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Application of Complex Polytopic Fuzzy Information Systems in Knowledge Engineering: Decision Support for COVID-19 Vaccine Selection

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Abstract: This paper aims to introduce the concepts of complex Polytopic fuzzy sets (CPoFSs) and complex Polytopic fuzzy numbers (CPoFNs), advancing the field of fuzzy logic. Three innovative aggregation operators based on CPoFNs are presented: The complex Polytopic fuzzy weighted averaging aggregation (CPoFWAA) operator, the complex Polytopic fuzzy ordered weighted averaging aggregation (CPoFOWAA) operator, and the complex Polytopic fuzzy hybrid averaging aggregation (CPoFHAA) operator. A significant application of these complex Polytopic fuzzy sets is their integration into decision-making processes, particularly in identifying the most suitable COVID-19 vaccines for patients. This application highlights the practical relevance and the innovative nature of the proposed methods. The paper further demonstrates the efficacy and efficiency of these methods through a comprehensive example provided towards the end, underscoring their potential in real-world scenarios.

Keywords: CPoFSs; CPoFWAA operator; CPoFOWAA operator; CPoFHAA operator; Decision-making

1 Introduction

Historically, the foundation of decision-making problems was predicated on the assumption that alternatives, crucial for rendering informed decisions, were comprised of definitive numerical values. However, it has been recognized that the majority of decision-making occurs under conditions of ambiguity and poorly delineated objectives. In response to these real-world complexities, a spectrum of theoretical frameworks has been developed to address the uncertainty inherent in various scenarios. Soft set (SS) theory, introduced by Molodtsov [1], represents an evolution of classical set theory, providing a versatile framework for the resolution of ambiguities encountered in decision-making and information processing. This theory has been instrumental in introducing flexibility and intuition into the handling of indeterminate data. In parallel, the fuzzy set (FS) theory, formulated by Zadeh [2], has offered a robust mathematical infrastructure for grappling with indeterminacy and imprecision. It has established itself as a fundamental approach for the manipulation, management, and depiction of information that may not be precisely defined, a common occurrence in practical situations. Complementing these, the rough set (RS) theory, developed by Pawlak [3], addresses the challenges of uncertainty, vagueness, and incomplete knowledge within data analysis and decision-making processes. Through the application of discernibility and equivalence relations, rough sets provide a formal apparatus for delineating sets in the presence of imprecise information.

Each theoretical model presented in above mentioned literatures has been distinguished by its unique, significant applications. It is important to acknowledge that the inception of FS theory by Zadeh [2] introduced a paradigm where the concept of a membership function was employed to quantify an object's level of satisfaction. However, it was soon identified that FS theory exhibited limitations, particularly its inability to concurrently handle information pertaining to both satisfaction and dissatisfaction. In order to report this weakness, Atanassov [4] generalized FSs to intuitionistic fuzzy (IF) sets where each element mathematically presented as: (μ, ν) , with $0 \leq \mu + \nu \leq 1$. Intuitionistic fuzzy sets aim to capture and represent uncertainty, vagueness, and hesitation in a more comprehensive manner compared to traditional fuzzy sets. Later on, Yager [5] presented Pythagorean fuzzy (PyF) sets which relaxes the limitation of IFSs, such as $0 \leq \mu+\nu \leq 1$ to $0 \leq \mu^2+\nu^2 \leq 1$. After that Sanapati and Yagar [6] presented the idea of Fermatean fuzzy (FeF) sets, which reduces the limitation of PyFSs, such as $0 \leq \mu^2+\nu^2 \leq 1$ to $0 \leq \mu^3+\nu^3 \leq 1$.

Similarly, Yager [7] developed q-rung orthopair fuzzy sets (q-ROF) sets, which reduces the limitations some previous model, such as PyF-sets and FeF-sets and $0 \leq \mu^3 + \nu^3 \leq 1$ to $0 \leq \mu^q + \nu^q \leq 1$ respectively.

There are several cases, where the above models failed, due to the neutral membership degree. Therefore, Cuong et al. [8] presented a new model known as picture fuzzy sets (PcFSs) to deal with the occurring problems. In PcFSs each element mathematically may be presented as: (μ, ℓ, ν) with $0 \leq \mu + \ell + \nu \leq 1$. Ashraf et al. [9] introduced spherical fuzzy sets (SpFSs), which reduces the limitation of PcFSs to $0 \leq \mu^2 + \ell^2 + \nu^2 \leq 1$. Later on, Beg et al. [10] presented Polytopic fuzzy sets (PoFSs), which reduces the limitation of PcFSs and SpFSs to $0 \leq \mu^q + \ell^q + \nu^q \leq 1$. Later on, some scholars [11–16] using IF numbers and presented many operators and their applications in real life. Rahman et al. [17–20] presented various operators using PyF numbers. Garg [21, 22] are presented Einstein t-norm and t-conorm based on PyF numbers. Liu and Wang [23] developed q-ROFWA method and the q-ROFWG method. Peng and Liu [24] introduced inclusion, entropy, measures and distance measures. Garg [25], Beg et al. [26] and introduced several techniques and their applications on in real life problems.

The above models are unable to depict the data's partial ignorance and how it changes over a certain time period. Therefore, Ramot et al. [27] introduced complex fuzzy (CF) sets, which is an extension of traditional fuzzy set theory that introduces complex-valued membership function to handle complex. Complex fuzzy sets provide a more expressive representation of uncertainty and ambiguity in complex systems or data domains. Later on, Alkouri and Salleh [28] presented complex intuitionistic fuzzy (CIF) sets, in which each element mathematically presented as: $\left(\mu e^{i2\pi m}, \nu e^{i2\pi n}\right)$, where $\mu \in [0,1]$, $\nu \in [0,1]$, $m \in [0, 2\pi]$ and $n \in [0, 2\pi]$ with $0 \leq \mu + \nu \leq 1$ and $0 < \frac{m}{2\pi} + \frac{n}{2\pi} \leq 1$. Ma et al. [29], Dick et al. [30], Liu and Zhang [31], Rani and Garg [32], Garg and Reni [33], Kumer and Bajej [34] presented some related work and techniques in CIF environments. Greenfield et al. [35] presented complex interval-valued fuzzy (CIVF) sets. Ullah et al. [36] presented complex Pythagorean fuzzy (CPyF) sets, which reduces the limitation of CIF-sets. Liu et al. [37] introduced complex q-rung orthopair fuzzy (Cq-ROF) sets. Rahman et al. [38] and Hezam et al. [39] presented several operational laws under CPyF environment. Thus keeping the advantages of the above mentioned models, in this paper we introduce CPoF sets and their corresponding aggregation along with their structure properties.

Next, the Paper is ordered in the following form: Section 2 presents existing models, namely CF sets, CIF sets, CPyF sets. Section 3 presents the CPoF sets and CPoF numbers. Section 4 presents some novel techniques based on CPoFNs. Section 5 presents emergency decision-making model. Section 6 presents an example. Section 7 present conclusions of the new research.

2 Preliminaries

Definition 1: [27] Let X be a universal set, then complex fuzzy set C on X can be defined as follows: $C = \left\{x, \mu_C(x)e^{im_C(x)} | x \in X\right\}$, where $\mu_C(x) : X \to [0,1]$ and $\mu_C(x)$ is called the membership function of x in the complex plane.

Definition 2: [28] Let X be a universal set, then complex intuitionistic fuzzy set D on X can be defined as: $D = \left\{\langle x, \mu_D(x)e^{im_D(x)}, \nu_D(x)e^{in_D(x)}\rangle | x \in X\right\}$, where $\mu_D(x) : X \to [0,1]$, $\nu_D(x) : X \to [0,1]$, $m_D(x) \in [0, 2\pi]$ and $n_D(x) \in [0, 2\pi]$ under the conditions: $0 < \mu_D(x) + \nu_D(x) \leq 1$ and $0 < \frac{m_D(x)}{2\pi} + \frac{n_D(x)}{2\pi} \leq 1$.

Definition 3: [36] Let X be a universal set, then complex Pythagorean fuzzy set V can be defined as: $V = \left\{\langle x, \mu_V(x)e^{im_V(x)}, \nu_V(x)e^{in_V(x)}\rangle | x \in X\right\}$, where $\mu_V(x) : X \to [0,1]$, $\nu_V(x) : X \to [0,1]$, $m_V(x) \in [0, 2\pi]$ and $n_V(x) \in [0, 2\pi]$ under the conditions: $0 < (\mu_V(x))^2 + (\nu_V(x))^2 \leq 1$ and $0 < \left(\frac{m_V(x)}{2\pi}\right)^2 + \left(\frac{n_V(x)}{2\pi}\right)^2 \leq 1$.

3 Complex Polytopic Fuzzy Sets

Definition 4: Let X be a universal set, then complex Polytopic fuzzy set S can be defined on X as follows: $V = \left\{\langle x, \mu_S(x)e^{im_S(x)}, \ell_S(x)e^{ir_S(x)}, \nu_S(x)e^{in_S(x)}\rangle | x \in X\right\}$ where $\mu_S(x) : X \to [0,1]$, $\ell_S(x) : X \to [0,1]$, $\nu_S(x) : X \to [0,1]$, $m_S(x) \in [0, 2\pi]$, $r_S(x) \in [0, 2\pi]$ and $n_S(x) \in [0, 2\pi]$ under the conditions:$0 < (\mu_S(o))^q + (\ell_S(o))^q + (v_S(o))^q \leq 1 (1 \leq q)$ and $0 < \left(\frac{m_S(o)}{2\pi}\right)^2 + \left(\frac{r_S(o)}{2\pi}\right)^2 + \left(\frac{n_S(o)}{2\pi}\right)^2 \leq 1$.

Definition 5: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(j = 1, 2)$ be a family of CPoFNs and $p > 0$, then

i) $$R_1 \oplus R_2 = \begin{pmatrix} (\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q)^{\frac{1}{q}} e^{i2\pi\left(\left(\frac{m_1}{2\pi}\right)^q + \left(\frac{m_2}{2\pi}\right)^q - \left(\frac{m_1}{2\pi}\right)^q \left(\frac{m_2}{2\pi}\right)^q\right)^{\frac{1}{q}}}, \\ (\ell_1 \ell_2) e^{i2\pi\left(\frac{r_1}{2\pi}\right)\left(\frac{r_2}{2\pi}\right)}, (\nu_1 \nu_2) e^{i2\pi\left(\frac{n_1}{2\pi}\right)\left(\frac{n_2}{2\pi}\right)} \end{pmatrix}$$

ii) $$p(R) = \left((1 - (1 - \mu^q)^p)^{\frac{1}{q}} e^{i2\pi\left(1 - \left(1 - \left(\frac{m}{2\pi}\right)^q\right)^p\right)^{\frac{1}{q}}}, (\ell)^p e^{i2\pi\left(\frac{r}{2\pi}\right)^p}, (\nu)^p e^{i2\pi\left(\frac{n}{2\pi}\right)^p}\right)$$

Definition 6: Let $R = \left(\mu e^{im}, \ell e^{ir}, \nu e^{in}\right)$ be a CPoFN, then its score $S(R)$ and accuracy $A(R)$ can be defined as: $S(R) = \frac{1}{3}\left[(1 + \mu^q + \ell^q - \nu^q) + (1 + m^q + r^q - n^q)\right]$ with condition: $S(R) \in [2, 2]$ and $A(R) = \frac{1}{2}\left[(1 + max(\mu^q, \ell^q) - \nu^q) + (1 + max(m^q, r^q) - n^q)\right]$ with condition: $A(R) \in [0, 2]$ respectively.

Definition 7: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ be a family of CPoFNs, then

1) If, $S(R_1) > S(R_2)$, this means that $R_1 > R_2$

2) If, $S(R_1) < S(R_2)$, this means that $R_1 < R_2$

3) If, $S(R_1) = S(R_2)$, then we have three conditions as given below:

i) If, $A(R_1) > S(R_2)$, this means that $R_1 > R_2$

ii) If, $A(R_1) > S(R_2)$, this means that $R_1 > R_2$

iii) If, $A(R_1) = S(R_2)$, this means that $R_1 = R_2$

Theorem 1: Symmetry property: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq 2)$ be a group of CPoFNs and $(R_j)^c = \left(\nu_j e^{in_j}, \ell_j e^{ir_j}, \mu_j e^{im_j}\right)$ $(1 \leq j \leq 2)$ be their corresponding complement respectively, then $S(R_1) \leq S(R_2) \Leftrightarrow S(R_1)^c \geq S(R_2)^c$.

Proof. By Definition 6, we have

$S(R_1) = \frac{1}{3}\left[(1 + \mu_1^q + \ell_1^q - \nu_1^q) + (1 + m_1^q + r_1^q - n_1^q)\right]$

and

$S(R_2) = \frac{1}{3}\left[(1 + \mu_2^q + \ell_2^q - \nu_2^q) + (1 + m_2^q + r_2^q - n_2^q)\right]$.

Since $S(R_1) \leq S(R_2)$, then we have

$$
\begin{aligned}
&\Leftrightarrow S(R_1) = \frac{1}{3}\left[(1 + \mu_1^q + \ell_1^q - \nu_1^q) + (1 + m_1^q + r_1^q - n_1^q)\right] \\
&\leq S(R_2) = \frac{1}{3}\left[(1 + \mu_2^q + \ell_2^q - \nu_2^q) + (1 + m_2^q + r_2^q - n_2^q)\right] \\
&\Leftrightarrow S(R_1) = \frac{1}{3}\left[(1 - \mu_1^q + \ell_1^q + \nu_1^q) + (1 - m_1^q + r_1^q + n_1^q)\right] \\
&\geq S(R_2) = \frac{1}{3}\left[(1 - \mu_2^q + \ell_2^q + \nu_2^q) + (1 - m_2^q + r_2^q + n_2^q)\right] \\
&\Leftrightarrow S(R_1) = \frac{1}{3}\left[(1 + \nu_1^q + \ell_1^q - \mu_1^q) + (1 + n_1^q + r_1^q - m_1^q)\right] \\
&\geq S(R_2) = \frac{1}{3}\left[(1 + \nu_2^q + \ell_2^q - \mu_2^q) + (1 + n_2^q + r_2^q - m_2^q)\right] \\
&\Leftrightarrow S(R_1)^c \geq S(R_2)^c
\end{aligned}
$$

Theorem 2: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq 3)$ be a family of CPoFNs, then

1) Commutative laws:

i) $R_1 \oplus R_2 = R_2 \oplus R_1$

ii) $R_1 \otimes R_2 = R_2 \otimes R_1$

2) Associative laws:

i) $(R_1 \oplus R_2) \oplus R_3 = R_1 \oplus (R_2 \oplus R_3)$

ii) $(R_1 \otimes R_2) \otimes R_3 = R_1 \otimes (R_2 \otimes R_3)$

3) Distributive laws:

i) $R_1 \otimes (R_2 \oplus R_3) = (R_1 \otimes R_2) \oplus (R_1 \otimes R_3)$

ii) $(R_1 \oplus R_2) \otimes R_3 = (R_1 \otimes R_3) \oplus (R_2 \otimes R_3)$

Proof. i) Since $R_1 = \left(\mu_1 e^{im_1}, \ell_1 e^{ir_1}, \nu_1 e^{in_1}\right)$ and $R_2 = \left(\mu_2 e^{im_2}, \ell_2 e^{ir_2}, \nu_2 e^{in_2}\right)$ are two CPoFNs, then by Definition 5, we can prove it and the other parts can be proved by applying the same process. $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(j = 1, 2)$ be a family of CPoFNs and $p > 0$, then

$$\text{i)} \quad R_1 \oplus R_2 = \begin{pmatrix} (\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q)^{\frac{1}{q}} e^{i2\pi\left((\frac{m_1}{2\pi})^q + (\frac{m_2}{2\pi})^q - (\frac{m_1}{2\pi})^q (\frac{m_2}{2\pi})^q\right)^{\frac{1}{q}}}, \\ (\ell_1 \ell_2) e^{i2\pi(\frac{r_1}{2\pi})(\frac{r_2}{2\pi})}, (\nu_1 \nu_2) e^{i2\pi(\frac{n_1}{2\pi})(\frac{n_2}{2\pi})} \end{pmatrix}$$

$$= \begin{pmatrix} (\mu_2^q + \mu_1^q - \mu_2^q \mu_1^q)^{\frac{1}{q}} e^{i2\pi\left((\frac{m_2}{2\pi})^q + (\frac{m_1}{2\pi})^q - (\frac{m_2}{2\pi})^q (\frac{m_1}{2\pi})^q\right)^{\frac{1}{q}}}, \\ (\ell_2 \ell_1) e^{i2\pi(\frac{r_2}{2\pi})(\frac{r_1}{2\pi})}, (\nu_2 \nu_1) e^{i2\pi(\frac{n_2}{2\pi})(\frac{n_1}{2\pi})} \end{pmatrix}$$

$$= \quad R_2 \oplus R_1$$

Theorem 3: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq 2)$ be a family of CPoFNs, and $p > 0$, then the resulting values of: i) $R_1 \otimes R_2$, ii) $R_1 \oplus R_2$, iii) $(R)^p$, iv) $p(R)$ are also CPoFNs.

Proof. The proof is straight forward and can easily be obtained by using the above stated Definitions. Therefore, it is omitted here.

Theorem 4: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq 3)$ be a family of CPoFNs, $p, p_1, p_2 > 0$, then

i) $p(R_1 \oplus R_2) = p(R_1) \oplus p(R_2)$

ii) $(R_1 \oplus R_2)^p = (R_1)^p \oplus (R_2)^p$

iii) $p_1(R_1) \oplus p_2(R_1) = (p_1 \oplus p_2)R_1$

iv) $(R_1)^{p_1} \otimes (R_1)^{p_2} = (R_1)^{p_1 \oplus p_2}$

Proof. We prove only (i) and the reaming parts can be easily proved by the same process

$$p(R_1 \oplus R_2) = \begin{pmatrix} \left(1 - \prod_{j=1}^{2}(1 - \mu_j^q)^p\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \prod_{j=1}^{2}(1 - (\frac{m_j}{2\pi})^q)^p\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{2} \ell_j^p e^{i2\pi\left(\prod_{j=1}^{2}(\frac{r_j}{2\pi})^p\right)^{\frac{1}{q}}}, \prod_{j=1}^{2} \nu_j^p e^{i2\pi\left(\prod_{j=1}^{2}(\frac{n_j}{2\pi})^p\right)^{\frac{1}{q}}} \end{pmatrix}$$

$$= \left((1 - (1 - \mu_1^q)^p)^{\frac{1}{q}} e^{i2\pi\left(1 - (1 - (\frac{m_1}{2\pi})^q)^p\right)^{\frac{1}{q}}}, \ell_1^p e^{\frac{1}{q}} e^{i2\pi\left((\frac{r_1}{2\pi})^p\right)}, \nu_1^p e^{\frac{1}{q}} e^{i2\pi\left((\frac{n_1}{2\pi})^p\right)} \right)$$

$$+ \left((1 - (1 - \mu_2^q)^p)^{\frac{1}{q}} e^{i2\pi\left(1 - (1 - (\frac{m_2}{2\pi})^q)^p\right)^{\frac{1}{q}}}, \ell_2^p e^{\frac{1}{q}} e^{i2\pi\left((\frac{r_2}{2\pi})^p\right)}, \nu_2^p e^{\frac{1}{q}} e^{i2\pi\left((\frac{n_2}{2\pi})^p\right)}\right)$$

$$= p(R_1) \oplus p(R_2)$$

Theorem 5: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq 3)$ be a family of CPoFNs, then

i) $(R_1 \cup R_2) \cap R_1 = R_1$

ii) $(R_1 \cap R_2) \cup R_1 = R_1$

i) $(R_1 \cup R_2) \oplus (R_1 \cap R_2) = R_1 \oplus R_2$

Proof. We prove part (i), the remaining part can be prove by the same process.

$$(R_1 \cup R_2) \cap R_1 = \begin{pmatrix} max\{\mu_1, \mu_2\} e^{i(max\{m_1, m_2\})}, max\{\ell_1, \ell_2\} e^{i(max\{r_1, r_2\})}, \\ min\{\nu_1, \nu_2\} e^{i(min\{n_1, n_2\})} \cap \left(\mu_1 e^{im_1}, \ell_1 e^{ir_1}, \nu_1 e^{in_1}\right) \end{pmatrix}$$

$$= \begin{pmatrix} min\{max\{\mu_1, \mu_2\}, \mu_1\} e^{i(min\{(max\{m_1, m_2\}, m_1\})}, \\ min\{max\{\ell_1, \ell_2\}, \ell_1\} e^{i(min\{(max\{r_1, r_2\}, m_1\})}, \\ min\{max\{\nu_1, \nu_2\}, \nu_1\} e^{i(min\{(max\{n_1, n_2\}, n_1\})} \end{pmatrix}$$

$$= \left(\mu_1 e^{im_1}, \ell_1 e^{ir_1}, \nu_1 e^{in_1}\right) = R_1$$

4 Aggregation Operators under Complex Polytopic Fuzzy Information

Definition 8: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq n)$ be a family of CPoFNs, with their weighted vector $\upsilon = (\upsilon_1, \upsilon_2, ..., \upsilon_n)^T$ with $(1 \leq \upsilon_j \leq n)$ and $\sum_{j=1}^{n} \upsilon_j = 1$. Then CPoFWAA operator can be defined as:

$$\begin{aligned} & CPoFWAA_\upsilon \left(R_1, R_2, R_3, ..., R_n\right) \\ &= \left(\begin{array}{c} \left(1 - \prod_{j=1}^{n} (1-\mu_j^q)^{\upsilon_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{n}(1-(\frac{m_j}{2\pi})^q)^{\upsilon_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n} (\ell_j)^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{n}(\frac{r_j}{2\pi})^{\upsilon_j}\right)}, \prod_{j=1}^{n} (\nu_j)^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{n}(\frac{n_j}{2\pi})^{\upsilon_j}\right)} \end{array} \right) \end{aligned} \tag{1}$$

Example 1: We consider the following four CPoFNs, with weights $\upsilon = (0.10, 0.20, 0.30, 0.40)$ and $q = 3$.

$$\begin{aligned} R_1 &= \left(0.50e^{i2\pi(0.70)}, 0.80e^{i2\pi(0.50)}, 0.70e^{i2\pi(0.60)}\right) \\ R_2 &= \left(0.50e^{i2\pi(0.70)}, 0.80e^{i2\pi(0.50)}, 0.70e^{i2\pi(0.60)}\right) \\ R_3 &= \left(0.70e^{i2\pi(0.60)}, 0.50e^{i2\pi(0.40)}, 0.40e^{i2\pi(0.80)}\right) \\ R_4 &= \left(0.90e^{i2\pi(0.60)}, 0.40e^{i2\pi(0.50)}, 0.50e^{i2\pi(0.70)}\right) \end{aligned}$$

First, we calculate the required values as below:

$$\left(1 - \prod_{j=1}^{4} (1-\mu_j^q)^{\upsilon_j}\right)^{\frac{1}{q}} = \left(1 - \left(1-(0.50)^3\right)^{0.10} \left(1-(0.80)^3\right)^{0.20} \left(1-(0.70)^3\right)^{0.30} \left(1-(0.90)^3\right)^{0.40}\right)^{\frac{1}{3}} = 0.82$$

$$\left(1 - \prod_{j=1}^{4} (1-(\frac{m_j}{2\pi})^q)^{\upsilon_j}\right)^{\frac{1}{q}} = \left(1 - \left(1-(0.70)^3\right)^{0.10} \left(1-(0.60)^3\right)^{0.20} \left(1-(0.60)^3\right)^{0.30} \left(1-(0.60)^3\right)^{0.40}\right)^{\frac{1}{3}} = 0.82$$

$$\begin{aligned} \prod_{j=1}^{4} (\ell_j)^{\upsilon_j} &= (0.80)^{0.10}(0.50)^{0.20}(0.50)^{0.30}(0.40)^{0.40} = 0.47 \\ \prod_{j=1}^{4} (\frac{r_j}{2\pi})^{\upsilon_j} &= (0.50)^{0.10}(0.70)^{0.20}(0.40)^{0.30}(0.50)^{0.40} = 0.50 \\ \prod_{j=1}^{4} (\nu_j)^{\upsilon_j} &= (0.70)^{0.10}(0.60)^{0.20}(0.40)^{0.30}(0.50)^{0.40} = 0.50 \\ \prod_{j=1}^{4} (\frac{n_j}{2\pi})^{\upsilon_j} &= (0.60)^{0.10}(0.60)^{0.20}(0.80)^{0.30}(0.70)^{0.40} = 0.69 \end{aligned}$$

Now applying the CPoFWAA operator, we have

$$\begin{aligned} & CPoFWAA_\upsilon \left(R_1, R_2, R_3, R_4\right) \\ &= \left(\begin{array}{c} \left(1 - \prod_{j=1}^{4} (1-\mu_j^q)^{\upsilon_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{4}(1-(\frac{m_j}{2\pi})^q)^{\upsilon_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{4} (\ell_j)^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{4}(\frac{r_j}{2\pi})^{\upsilon_j}\right)}, \prod_{j=1}^{4} (\nu_j)^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{4}(\frac{n_j}{2\pi})^{\upsilon_j}\right)} \end{array} \right) \\ &= \left(0.82e^{2\pi(0.61)}, 0.47e^{2\pi(0.50)}, 0.50e^{2\pi(0.69)}\right) \end{aligned}$$

Property I (Idempotency): Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq n)$ be a family of CPoFNs, with their weighted vector $\upsilon = (\upsilon_1, \upsilon_2, ..., \upsilon_n)^T$ with the conditions $(1 \leq \upsilon_j \leq n)$ and $\sum_{j=1}^{n} \upsilon_j = 1$ and $R_j = R$, then

$$CPoFWAA_\upsilon \left(R_1, R_2, ..., R_n\right) = \upsilon_1 R_1 \oplus \upsilon_2 R_2 \oplus ... \oplus \upsilon_n R_n = R \tag{2}$$

Property II (Boundedness): Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq n)$ be a family of CPoFNs and let $R_{max} = \left(\mu_{max} e^{im_{max}}, \ell_{max} e^{ir_{max}}, \nu_{max} e^{in_{max}}\right)$ with conditions, such as: $\mu_{max} = max_j\{\mu_j\}$, $m_{max} = max_j\{m_j\}$, $\ell_{max} = max_j\{\ell_j\}$, $r_{max} = max_j\{r_j\}$, $\nu_{max} = max_j\{\nu_j\}$, $n_{max} = max_j\{n_j\}$. Similarly on the same way, we can process for minimum values with we have $R_{min} = \left(\mu_{min} e^{im_{min}}, \ell_{min} e^{ir_{min}}, \nu_{min} e^{in_{min}}\right)$ with $\mu_{min} = min_j\{\mu_j\}$, $m_{min} = min_j\{m_j\}$, $\ell_{min} = min_j\{\ell_j\}$, $r_{min} = min_j\{r_j\}$, $\nu_{min} = min_j\{\nu_j\}$, $n_{min} = min_j\{n_j\}$, then , we have

$$R_{min} \leq CPoFWAA_\upsilon \left(R_1, R_2, ..., R_n\right) \leq R_{max} \tag{3}$$

Property III (Monotonicity): Let there are two any families of CPoFNs, such that $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq n)$ and $R_j^\circ = \left(\mu_j^\circ e^{im_j^\circ}, \ell_j^\circ e^{ir_j^\circ}, \nu_j^\circ e^{in_j^\circ}\right)(1 \leq j \leq n)$ satisfying the conditions: $\mu_J \leq \mu_j^\circ$, $m_j \leq m_j^\circ$, $\ell_j \leq \ell_j^\circ, r_j \leq r_j^\circ, \nu_j \geq \nu_j^\circ, n_j \geq n_j^\circ$, then

$$CPoFWAA_\upsilon \left(R_1, R_2, R_3, ..., R_n\right) \leq CPoFWAA_\upsilon \left(R_1^\circ, R_2^\circ, R_3^\circ, ..., R_n^\circ\right) \tag{4}$$

Definition 9: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \leq j \leq n)$ be a family of CPoFNs, with their weighted vector $\upsilon = (\upsilon_1, \upsilon_2, ..., \upsilon_n)^T$ with $(1 \leq \upsilon_j \leq n)$ and $\sum_{j=1}^{n} \upsilon_j = 1$, where $(\propto(1), \propto(2), ..., \propto(n))$, is any reordered of $(1, 2, ..., n)$ with $R_{\propto(j)} \leq R_{\propto(j-1)}$. Then the CPoFOWAA operator can be presented as follows:

$$\begin{aligned} &CPoFWAA_\upsilon \left(R_1, R_2, R_3, ..., R_n\right) \\ &= \begin{pmatrix} \left(1 - \prod_{j=1}^{n}(1 - \mu_{\propto(j)}^q)^{\upsilon_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1 - \prod_{j=1}^{n}(1 - (\frac{m_{\propto(j)}}{2\pi})^q)^{\upsilon_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n}(\ell_{\propto(j)})^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{n}(\frac{r_{\propto(j)}}{2\pi})^{\upsilon_j}\right)}, \prod_{j=1}^{n}(\nu_{\propto(j)})^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{n}(\frac{n_{\propto(j)}}{2\pi})^{\upsilon_j}\right)} \end{pmatrix} \end{aligned} \tag{5}$$

Example 2: To develop the above aggregation operators, we consider an example. For this, we consider the following four CPoFNs, along with their weighted vector $\upsilon = (0.10, 0.20, 0.30, 0.40)$ and $q = 4$.

$$\begin{aligned} R_1 &= \left(0.90e^{i2\pi(0.60)}, 0.40e^{i2\pi(0.50)}, 0.50e^{i2\pi(0.70)}\right) \\ R_2 &= \left(0.80e^{i2\pi(0.60)}, 0.50e^{i2\pi(0.70)}, 0.60e^{i2\pi(0.60)}\right) \\ R_3 &= \left(0.70e^{i2\pi(0.60)}, 0.50e^{i2\pi(0.40)}, 0.40e^{i2\pi(0.80)}\right) \\ R_4 &= \left(0.50e^{i2\pi(0.70)}, 0.80e^{i2\pi(0.50)}, 0.70e^{i2\pi(0.60)}\right) \end{aligned}$$

First, by calculating the scores, and get the results bellow:

$$\begin{aligned} S(R_1) &= \frac{1}{3}\left[\left(1 + (0.90)^4 + (0.40)^4 - (0.50)^4\right) + \left(1 + (0.60)^4 + (0.50)^4 - (0.70)^4\right)\right] \\ S(R_2) &= \frac{1}{3}\left[\left(1 + (0.80)^4 + (0.50)^4 - (0.60)^4\right) + \left(1 + (0.60)^4 + (0.70)^4 - (0.60)^4\right)\right] \\ S(R_3) &= \frac{1}{3}\left[\left(1 + (0.70)^4 + (0.50)^4 - (0.40)^4\right) + \left(1 + (0.60)^4 + (0.40)^4 - (0.80)^4\right)\right] \\ S(R_4) &= \frac{1}{3}\left[\left(1 + (0.50)^4 + (0.80)^4 - (0.70)^4\right) + \left(1 + (0.70)^4 + (0.50)^4 - (0.60)^4\right)\right] \end{aligned}$$

Thus, we have the following values

$$R_{\propto(1)} = \left(0.80e^{i2\pi(0.60)}, 0.50e^{i2\pi(0.70)}, 0.60e^{i2\pi(0.60)}\right)$$
$$R_{\propto(2)} = \left(0.90e^{i2\pi(0.60)}, 0.40e^{i2\pi(0.50)}, 0.50e^{i2\pi(0.70)}\right)$$
$$R_{\propto(3)} = \left(0.50e^{i2\pi(0.70)}, 0.80e^{i2\pi(0.50)}, 0.70e^{i2\pi(0.60)}\right)$$
$$R_{\propto(4)} = \left(0.70e^{i2\pi(0.60)}, 0.50e^{i2\pi(0.40)}, 0.40e^{i2\pi(0.80)}\right)$$

$$\left(1-\prod_{j=1}^{4}(1-\mu_j^q)^{\upsilon_j}\right)^{\frac{1}{q}} = \left(\begin{array}{l} 1-\left(1-(0.80)^4\right)^{0.10}\left(1-(0.90)^4\right)^{0.20}\left(1-(0.50)^4\right)^{0.30} \\ \left(1-(0.70)^4\right)^{0.40} \end{array}\right)^{\frac{1}{4}}$$
$$= 0.96$$

$$\left(1-\prod_{j=1}^{4}(1-(\frac{m_j}{2\pi})^q)^{\upsilon_j}\right)^{\frac{1}{q}} = \left(\begin{array}{l} 1-\left(1-(0.60)^4\right)^{0.10}\left(1-(0.60)^4\right)^{0.20}\left(1-(0.70)^4\right)^{0.30} \\ \left(1-(0.60)^4\right)^{0.40} \end{array}\right)^{\frac{1}{4}}$$
$$= 0.84$$

$$\prod_{j=1}^{4}(\ell_j)^{\upsilon_j} = (0.50)^{0.10}(0.40)^{0.20}(0.80)^{0.30}(0.50)^{0.40} = 0.55$$
$$\prod_{j=1}^{4}(\frac{r_j}{2\pi})^{\upsilon_j} = (0.70)^{0.10}(0.50)^{0.20}(0.50)^{0.30}(0.40)^{0.40} = 0.47$$
$$\prod_{j=1}^{4}(\nu_j)^{\upsilon_j} = (0.60)^{0.10}(0.50)^{0.20}(0.70)^{0.30}(0.40)^{0.40} = 0.51$$
$$\prod_{j=1}^{4}(\frac{n_j}{2\pi})^{\upsilon_j} = (0.60)^{0.10}(0.70)^{0.20}(0.60)^{0.30}(0.80)^{0.40} = 0.69$$

By using CPoFOWAA operator, we have

$$CPoFWAA_{\upsilon}\left(R_1, R_2, R_3, R_4\right)$$
$$= \left(\begin{array}{l} \left(1-\prod_{j=1}^{4}(1-\mu_{\propto(j)}^q)^{\upsilon_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{4}(1-(\frac{m_{\propto(j)}}{2\pi})^q)^{\upsilon_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{4}(\ell_{\propto(j)})^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{4}(\frac{r_{\propto(j)}}{2\pi})^{\upsilon_j}\right)}, \prod_{j=1}^{4}(\nu_{\propto(j)})^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{4}(\frac{n_{\propto(j)}}{2\pi})^{\upsilon_j}\right)} \end{array}\right)$$
$$= \left(0.96e^{2\pi(0.84)}, 0.55e^{2\pi(0.47)}, 0.51e^{2\pi(0.69)}\right)$$

Definition 10: Let $R_j = \left(\mu_j e^{im_j}, \ell_j e^{ir_j}, \nu_j e^{in_j}\right)$ $(1 \le j \le n)$ be a family of CPoFNs, be a family of CPoFNs, along with their associated vector, such as $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ under conditions $(0 \le \varpi \le 1)$, $\sum_{j=1}^{n}\varpi_j = 1$. Similarly, their weighted vector $\upsilon = (\upsilon_1, \upsilon_2, ..., \upsilon_n)^T$ under conditions, such as $(1 \le \upsilon_j \le n)$, $\sum_{j=1}^{n}\upsilon_j = 1$. And $R_{\propto(j)} = n\varpi(R_j)$, where $R_{\propto(j)}$ be the maximum value, and n is known as the balancing coefficient, which show a vigorous role to balance the equation. If $\varpi = (\varpi_1, \varpi_2, ..., \varpi_n)^T$ approaches to $\left(\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n}\right)$ then $(n\varpi_1 R_1, n\varpi_2 R_2, ..., n\varpi_n R_n)$ approaches to $(R_1, R_2, ..., R_n)$. Then the CPoFHAA operator mathematically can be presented as follows:

$$CPoFHAA_{\varpi,\upsilon}\left(R_1, R_2, R_3, ..., R_n\right)$$
$$= \left(\begin{array}{l} \left(1-\prod_{j=1}^{n}\left(1-\left(\mu_{\dot{R}_{\propto(j)}}\right)^q\right)^{\upsilon_j}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\prod_{j=1}^{n}\left(1-\left(\frac{m_{\dot{R}\propto(j)}}{2\pi}\right)^q\right)^{\upsilon_j}\right)^{\frac{1}{q}}}, \\ \prod_{j=1}^{n}(\ell_{\dot{R}\propto(j)})^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{n}\left(\frac{r_{\dot{R}\propto(j)}}{2\pi}\right)^{\upsilon_j}\right)}, \prod_{j=1}^{n}(\nu_{\dot{R}\propto(j)})^{\upsilon_j} e^{i2\pi\left(\prod_{j=1}^{n}\left(\frac{n_{\dot{R}\propto(j)}}{2\pi}\right)^{\upsilon_j}\right)} \end{array}\right) \quad (6)$$

5 An Application of the Proposed Approaches

Algorithm: Let $\mathrm{A} = \{\mathrm{A}_1, \mathrm{A}_2, ..., \mathrm{A}_n\}$ be a fixed set of m alternative and $\mathrm{C} = \{\mathrm{C}_1, \mathrm{C}_2, ..., \mathrm{C}_n\}$ be a fixed set of n criteria, whose weighted vector is $\upsilon = (\upsilon_1, \upsilon_2, ..., \upsilon_n)$ under conditions, such as $(1 \leq \upsilon_j \leq n)$ and $\sum_{j=1}^{n} \upsilon_j = 1$. Let $\mathrm{E} = \{\mathrm{E}_1, \mathrm{E}_2, ..., \mathrm{E}_n\}$ group of k experts, whose weighted vector is $\omega = (\omega_1, \omega_2, ..., \omega_n)$ with $(1 \leq \omega_j \leq n)$ and $\sum_{j=1}^{n} \omega_j = 1$.

Step 1: collect all data in matrices form.

Step 2: Aggregate all individuals' matrices into a single collective decision matrix.

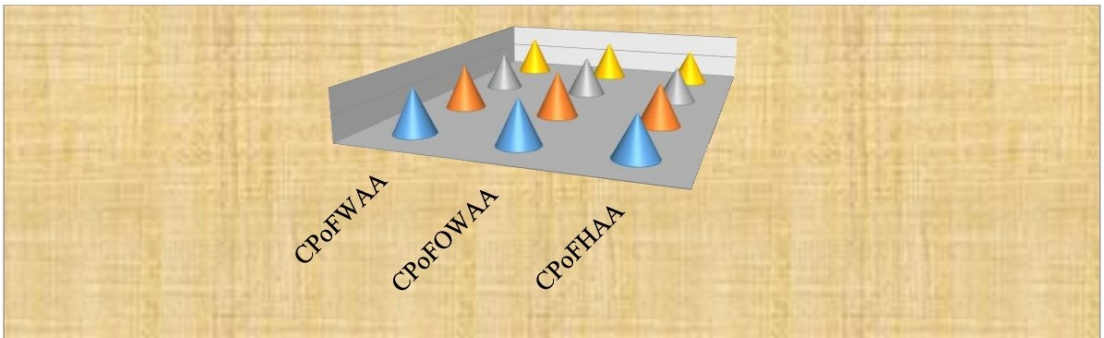
Step 3: Again utilize all the proposed techniques to computing the preference values.

Step 4: Calculate the score function.

Step 5: Sort out according to the score and choose the one with the highest score value.

6 Illustrative Example

The outbreak of a novel coronavirus, subsequently named COVID-19, originated in China in late December 2019. The first incidence in Pakistan was reported in March 2020. The nation has since been grappling with the ramifications of the pandemic. To mitigate the rapid propagation of the virus across the country, the Pakistani government sought to identify the most efficacious vaccine for its populace. The government of Pakistan assigned this task to a committee of four experts/ decision-makers to select the more suitable vaccine for COVID-19 patients in Pakistan whose weighted vector is $\omega = (0.2, 0.1, 0.4, 0.3)$. Many vaccines can be used for COVID-19 patients, but hare the experts considered only the four vaccines to control the spreading rate COVID-19 such as: A_1 Pfizer-BioNTech, A_2 Johnson and Johnson's Janssen vaccine, A_3 AstraZeneca, A_4 Moderna: The decision makers take decision about the above four short listed vaccine according to the following four criteria, whose weighted vector is $\upsilon = (0.3, 0.3, 0.2, 0.2)$ and $q = 4$. C_1 : Availability of the vaccine, C_2 : Age and Health Status, C_3 : Travel Considerations, C_4 : Efficacy of the vaccine. All information of about COVID-19 IN Pakistan are in the following: (Figure 1, Figure 2, Figure 3).



	CPoFWAA	CPoFOWAA	CPoFHAA
Pfizer BioNTech	0.88	0.87	0.78
Johnson	0.89	0.9	0.83
Astra Zeneca	0.79	0.82	0.74
Moderna	0.86	0.85	0.76

Figure 1. Ranking of all methods

Step 1: All information pertaining to the decision-makers are systematically organized into matrix form, as depicted in Tables 1- 4.

Step 2: Combine all the individual matrices into a single matrix using the CPoFWAA operator, with $\omega = (0.2, 0.1, 0.4, 0.3)$ and $q = 4$ (Table 5).

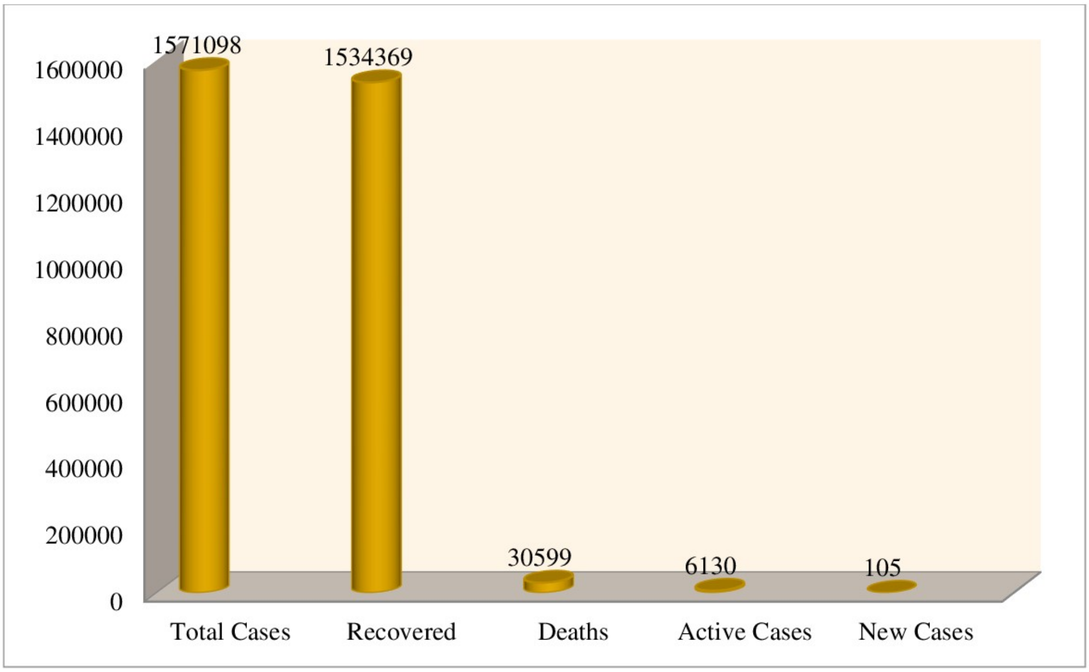


Figure 2. Confirmed cases distribution in Pakistan

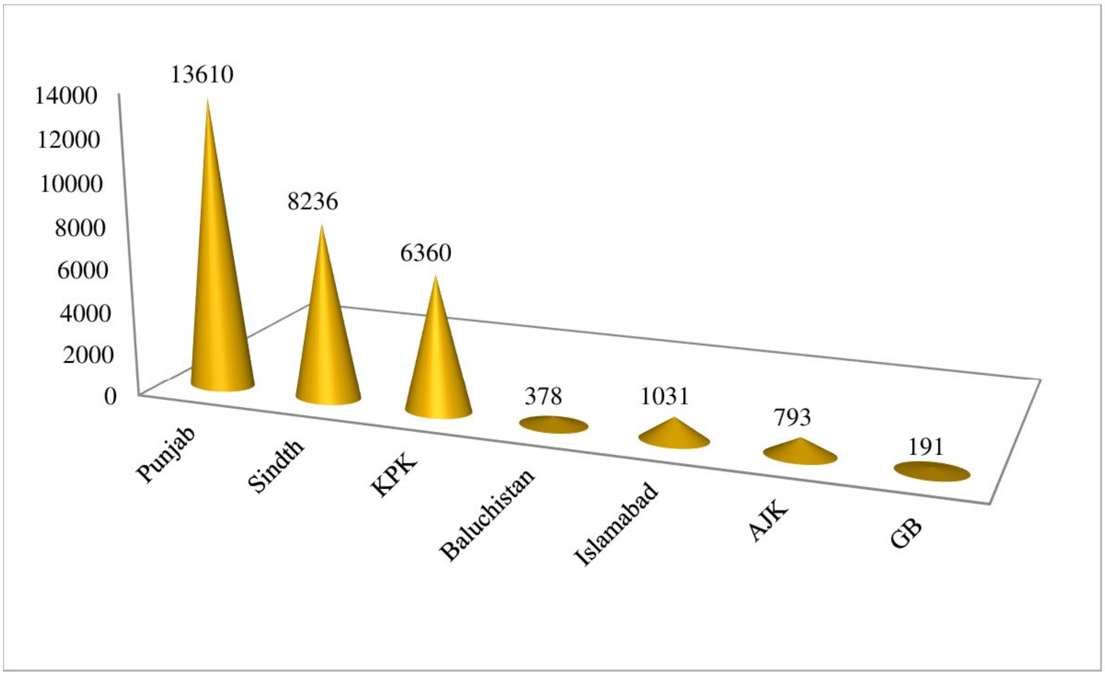


Figure 3. Province wise confirmed deaths cases in Pakistan

Step 3: Next, by using CPoFWAA operator, with $\upsilon = (0.3, 0.3, 0.2, 0.2)$, the preference values are attained:

$$r_1 = \left(0.86e^{i2\pi(0.66)}, 0.69e^{i2\pi(0.58)}, 0.64e^{i2\pi(0.70)}\right)$$
$$r_2 = \left(0.82e^{i2\pi(0.64)}, 0.74e^{i2\pi(0.62)}, 0.70e^{i2\pi(0.61)}\right)$$
$$r_3 = \left(0.78e^{i2\pi(0.66)}, 0.74e^{i2\pi(0.51)}, 0.81e^{i2\pi(0.58)}\right)$$
$$r_4 = \left(0.79e^{i2\pi(0.77)}, 0.75e^{i2\pi(0.66)}, 0.78e^{i2\pi(0.71)}\right)$$

Table 1. Decision of the 1st expert

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
$\mathbf{A_1}$	$\begin{pmatrix}0.48e^{i2\pi(0.54)},\\ 0.77e^{i2\pi(0.57)},\\ 0.38e^{i2\pi(0.49)}\end{pmatrix}$	$\begin{pmatrix}0.48e^{i2\pi(0.54)},\\ 0.77e^{i2\pi(0.57)},\\ 0.38e^{i2\pi(0.49)}\end{pmatrix}$	$\begin{pmatrix}0.62e^{i2\pi(0.63)},\\ 0.65e^{i2\pi(0.81)},\\ 0.71e^{i2\pi(0.56)}\end{pmatrix}$	$\begin{pmatrix}0.48e^{i2\pi(0.54)},\\ 0.77e^{i2\pi(0.57)},\\ 0.38e^{i2\pi(0.49)}\end{pmatrix}$
$\mathbf{A_2}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\ 0.58e^{i2\pi(0.46)},\\ 0.56e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.90e^{i2\pi(0.70)},\\ 0.60e^{i2\pi(0.50)},\\ 0.50e^{i2\pi(0.90)}\end{pmatrix}$	$\begin{pmatrix}0.80e^{i2\pi(0.60)},\\ 0.50e^{i2\pi(0.70)},\\ 0.60e^{i2\pi(0.70)}\end{pmatrix}$
$\mathbf{A_3}$	$\begin{pmatrix}0.75e^{i2\pi(0.62)},\\ 0.38e^{i2\pi(0.67)},\\ 0.72e^{i2\pi(0.45)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\ 0.58e^{i2\pi(0.46)},\\ 0.56e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_4}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\ 0.58e^{i2\pi(0.46)},\\ 0.56e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$

Table 2. Decision of the 2nd expert

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
$\mathbf{A_1}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\ 0.58e^{i2\pi(0.46)},\\ 0.56e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_2}$	$\begin{pmatrix}0.54e^{i2\pi(0.68)},\\ 0.55e^{i2\pi(0.63)},\\ 0.57e^{i2\pi(0.64)}\end{pmatrix}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\ 0.58e^{i2\pi(0.46)},\\ 0.56e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.54e^{i2\pi(0.68)},\\ 0.55e^{i2\pi(0.63)},\\ 0.57e^{i2\pi(0.64)}\end{pmatrix}$
$\mathbf{A_3}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\ 0.58e^{i2\pi(0.46)},\\ 0.56e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_4}$	$\begin{pmatrix}0.75e^{i2\pi(0.62)},\\ 0.38e^{i2\pi(0.67)},\\ 0.72e^{i2\pi(0.45)}\end{pmatrix}$	$\begin{pmatrix}0.72e^{i2\pi(0.63)},\\ 0.65e^{i2\pi(0.81)},\\ 0.51e^{i2\pi(0.56)}\end{pmatrix}$	$\begin{pmatrix}0.54e^{i2\pi(0.68)},\\ 0.55e^{i2\pi(0.63)},\\ 0.57e^{i2\pi(0.64)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$

Table 3. Decision of the 3rd expert

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
$\mathbf{A_1}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.54e^{i2\pi(0.68)},\\ 0.55e^{i2\pi(0.63)},\\ 0.57e^{i2\pi(0.64)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.70e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_2}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_3}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_4}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\ 0.47e^{i2\pi(0.45)},\\ 0.56e^{i2\pi(0.72)}\end{pmatrix}$

Step 4: Computing the score functions as:

$$S(r_1) = \frac{1}{3}\left[\left(1+(0.86)^4+(0.69)^4-(0.64)^4\right)+\left(1+(0.66)^4+(0.58)^4-(0.70)^4\right)\right]$$
$$S(r_2) = \frac{1}{3}\left[\left(1+(0.82)^4+(0.74)^4-(0.70)^4\right)+\left(1+(0.64)^4+(0.62)^4-(0.61)^4\right)\right]$$
$$S(r_3) = \frac{1}{3}\left[\left(1+(0.78)^4+(0.74)^4-(0.81)^4\right)+\left(1+(0.65)^4+(0.51)^4-(0.58)^4\right)\right]$$
$$S(r_4) = \frac{1}{3}\left[\left(1+(0.70)^4+(0.75)^4-(0.78)^4\right)+\left(1+(0.77)^4+(0.66)^4-(0.71)^4\right)\right]$$

Table 4. Decision of the 4^{th} expert

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
$\mathbf{A_1}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\0.47e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.80e^{i2\pi(0.60)},\\0.40e^{i2\pi(0.40)},\\0.70e^{i2\pi(0.50)}\end{pmatrix}$	$\begin{pmatrix}0.51e^{i2\pi(0.46)},\\0.57e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\0.47e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_2}$	$\begin{pmatrix}0.51e^{i2\pi(0.46)},\\0.57e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.51e^{i2\pi(0.46)},\\0.57e^{i2\pi(0.45)},\\0.46e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\0.47e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\0.58e^{i2\pi(0.46)},\\0.56e^{i2\pi(0.68)}\end{pmatrix}$
$\mathbf{A_3}$	$\begin{pmatrix}0.80e^{i2\pi(0.60)},\\0.40e^{i2\pi(0.40)},\\0.70e^{i2\pi(0.50)}\end{pmatrix}$	$\begin{pmatrix}0.51e^{i2\pi(0.46)},\\0.57e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\0.47e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\0.47e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$
$\mathbf{A_4}$	$\begin{pmatrix}0.73e^{i2\pi(0.51)},\\0.58e^{i2\pi(0.46)},\\0.56e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.46)},\\0.47e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.51e^{i2\pi(0.46)},\\0.57e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$	$\begin{pmatrix}0.58e^{i2\pi(0.46)},\\0.47e^{i2\pi(0.45)},\\0.56e^{i2\pi(0.72)}\end{pmatrix}$

Table 5. Combined decision of all decision-makers

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
$\mathbf{A_1}$	$\begin{pmatrix}0.72e^{i2\pi(0.58)},\\0.73e^{i2\pi(0.61)},\\0.84e^{i2\pi(0.67)}\end{pmatrix}$	$\begin{pmatrix}0.70e^{i2\pi(0.68)},\\0.67e^{i2\pi(0.42)},\\0.69e^{i2\pi(0.54)}\end{pmatrix}$	$\begin{pmatrix}0.67e^{i2\pi(0.62)},\\0.79e^{i2\pi(0.46)},\\0.78e^{i2\pi(0.70)}\end{pmatrix}$	$\begin{pmatrix}0.81e^{i2\pi(0.53)},\\0.67e^{i2\pi(0.75)},\\0.85e^{i2\pi(0.71)}\end{pmatrix}$
$\mathbf{A_2}$	$\begin{pmatrix}0.71e^{i2\pi(0.65)},\\0.56e^{i2\pi(0.55)},\\0.80e^{i2\pi(0.62)}\end{pmatrix}$	$\begin{pmatrix}0.57e^{i2\pi(0.55)},\\0.68e^{i2\pi(0.61)},\\0.81e^{i2\pi(0.70)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.68)},\\0.55e^{i2\pi(0.49)},\\0.90e^{i2\pi(0.59)}\end{pmatrix}$	$\begin{pmatrix}0.83e^{i2\pi(0.44)},\\0.65e^{i2\pi(0.56)},\\0.78e^{i2\pi(0.62)}\end{pmatrix}$
$\mathbf{A_3}$	$\begin{pmatrix}0.94e^{i2\pi(0.63)},\\0.72e^{i2\pi(0.34)},\\0.86e^{i2\pi(0.60)}\end{pmatrix}$	$\begin{pmatrix}0.96e^{i2\pi(0.58)},\\0.72e^{i2\pi(0.67)},\\0.68e^{i2\pi(0.45)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.75)},\\0.59e^{i2\pi(0.64)},\\0.86e^{i2\pi(0.48)}\end{pmatrix}$	$\begin{pmatrix}0.65e^{i2\pi(0.74)},\\0.75e^{i2\pi(0.53)},\\0.86e^{i2\pi(0.92)}\end{pmatrix}$
$\mathbf{A_4}$	$\begin{pmatrix}0.89e^{i2\pi(0.74)},\\0.77e^{i2\pi(0.47)},\\0.94e^{i2\pi(0.64)}\end{pmatrix}$	$\begin{pmatrix}0.89e^{i2\pi(0.77)},\\0.75e^{i2\pi(0.55)},\\0.96e^{i2\pi(0.68)}\end{pmatrix}$	$\begin{pmatrix}0.89e^{i2\pi(0.54)},\\0.75e^{i2\pi(0.68)},\\0.96e^{i2\pi(0.45)}\end{pmatrix}$	$\begin{pmatrix}0.68e^{i2\pi(0.43)},\\0.95e^{i2\pi(0.72)},\\0.73e^{i2\pi(0.56)}\end{pmatrix}$

Step 5: Thus, the best option is A_2.

Score functions of the all novel methods are presented (Table 6, Table 7)

Table 6. Scores of the novel approaches

Operators	$\mathrm{A_1}$	$\mathrm{A_2}$	$\mathrm{A_3}$	$\mathrm{A_4}$
CPoFWAA	0.88	0.89	0.79	0.86
CPoFOWAA	0.87	0.90	0.82	0.85
CPoFHAA	0.78	0.83	0.74	0.76

Table 7. Ranking of various approaches

Operators	**Score Functions**	**Ranking**
CPoFWAA	$S(r_2) \succ S(r_1) \succ S(r_4) \succ S(r_3)$	$A_3 \prec A_4 \prec A_1 \prec A_2$
CPoFOWAA	$S(r_2) \succ S(r_1) \succ S(r_4) \succ S(r_3)$	$A_3 \prec A_4 \prec A_1 \prec A_2$
CPoFHAA	$S(r_2) \succ S(r_1) \succ S(r_4) \succ S(r_3)$	$A_3 \prec A_4 \prec A_1 \prec A_2$

7 Conclusions

Complex Polytopic fuzzy sets are a generalization of complex fuzzy sets where the membership grade is defined by a collection of fuzzy membership grades associated with different subregions or regions in the universe of discourse. In this paper, we have presented a novel model, such as complex Polytopic fuzzy set, complex Polytopic fuzzy numbers and some of their basic operational laws. We have also introduced several averaging novel techniques with examples, namely CPoFWAA operator, CPoFOWAA operator, and CPoFHAA operator. We have also developed some of their structure properties, such as idempotency, monotonicity and boundedness. This new model is explained with an illustrative example associated to the assortment of the more suitable option among the existing options. Finally, comparison and sensitivity analysis are presented to show their effectiveness and proficiency.

The potential applications of this research are expansive, offering avenues for further exploration into related domains such as complex Fermatean fuzzy sets, complex Hamacher techniques, and various complex interval-valued and logarithmic techniques. It is projected that theprinciples established within this framework will pave the way for significant advancements in multiple fields requiring intricate decision-making models. In this conclusion, a synthesis of the research findings is presented, illustrating the robustness and versatility of the proposed models and techniques. The practical applications and theoretical implications underscore the contributions to the body of knowledge in complex fuzzy logic systems.

Data Availability

The data used to support the findings of this study are included in this paper.

Conflicts of Interest

The authors declare no conflict of interest.

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