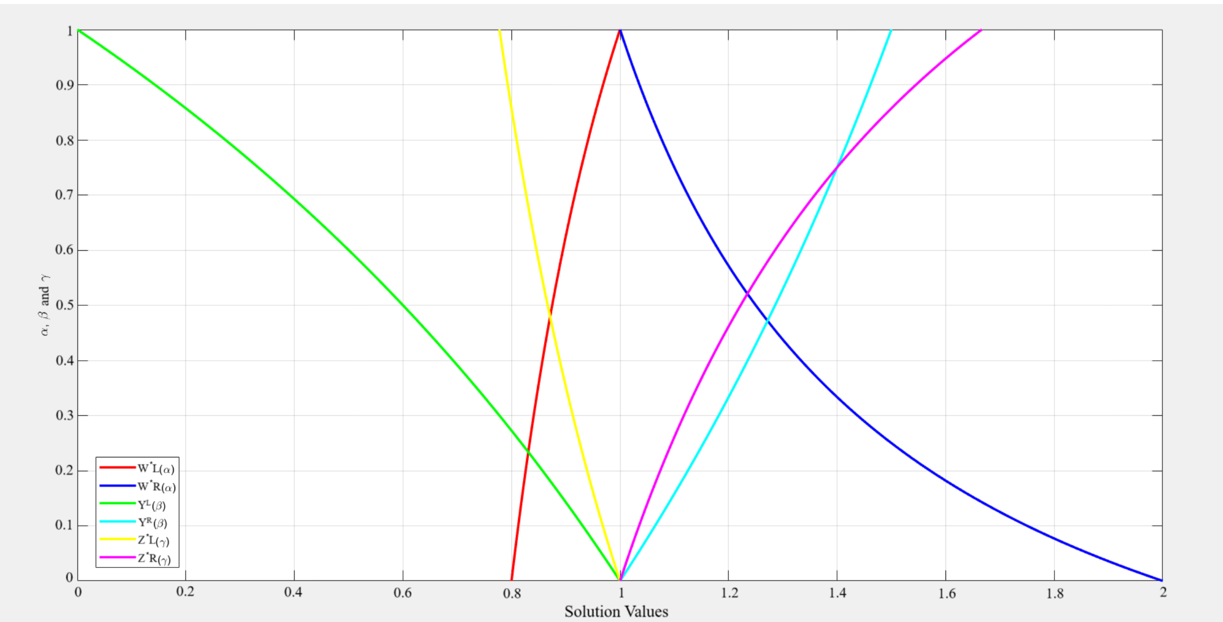

Figure 1. Membership, indeterminacy, non-membership curves of the solutions of Type 1

Remark 10. Figure 1 graphically illustrates that the membership function is increasing but both the indeterminacy and non-membership functions are decreasing. After reaching the maximum value 1, the membership function will decrease and the rest will increase.

Remark 11. When $\alpha = 0.9$ then $\mathcal{W}_L^*(\alpha) > 0$, when $\beta = 0.9$ then $\mathcal{Y}_L(\beta) < 0$ and when $\gamma = 0.9$ then $\mathcal{Z}_L^*(\gamma) < 0$, simultaneously, when $\alpha = 0.5$ then $\mathcal{W}_R^*(\alpha) < 0$, when $\beta = 0.5$ then $\mathcal{Y}_R(\beta) > 0$ and when $\gamma = 0.5$ then $\mathcal{Z}_R^*(\gamma) > 0$. Then, the solution of Eq. (71) is Eqs. (79), (80), and (82), respectively.

Remark 12. When $[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)]$, $[\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)]$ and $[\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]$ are partially positive or partially negative in interval for $0 \le \alpha, \beta, \gamma \le 1$, then the solution may differ.

5.2 Numerical Example of Neutrosophic Equation (Type 2)

Consider the neutrosophic equation Type 2 shown in Eq. (37) as follows:

$$\tilde{\mathcal{A}}\tilde{\mathcal{X}} = \tilde{\mathcal{C}} - \tilde{\mathcal{B}} \tag{83}$$

and we consider the constants
$\tilde{\mathcal{A}} = \{1, 3, 5; 2, 3, 4; 1.5, 3, 4.5\}$,
$\tilde{\mathcal{B}} = \{2, 4, 6; 3, 4, 5; 2.5, 4, 5.5\}$,
$\tilde{\mathcal{C}} = \{4, 7, 10; 3, 7, 11; 5, 7, 9\}$ are three with neutrosophic numbers, and the variable,
$\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$.

In this study, we only consider triangular neutrosophic numbers as coefficients and variables. Now, we take the $(\alpha, \beta, \gamma)-$cut of $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}$, $\tilde{\mathcal{C}}$ and $\tilde{\mathcal{X}}$ with the help of Subsection 3.8, as follows

$$\begin{aligned}
\mathcal{A}(\alpha) &= [1 + 2\alpha, 5 - 2\alpha] \\
\mathcal{B}(\alpha) &= [2 + 2\alpha, 6 - 2\alpha] \\
-\mathcal{B}(\alpha) &= [-(6 - 2\alpha), -(2 + 2\alpha)] = [-6 + 2\alpha, -2 - 2\alpha] \\
\mathcal{C}(\alpha) &= [4 + 3\alpha, 10 - 3\alpha] \\
\mathcal{X}(\alpha) &= [\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)]
\end{aligned}$$

where, $0 \le \alpha \le 1$ and

$$\begin{aligned}
\mathcal{A}(\beta) &= [3 - \beta, 3 + \beta] \\
\mathcal{B}(\beta) &= [4 - \beta, 4 + \beta] \\
-\mathcal{B}(\beta) &= [-(4 + \beta), -(4 - \beta)] = [-4 - \beta, -4 + \beta] \\
\mathcal{C}(\beta) &= [7 - 4\beta, 7 + 4\beta] \\
\mathcal{Y}(\beta) &= [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)]
\end{aligned}$$

where, $0 \le \beta \le 1$ and

$$\begin{aligned}
\mathcal{A}(\gamma) &= [3 - 1.5\gamma, 3 + 1.5\gamma] \\
\mathcal{B}(\gamma) &= [4 - 1.5\gamma, 4 + 1.5\gamma] \\
-\mathcal{B}(\gamma) &= [-(4 + 1.5\gamma), -(4 - 1.5\gamma)] = [-4 - 1.5\gamma, -4 + 1.5\gamma] \\
\mathcal{C}(\gamma) &= [7 - 2\gamma, 7 + 2\gamma] \\
\mathcal{Z}(\gamma) &= [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]
\end{aligned}$$

where, $0 \le \gamma \le 1$ and $0 \le \alpha + \beta + \gamma \le 3$. We consider here the basic concept of Hukuhara difference and characterisation theorem with Eq. (83) and it is converted as,

$$\begin{cases}\min\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\} = (4+3\alpha) + (-6+2\alpha) \\ \max\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\} = (10-3\alpha) + (-2-2\alpha)\end{cases} \tag{84}$$

and

$$\begin{cases}\min\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\} = (7-4\beta) + (-4-\beta) \\ \max\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\} = (7+4\beta) + (-4+\beta)\end{cases} \tag{85}$$

and

$$\begin{cases}\min\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\} = (7-2\gamma) + (-4-1.5\gamma) \\ \max\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\} = (7+2\gamma) + (-4+1.5\gamma)\end{cases} \tag{86}$$

After applying the interval arithmetic described in paper [74] and we already know that $0 \leq \alpha, \beta, \gamma \leq 1$. Here, we consider $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are always positive $\forall \alpha \in [0,1]$. Now, using Interval arithmetic, we get from Eq. (84), as

$$\begin{cases}(1+2\alpha)\mathcal{W}_L(\alpha) = (4+3\alpha)+(-6+2\alpha)\\(5-2\alpha)\mathcal{W}_R(\alpha) = (10-3\alpha)+(-2-2\alpha)\end{cases} \tag{87}$$

From the basic interval arithmetic operation (multiplication), we know that,
$[\min\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\}$,
$\max\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\}]$ is
$[(1+2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)]$. We choose $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are always positive $\forall \beta \in [0,1]$ and form Eq. (85),

$$\begin{cases}(3-\beta)\mathcal{Y}_L(\beta) = (7-4\beta)+(-4-\beta)\\(3+\beta)\mathcal{Y}_R(\beta) = (7+4\beta)+(-4+\beta)\end{cases} \tag{88}$$

From the basic interval arithmetic operation (multiplication), we know that,
$[\min\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\}$,
$\max\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\}]$ is $[(3-\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)]$.
So, we consider here $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are always positive $\forall \gamma \in [0,1]$ and form Eq. (86),

$$\begin{cases}(3-1.5\gamma)\mathcal{Z}_L(\gamma) = (7-2\gamma)+(-4-1.5\gamma)\\(3+1.5\gamma)\mathcal{Z}_R(\gamma) = (7+2\gamma)+(-4+1.5\gamma)\end{cases} \tag{89}$$

From the basic interval arithmetic operation (multiplication), we know that,
$[\min\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\}$,
$\max\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\}]$ is
$[(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)]$.
Then, from Eq. (87),

$$\begin{cases}\mathcal{W}_L(\alpha) = \dfrac{(4+3\alpha)+(-6+2\alpha)}{(1+2\alpha)}\\ \qquad\quad = \dfrac{(-2+5\alpha)}{(1+2\alpha)}\\ \mathcal{W}_R(\alpha) = \dfrac{(10-3\alpha)+(-2-2\alpha)}{(5-2\alpha)}\\ \qquad\quad = \dfrac{8-5\alpha}{(5-2\alpha)}\end{cases} \tag{90}$$

From Eq. (90), $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha)) > 0$. So, $(\mathcal{W}_L(\alpha))$ be a monotonically increasing function of α when $0 \leq \alpha \leq 1$ and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha)) < 0$. So, $(\mathcal{W}_R(\alpha))$ be a monotonically decreasing function of α when $0 \leq \alpha \leq 1$.
Then, from Eq. (88), we get

$$\begin{cases}\mathcal{Y}_L(\beta) = \dfrac{(7-4\beta)+(-4-\beta)}{(3-\beta)}\\ \qquad\quad = \dfrac{3-5\beta}{(3-\beta)}\\ \mathcal{Y}_R(\beta) = \dfrac{(7+4\beta)+(-4+\beta)}{(3+\beta)}\\ \qquad\quad = \dfrac{3+5\beta}{(3+\beta)}\end{cases} \tag{91}$$

From Eq. (91), $\frac{d}{d\beta}(\mathcal{Y}_L(\beta)) < 0$. So, $(\mathcal{Y}_L(\beta))$ be a monotonically decreasing function of β when $0 \leq \beta \leq 1$ and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta)) > 0$. So, $(\mathcal{Y}_R(\beta))$ be a monotonically increasing function of β when $0 \leq \beta \leq 1$ and from Eq. (89),

$$\begin{cases}\mathcal{Z}_L(\gamma) = \dfrac{(7-2\gamma)+(-4-1.5\gamma)}{(3-1.5\gamma)}\\ \qquad\quad = \dfrac{(3-3.5\gamma)}{(3-1.5\gamma)}\\ \mathcal{Z}_R(\gamma) = \dfrac{(7+2\gamma)+(-4+1.5\gamma)}{(3+1.5\gamma)}\\ \qquad\quad = \dfrac{(3+3.5\gamma)}{(3+1.5\gamma)}\end{cases} \tag{92}$$

From Eq. (92), $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma)) < 0$. So, $(\mathcal{Z}_L(\gamma))$ be a monotonically decreasing function of γ when $0 \leq \gamma \leq 1$ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma)) > 0$. So, $(\mathcal{Z}_R(\gamma))$ be a monotonically increasing function of γ when $0 \leq \gamma \leq 1$.

Here, the solution of $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$; $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$; $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are confirmed using (α, β, γ)-cut. Now, Figure 2 shows the graphical structure of membership, indeterminacy and non-membership curves of the solutions of the Type 2 example of the neutrosophic equation.

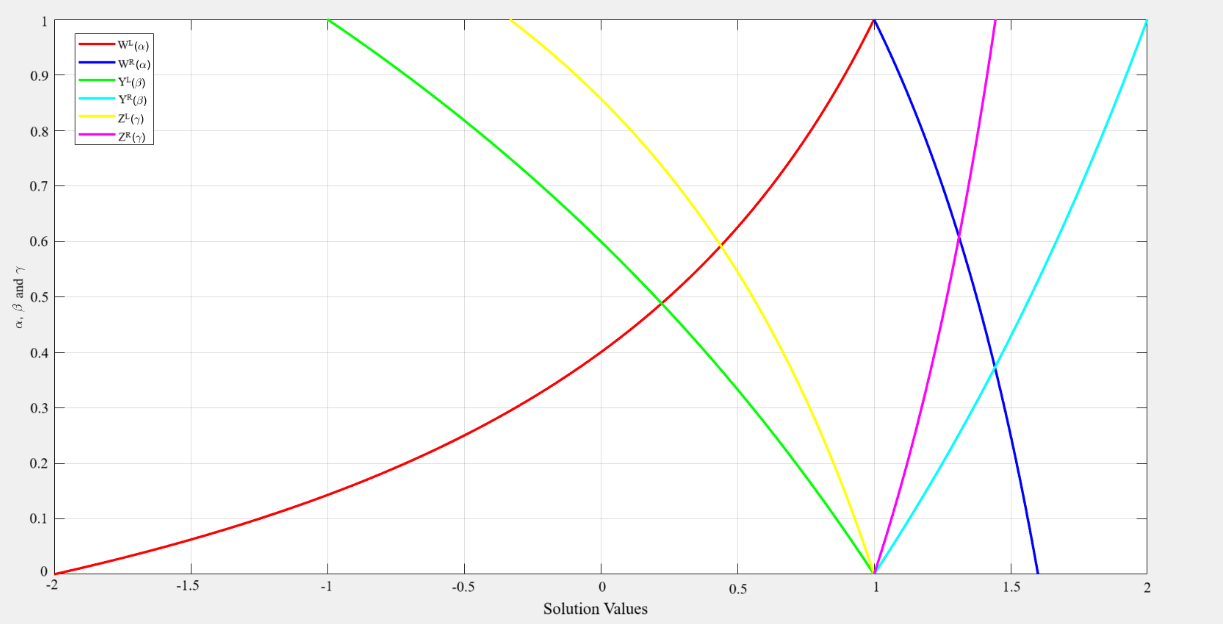

Figure 2. Membership, indeterminacy, non-membership curves of the solutions of Type 2

Remark 13. Figure 2 indicates that the membership function is increasing but both the indeterminacy and non-membership functions are decreasing. After reaching the maximum value 1, the membership function is decreasing and the indeterminacy and non-membership functions are increasing. This actually makes for the perfect visualisation.

5.3 Numerical Example of Neutrosophic Equation (Type 3)

Consider the neutrosophic equation Type 3 shown in Eq. (37) as follows:

$$\tilde{\mathcal{A}}\tilde{\mathcal{X}} - \tilde{\mathcal{C}} = -\tilde{\mathcal{B}} \tag{93}$$

and we consider the constants

$\tilde{\mathcal{A}} = \{1, 3, 5; 2, 3, 4; 1.5, 3, 4.5\}$,

$\tilde{\mathcal{B}} = \{2, 4, 6; 3, 4, 5; 2.5, 4, 5.5\}$,

$\tilde{\mathcal{C}} = \{4, 7, 10; 3, 7, 11; 5, 7, 9\}$ are three with neutrosophic numbers, and the variable,

$\tilde{\mathcal{X}} = \{[\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)], [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)], [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]\}$.

In this study, we only consider triangular neutrosophic numbers as coefficients and variables. Now, we take the $(\alpha, \beta, \gamma)-$cut of $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}$, $\tilde{\mathcal{C}}$ and $\tilde{\mathcal{X}}$ with the help of Subsection 3.8, as follows

$$\begin{aligned}
\mathcal{A}(\alpha) &= [1 + 2\alpha, 5 - 2\alpha] \\
\mathcal{B}(\alpha) &= [2 + 2\alpha, 6 - 2\alpha] \\
-\mathcal{B}(\alpha) &= [-(6 - 2\alpha), -(2 + 2\alpha)] = [-6 + 2\alpha, -2 - 2\alpha] \\
\mathcal{C}(\alpha) &= [4 + 3\alpha, 10 - 3\alpha] \\
-\mathcal{C}(\alpha) &= [-(10 - 3\alpha), -(4 + 3\alpha)] = [-10 + 3\alpha, -4 - 3\alpha] \\
\mathcal{W}(\alpha) &= [\mathcal{W}_L(\alpha), \mathcal{W}_R(\alpha)]
\end{aligned}$$

where, $0 \leq \alpha \leq 1$ and

$$\begin{aligned}
\mathcal{A}(\beta) &= [3-\beta, 3+\beta] \\
\mathcal{B}(\beta) &= [4-\beta, 4+\beta] \\
-\mathcal{B}(\beta) &= [-(4+\beta), -(4-\beta)] = [-4-\beta, -4+\beta] \\
\mathcal{C}(\beta) &= [7-4\beta, 7+4\beta] \\
-\mathcal{C}(\beta) &= [-(7+4\beta), -(7-4\beta)] = [-7-4\beta, -7+4\beta] \\
\mathcal{Y}(\beta) &= [\mathcal{Y}_L(\beta), \mathcal{Y}_R(\beta)]
\end{aligned}$$

where, $0 \leq \beta \leq 1$ and

$$\begin{aligned}
\mathcal{A}(\gamma) &= [3-1.5\gamma, 3+1.5\gamma] \\
\mathcal{B}(\gamma) &= [4-1.5\gamma, 4+1.5\gamma] \\
-\mathcal{B}(\gamma) &= [-(4+1.5\gamma), -(4-1.5\gamma)] = [-4-1.5\gamma, -4+1.5\gamma] \\
\mathcal{C}(\gamma) &= [7-2\gamma, 7+2\gamma] \\
-\mathcal{C}(\gamma) &= [-(7+2\gamma), -(7-2\gamma)] = [-7-2\gamma, -7+2\gamma] \\
\mathcal{Z}(\gamma) &= [\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)]
\end{aligned}$$

where, $0 \leq \gamma \leq 1$ and $0 \leq \alpha+\beta+\gamma \leq 3$.

We consider here the basic concept of Hukuhara difference and characterisation of the theorem with Eq. (93) and it is converted as,

$$\begin{cases}\min\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\} - (10-3\alpha) = -(6-2\alpha) \\ \max\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\} - (4+3\alpha) = -(2+2\alpha)\end{cases} \tag{94}$$

and

$$\begin{cases}\min\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\alpha), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\} - (7+4\beta) = -(4+\beta) \\ \max\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\alpha), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\} - (7-4\beta) = -(4-\beta)\end{cases} \tag{95}$$

and

$$\begin{cases}\min\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\} - (7+2\gamma) = -(4+1.5\gamma) \\ \max\{(3-1.5\gamma)\mathcal{Z}_L(\gamma), (3-1.5\gamma)\mathcal{Z}_R(\gamma), (3+1.5\gamma)\mathcal{Z}_L(\gamma), (3+1.5\gamma)\mathcal{Z}_R(\gamma)\} - (7-2\gamma) = -(4-1.5\gamma)\end{cases} \tag{96}$$

After applying the interval arithmetic described in paper [74] and we already know that $0 \leq \alpha, \beta, \gamma \leq 1$. Here, we consider $\mathcal{W}_L(\alpha)$ and $\mathcal{W}_R(\alpha)$ are always positive $\forall \alpha \in [0,1]$. Now, using Interval arithmetic, we get form Eq. (94), as

$$\begin{cases}(1+2\alpha)\mathcal{W}_L(\alpha) - (10-3\alpha) = -(6-2\alpha) \\ (5-2\alpha)\mathcal{W}_R(\alpha) - (4+3\alpha) = -(2+2\alpha)\end{cases} \tag{97}$$

From the basic interval arithmetic operation (multiplication), we know that,
$[\min\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\},$
$\max\{(1+2\alpha)\mathcal{W}_L(\alpha), (1+2\alpha)\mathcal{W}_R(\alpha), (5-2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)\}]$ is
$[(1+2\alpha)\mathcal{W}_L(\alpha), (5-2\alpha)\mathcal{W}_R(\alpha)]$ when $(1+2\alpha) \geq 0, \mathcal{W}_L(\alpha) \leq 0$.
Now, we consider $\mathcal{Y}_L(\beta)$ and $\mathcal{Y}_R(\beta)$ are always positive $\forall \beta \in [0,1]$. Form Eq. (95),

$$\begin{cases}(3-\beta)\mathcal{Y}_L(\beta) - (7+4\beta) = -(4+\beta) \\ (3+\beta)\mathcal{Y}_R(\beta) - (7-4\beta) = -(4-\beta)\end{cases} \tag{98}$$

From the basic interval arithmetic operation (multiplication), we know that,
$[\min\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\},$
$\max\{(3-\beta)\mathcal{Y}_L(\beta), (3-\beta)\mathcal{Y}_R(\beta), (3+\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)\}]$ is $[(3-\beta)\mathcal{Y}_L(\beta), (3+\beta)\mathcal{Y}_R(\beta)]$ when $(3+\beta) \leq 0, \mathcal{Y}_L(\beta) \leq 0$.
Here, we consider $\mathcal{Z}_L(\gamma)$ and $\mathcal{Z}_R(\gamma)$ are always positive $\forall \gamma \in [0,1]$. Form Eq. (96),

$$\begin{cases}(3-1.5\gamma)\mathcal{Z}_L(\gamma) - (7+2\gamma) = -(4+1.5\gamma) \\ (3+1.5\gamma)\mathcal{Z}_R(\gamma) - (7-2\gamma) = -(4-1.5\gamma)\end{cases} \tag{99}$$

From the basic interval arithmetic operation (multiplication), we know that,
$[\min\{(3-1.5\gamma)\mathcal{Z}_L(\gamma),(3-1.5\gamma)\mathcal{Z}_R(\gamma),(3+1.5\gamma)\mathcal{Z}_L(\gamma),(3+1.5\gamma)\mathcal{Z}_R(\gamma)\},$
$\max\{(3-1.5\gamma)\mathcal{Z}_L(\gamma),(3-1.5\gamma)\mathcal{Z}_R(\gamma),(3+1.5\gamma)\mathcal{Z}_L(\gamma),(3+1.5\gamma)\mathcal{Z}_R(\gamma)\}]$ is
$[(3-1.5\gamma)\mathcal{Z}_L(\gamma),(3+1.5\gamma)\mathcal{Z}_R(\gamma)]$ when $(3+1.5\gamma)\le 0$, $\mathcal{Z}_L(\gamma)\le 0$.
Then, from Eq. (97),

$$\begin{cases}\mathcal{W}_L(\alpha)=&\dfrac{-(6-2\alpha)+(10-3\alpha)}{(1+2\alpha)}\\ &=\dfrac{4-\alpha}{(1+2\alpha)}\\ \mathcal{W}_R(\alpha)=&\dfrac{-(2+2\alpha)+(4+3\alpha)}{(5-2\alpha)}\\ &=\dfrac{2+\alpha}{(5-2\alpha)}\end{cases}\tag{100}$$

From Eq. (100), $\frac{d}{d\alpha}(\mathcal{W}_L(\alpha))<0$. So, $(\mathcal{W}_L(\alpha))$ be a monotonically decreasing function of α when $0\le\alpha\le 1$ and $\frac{d}{d\alpha}(\mathcal{W}_R(\alpha))>0$. So, $(\mathcal{W}_R(\alpha))$ be a monotonically increasing function of α when $0\le\alpha\le 1$.
This implies $\mathcal{W}_L(\alpha)>\mathcal{W}_R(\alpha)$, then the correct solution is, $\mathcal{W}_L^*(\alpha)=\min\{\mathcal{W}_L(\alpha),\mathcal{W}_R(\alpha)\}=\mathcal{W}_R(\alpha)$ and $\mathcal{W}_R*(\alpha)=\max\{\mathcal{W}_L(\alpha),\mathcal{W}_R(\alpha)\}=\mathcal{W}_L(\alpha)$. Now, the proper solution is,

$$\begin{cases}\mathcal{W}_L^*(\alpha)=\dfrac{(2+\alpha)}{(5-2\alpha)}\\ \mathcal{W}_R^*(\alpha)=\dfrac{(4-\alpha)}{(1+2\alpha)}\end{cases}\tag{101}$$

Then, from Eq. (98),

$$\begin{cases}\mathcal{Y}_L(\beta)=&\dfrac{-(4+\beta)+(7+4\beta)}{(3-\beta)}\\ &=\dfrac{3+3\beta}{(3-\beta)}\\ \mathcal{Y}_R(\beta)=&\dfrac{-(4-\beta)+(7-4\beta)}{(3+\beta)}\\ &=\dfrac{3-3\beta}{(3+\beta)}\end{cases}\tag{102}$$

From Eq. (102), $\frac{d}{d\beta}(\mathcal{Y}_L(\beta))>0$. So, $(\mathcal{Y}_L(\beta))$ be a monotonically increasing function of β when $0\le\beta\le 1$ and $\frac{d}{d\beta}(\mathcal{Y}_R(\beta))<0$. So, $(\mathcal{Y}_R(\beta))$ be a monotonically decreasing function of β when $0\le\beta\le 1$.
This implies $\mathcal{Y}_L(\beta)>\mathcal{Y}_R(\beta)$, then the correct solution is, $\mathcal{Y}_L^*(\beta)=\min\{\mathcal{Y}_L(\beta),\mathcal{Y}_R(\beta)\}=\mathcal{Y}_R(\beta)$ and $\mathcal{Y}_R*(\beta)=\max\{\mathcal{Y}_L(\beta),\mathcal{Y}_R(\beta)\}=\mathcal{Y}_L(\beta)$. Now, the modified solution is,

$$\begin{cases}\mathcal{Y}_L^*(\beta)=\dfrac{(3-3\beta)}{(3+\beta)}\\ \mathcal{Y}_R^*(\beta)=\dfrac{(3+3\beta)}{(3-\beta)}\end{cases}\tag{103}$$

From Eq. (99),

$$\begin{cases}\mathcal{Z}_L(\gamma)=&\dfrac{-(4+1.5\gamma)+(7+2\gamma)}{(3-1.5\gamma)}\\ &=\dfrac{3+0.5\gamma}{(3-1.5\gamma)}\\ \mathcal{Z}_R(\gamma)=&\dfrac{-(4-1.5\gamma)+(7-2\gamma)}{(3+1.5\gamma)}\\ &=\dfrac{3-0.5\gamma}{(3+1.5\gamma)}\end{cases}\tag{104}$$

From Eq. (104), $\frac{d}{d\gamma}(\mathcal{Z}_L(\gamma))>0$. So, $(\mathcal{Z}_L(\gamma))$ be a monotonically increasing function of γ when $0\le\gamma\le 1$ and $\frac{d}{d\gamma}(\mathcal{Z}_R(\gamma))<0$. So, $(\mathcal{Z}_R(\gamma))$ be a monotonically decreasing function of γ when $0\le\gamma\le 1$.

This implies $\mathcal{Z}_L(\gamma) > \mathcal{Z}_R(\gamma)$, then the correct solution is, $\mathcal{Z}_L^*(\gamma) = \min\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\} = \mathcal{Z}_R(\gamma)$ and $\mathcal{Z}_R * (\gamma) = \max\{\mathcal{Z}_L(\gamma), \mathcal{Z}_R(\gamma)\} = \mathcal{Z}_L(\gamma)$. Now, the modified solution is,

$$\begin{cases} \mathcal{Z}_L^*(\gamma) = \dfrac{(3 - 0.5\gamma)}{(3 + 1.5\gamma)} \\ \mathcal{Z}_R^*(\gamma) = \dfrac{(3 + 0.5\gamma)}{(3 - 1.5\gamma)} \end{cases} \tag{105}$$

Here, the solution of $\mathcal{W}_L^*(\alpha)$ and $\mathcal{W}_R^*(\alpha)$; $\mathcal{Y}_L^*(\beta)$ and $\mathcal{Y}_R^*(\beta)$; $\mathcal{Z}_L^*(\gamma)$ and $\mathcal{Z}_R^*(\gamma)$ are confirmed using (α, β, γ)-cut. And, Figure 3 demonstrates the membership, indeterminacy and non-membership curves of the solutions of the Type 3 example of the neutrosophic equation.

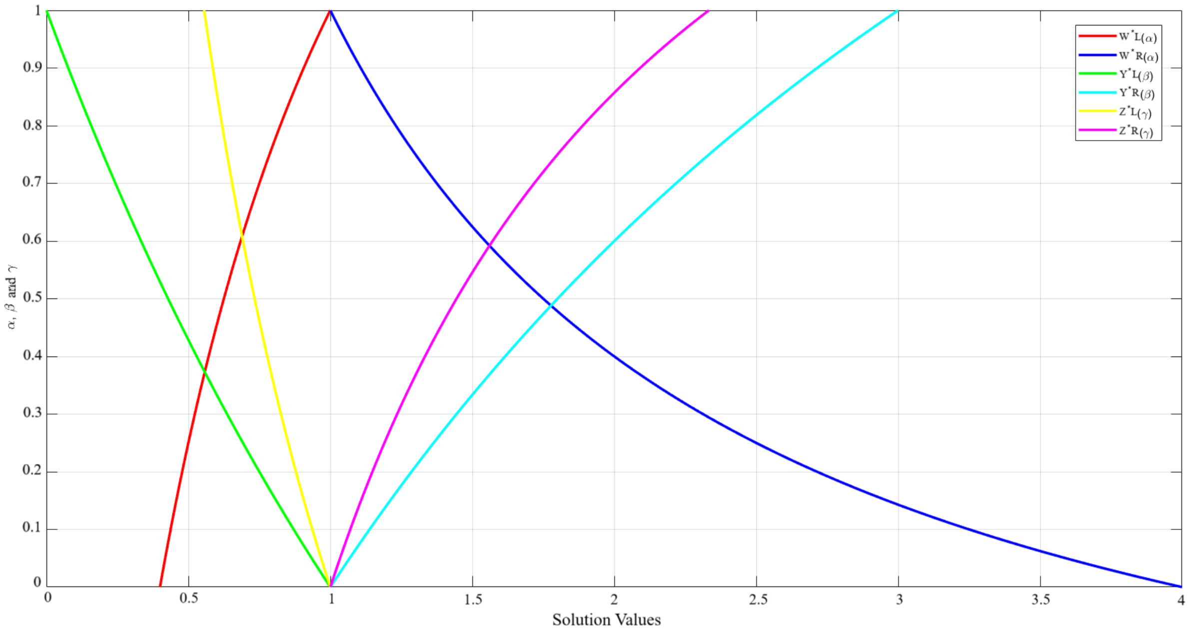

Figure 3. Membership, indeterminacy, non-membership curves of the solutions of Type 3

Remark 14. Figure 3 explains that the membership function is increasing but both the indeterminacy and non-membership functions are decreasing. And after touching the maximum point 1, the opposite happens.

6 Application

The application of this proposed model is presented in this section. These are presented as follows:

6.1 Budgeting and Finance

Problem statement: A pharmaceutical company needs to identify their profit ($\mathcal{P}$) and revenue ($\mathcal{R}$) after accounting for fixed and variable costs. Let, $\mathscr{X}$ be the number of products sold, $\mathscr{U}$ be the variable cost per product, $\mathscr{S}$ be the selling cost per unit and $\mathscr{V}$ be the maintenance costs. The company wants to earn $\mathscr{W}$, which is also known as the target revenue, at a certain time. The total cost can be computed with the equation $\mathscr{U}\mathscr{X} + \mathscr{V} = \mathscr{W}$. Now, apply the triangular neutrosophic numbers in the above equation and get $\tilde{\mathscr{U}}\tilde{\mathscr{X}} + \tilde{\mathscr{V}} = \tilde{\mathscr{W}}$. Now, considering both fixed and variable costs, the profit of this pharmaceutical company will be determined here.

Model formulation: The given total production cost of this pharmaceutical company is,

$$\tilde{\mathscr{U}}\tilde{X} + \tilde{\mathscr{V}} = \tilde{\mathscr{W}} \tag{106}$$

where, $\tilde{\mathscr{U}}\tilde{X}$ and $\tilde{\mathscr{V}}$ are the total variable cost and the fixed maintenance costs, respectively. Then, the total money earned from selling units is defined as the revenue cost ($\mathcal{R}$), i.e.,

$$\text{Revenue cost } (\tilde{\mathcal{R}}) = \text{ Selling cost per unit} * \tilde{X} = \mathscr{S}\tilde{X} \tag{107}$$

where, $\mathscr{S}$ is the company's cost of sales per unit in neutrosophic uncertain environment. And, the profit ($\tilde{\mathcal{P}}$) of this particular company is,

$$\begin{aligned} \text{Profit } (\tilde{\mathcal{P}}) =& \text{Revenue cost} - \text{Total cost} \\ =& \tilde{\mathcal{R}} - \tilde{\mathscr{W}} \\ =& \mathscr{S}\tilde{X} - (\tilde{\mathscr{U}}\tilde{X} + \tilde{\mathscr{V}}) \end{aligned} \tag{108}$$

Remark 15. In the above application, the break-even point originates when revenue cost and total cost are equal, which means no profit or loss will happen and if the revenue cost exceeds the total cost, then the pharmaceutical company makes a profit, otherwise it faces a loss. We consider all the co-efficienmt of Eq. (106) as neutrosophic fuzzy variable. So, the solution will also be fuzzy in nature.

Example 3. To bring into the uncertain environment, this is a numerical example to find the profit ($\mathcal{P}$) and revenue ($\mathcal{R}$) after accounting for fixed and variable costs of the pharma company. The above considered equation is converted into neutrosophic environment, let, $\tilde{\mathscr{U}}$, $\tilde{\mathscr{V}}$, $\tilde{\mathscr{W}}$ and $\tilde{\mathscr{X}}$ are triangular neutrosophic number, where, let, $\tilde{\mathscr{U}} = \{501, 503, 505; 502, 503, 504; 501.5, 503, 504.5\}$, $\tilde{\mathscr{V}} = \{502, 504, 506; 503, 504, 505; 502.5, 504, 505.5\}$ and $\tilde{\mathscr{W}} = \{504, 507, 510; 503, 507, 511; 505, 507, 509\}$ and $\tilde{\mathscr{X}}$. And, the selling price per unit be Rs. 20. After calculating by the similar way of Subsection 4.1, at first, we get the number of sold units by (α, β, γ)-cut in uncertain neutrosophic environment. Here, we use the basic concept of Hukuhara difference and characterisation theorem. So, the equations of total cost are,

$$\begin{cases} (501 + 2\alpha)\mathcal{W}_L(\alpha) + (506 - 2\alpha) = (504 + 3\alpha) \\ (505 - 2\alpha)\mathcal{W}_R(\alpha) + (502 + 2\alpha) = (510 - 3\alpha) \end{cases} \tag{109}$$

$$\begin{cases} (503 - \beta)\mathcal{Y}_R(\beta) + (504 + \beta) = (507 - 4\beta) \\ (503 + \beta)\mathcal{Y}_L(\beta) + (504 - \beta) = (507 + 4\beta) \end{cases} \tag{110}$$

$$\begin{cases} (503 - 1.5\gamma)\mathcal{Z}_R(\gamma) + (504 + 1.5\gamma) = (507 - 2\gamma) \\ (503 + 1.5\gamma)\mathcal{Z}_L(\gamma) + (504 - 1.5\gamma) = (507 + 2\gamma) \end{cases} \tag{111}$$

So, the required solutions of above Eqs. (109)-(111) are,

$$\begin{cases} \mathcal{W}_L(\alpha) = & \frac{-2+5\alpha}{(501+2\alpha)} \\ \mathcal{W}_R(\alpha) = & \frac{8-5\alpha}{(505-2\alpha)} \end{cases} \tag{112}$$

$$\begin{cases} \mathcal{Y}_R(\beta) = & \frac{3-5\beta}{(503-\beta)} \\ \mathcal{Y}_L(\beta) = & \frac{3+5\beta}{(503+\beta)} \end{cases} \tag{113}$$

$$\begin{cases} \mathcal{Z}_R(\gamma) = & \frac{3-3.5\gamma}{(503-1.5\gamma)} \\ \mathcal{Z}_L(\gamma) = & \frac{3+3.5\gamma}{(503+1.5\gamma)} \end{cases} \tag{114}$$

Here, Figure 4 displays the membership, indeterminacy and non-membership curves of Eqs. (112)-(114).
The revenue costs ($\mathcal{R}$) of this particular pharma company are,

$$\begin{cases} \mathcal{R}_1 = & 20 \times \frac{-2+5\alpha}{(501+2\alpha)} \\ \mathcal{R}_2 = & 20 \times \frac{8-5\alpha}{(505-2\alpha)} \end{cases} \tag{115}$$

$$\begin{cases} \mathcal{R}_3 = & 20 \times \frac{3-5\beta}{(503-\beta)} \\ \mathcal{R}_4 = & 20 \times \frac{3+5\beta}{(503+\beta)} \end{cases} \tag{116}$$

$$\begin{cases} \mathcal{R}_5 = & 20 \times \frac{3-3.5\gamma}{(503-1.5\gamma)} \\ \mathcal{R}_6 = & 20 \times \frac{3+3.5\gamma}{(503+1.5\gamma)} \end{cases} \tag{117}$$

Then, the profits ($\mathcal{P}$) of this company are,

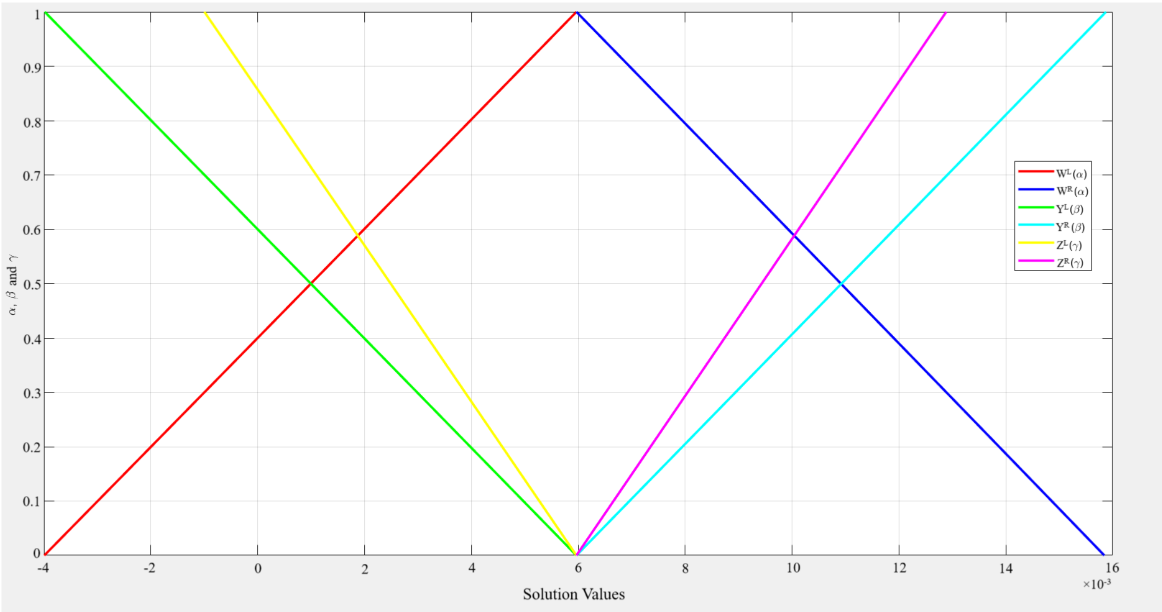

Figure 4. Membership, indeterminacy, non-membership curves of the solutions of the application

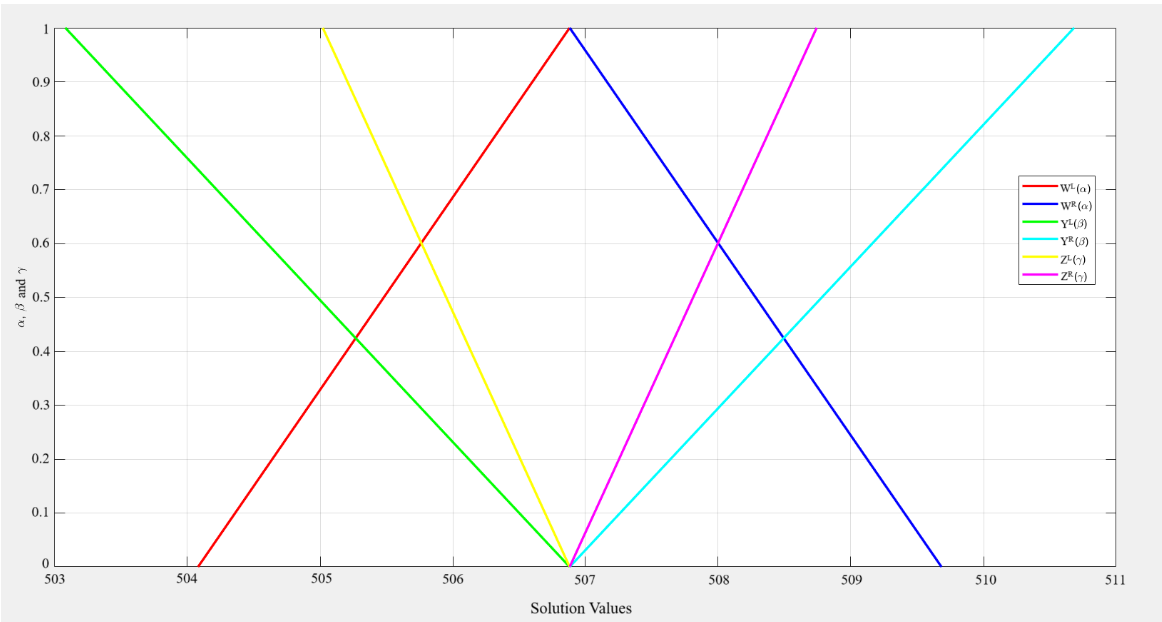

Figure 5. Graphical structure of the profits (properly define here in loss) of described pharmaceutical company

$$\begin{cases} \mathcal{P}_1 = & \left[20 \times \frac{-2+5\alpha}{(501+2\alpha)}\right] - (504+3\alpha) \\ \mathcal{P}_2 = & \left[20 \times \frac{8-5\alpha}{(505-2\alpha)}\right] - (510-3\alpha) \end{cases} \tag{118}$$

$$\begin{cases} \mathcal{P}_3 = \left[20 \times \frac{3-5\beta}{(503-\beta)}\right] - (507-4\beta)) \\ \mathcal{P}_4 = \left[20 \times \frac{3+5\beta}{(503+\beta)}\right] - (507+4\beta)) \end{cases} \tag{119}$$

$$\begin{cases} \mathcal{P}_5 = \left[20 \times \frac{3-3.5\gamma}{(503-1.5\gamma)}\right] - (507-2\gamma)) \\ \mathcal{P}_6 = \left[20 \times \frac{3+3.5\gamma}{(503+1.5\gamma)}\right] - (507+2\gamma)) \end{cases} \tag{120}$$

From Eqs. (118)-(120), we notice that the total cost exceeds the revenue cost here. So, the pharmaceutical company faces a loss actually.

So, the profit (properly define here in loss) of this particular company is,

$$\begin{cases} \mathcal{P}_L = (504+3\alpha) - \left[20 \times \frac{-2+5\alpha}{(501+2\alpha)}\right] \\ \mathcal{P}_R = (510-3\alpha) - \left[20 \times \frac{8-5\alpha}{(505-2\alpha)}\right] \end{cases} \tag{121}$$

$$\begin{cases} \mathcal{P}_L = (507-4\beta)) - \left[20 \times \frac{3-5\beta}{(503-\beta)}\right] \\ \mathcal{P}_R = (507+4\beta)) - \left[20 \times \frac{3+5\beta}{(503+\beta)}\right] \end{cases} \tag{122}$$

$$\begin{cases} \mathcal{P}_L = (507-2\gamma) - \left[20 \times \frac{3-3.5\gamma}{(503-1.5\gamma)}\right] \\ \mathcal{P}_R = (507+2\gamma) - \left[20 \times \frac{3+3.5\gamma}{(503+1.5\gamma)}\right] \end{cases} \tag{123}$$

Now, Figure 5 illustrates the graphical representation of the profits (properly define here in loss) of the pharmaceutical company described.

7 Research Findings

In this portion, we briefly discuss what we found from this total research paper. The study findings are described as below,

(a) Three different types of linear equations are solved in neutrosophic environment and their behaviour are also addressed.
(b) We convert the linear equation into a neutrosophic equation and obtain the final result with three different cases.
(c) A real-life application, like budgeting and finance, is depicted for a better understanding of neutrosophic equations.

8 Conclusions and Future Research Scope

Traditional fuzzy numbers are extended by NFN with incorporating degrees of membership, indeterminacy and non-membership, making them more appropriate for controlling ambiguous data. We have solved the linear fuzzy differential equation with the concept of neutrosophy. In various real-life applications, like decision-making, healthcare diagnosis and engineering related problems, these equations are highly productive. These equations deliver a more elaborate mathematical structural work by integrating neutrosophic concepts for handling tough situations with uncertain environments. Their applications exhibit increased accuracy and scalability compared to standard fuzzy set theory. Finally, neutrosophic fuzzy equations offer strong equipment for solving different kinds of problems, including uncertainty and ambiguity in several fields. We exhibit the required solution of the neutrosophic equation of the three different cases with the help of the basic concept of Hukuhara difference, neutrosophic characterization theorem and interval arithmetic operations, which are constructed in this research paper. The solutions of neutrosophic equation concepts are also applied here to real-life applications.

This research work has some limitations. So, its extension of future research work describes here. We use triangular neutrosophic number to solve the linear fuzzy equation. The solution of these equations can also be found with different types of neutrosophic numbers, s.t., non-linear neutrosophic numbers, trapezoidal neutrosophic numbers, single valued trapezoidal neutrosophic numbers, type-2 neutrosophic numbers, etc. We can also consider the non-linear, non-homogeneous neutrosophic equations and find more numerical and analytical methods to solve these. Moreover, we can solve more real-life oriented applications associated with uncertain data.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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