International Journal of Knowledge and Innovation Studies
https://www.acadlore.com/journals/IJKIS

Selection of Enhanced Security Systems Using Complex T-Spherical Fuzzy Models Within a Complex Fuzzy Environment

Muhammad Sajjad Ali Khan*

Department of Mathematics, Khushal Khan Khattak University Karak, 27200 Karak, Pakistan

* Correspondence: Muhammad Sajjad Ali Khan (sajjadalimath@yahoo.com)

Received: 09-19-2024 **Revised:** 11-06-2024 **Accepted:** 11-15-2024

Citation: M. S. A. Khan, "Selection of enhanced security systems using complex T-spherical fuzzy models within a complex fuzzy environment," *Int J. Knowl. Innov Stud.*, vol. 2, no. 4, pp. 208–221, 2024. https://doi.org/10.56578/ijkis020402.

Abstract: The theory of Complex T-Spherical Fuzzy Sets (CTSpFSs) is introduced along with their Einstein operational methods under induced variables. This research aims to extend the theoretical framework of complex fuzzy sets (CFSs) by exploring fundamental Einstein operational laws and proposing two novel aggregation operators: the induced complex T-spherical fuzzy Einstein ordered weighted averaging (I-CTSpFEOWA) operator and the induced complex T-spherical fuzzy Einstein hybrid averaging (I-CTSpFEHA) operator. Aggregation operators serve as powerful tools in data analysis, decision-making, and understanding complex systems by enabling the extraction of meaningful insights from large, multidimensional datasets. These operators contribute to the simplification of information, ultimately enhancing decision support in complex decision-making processes. The proposed operators, designed to handle complex and multidimensional fuzzy information, enhance the ability to refine these decision-making processes. Their effectiveness is demonstrated through the development of a numerical example, which illustrates their potential application in real-world scenarios. The proposed techniques not only improve the clarity and relevance of the aggregated information but also provide an efficient methodology for managing complex fuzzy environments, thus refining decision-making across diverse domains. By demonstrating the utility of the I-CTSpFEOWA and I-CTSpFEHA operators, the research highlights their practical application in systems where traditional fuzzy aggregation methods may fall short. This work contributes significantly to the field of fuzzy set theory by presenting advanced aggregation methods that support improved decision-making in environments characterised by uncertainty and complexity.

Keywords: Complex T-Spherical Fuzzy Sets (CTSpFSs); Induced complex T-spherical fuzzy Einstein ordered weighted averaging (I-CTSpFEOWA); Induced complex T-spherical fuzzy Einstein hybrid averaging (I-CTSpFEHA); Decision-making process

1 Introduction

In practice, as systems grow increasingly complex, decision-making experts often face heightened effort in identifying the ideal choice from a set of possible choices. While attaining a single objective can be challenging, it remains within reach. Organizations frequently grapple with intricate aims involving employee motivation, achievement, and the expansion of shared perspectives. Consequently, organizational decisions, whether made by individuals or groups, typically involve multiple concurrent objectives. The quest for practical and reliable decision-making methods has driven the development of adaptable approaches. Decision-making serves as a key tool, helping decision-makers select the best option by utilizing expert judgment.

To address the complexity inherent in such decision-making contexts, Zadeh [1] introduced fuzzy set (FS), a mathematical framework designed to capture uncertainty. FS contains only one element called the membership degree (MD). So it fails to handle data that contains the non-membership degree (NMD). To address this limitation, Atanassov [2] expanded upon FS, developing intuitionistic fuzzy sets (IFS), which incorporate both MD and NMD to represent uncertainty more comprehensively. In IFS, each element is written as (x, y) with $0 \leq x + y \leq 1$. But there are several cases where this condition fails. For example, if $(x, y) = (0.7, 0.6)$, then $0.7 + 0.6 = 1.3 \succ 1$. To overcome this limitation, Yager [3] introduced the Pythagorean fuzzy set (PyFS), with condition: $0 \leq x^2 + y^2 \leq 1$, then $(0.7)^2 + (0.6)^2 = 0.85 \prec 1$. These models, such as IFS and PyFS, focus only on MD and NMD while overlooking the important role of the neutral degree in real-life problems. However, the neutral degree significantly influences outcomes by balancing opposing factors. Incorporating it enhances analytical precision and provides a more comprehensive understanding of decision dynamics. This integration ensures a more balanced evaluation of choices.

To address these limitations, Cuong and Kreinovich [4] familiarized the Picture fuzzy set (PcFS), with condition: $0 \leq x+y+z \leq 1$, a more flexible model capable of accommodating this expanded range of responses. In practical scenarios, there are limitations to the application of PcFS, especially in cases where the sum of the three degrees of membership (x), neutral (y), and non-membership (z), exceeds 1 , that is $x+y+z \succ 1$. In such instances, PcFS struggles to yield an adequate solution. To address this, researchers in these studies [5, 6] independently familiarized spherical FS (SpFS), relaxing the constraint in PcFS by setting $0 \leq x^2+y^2+z^2 \leq 1$. Later, Mahmood et al. [7] expanded upon SpFS by proposing Tspherical FS (T-SpFS), which further relaxed the condition to $0 \leq x^t+y^t+z^t \leq 1$, where t is any natural number, thereby allowing for greater uncertainty and surpassing traditional fuzzy frameworks in flexibility. For example, if $(x, y, z)=(0.6,0.9,0.5)$, then $0.6+0.9+0.5=2.0 \succ 1$, $(0.6)^2+(0.9)^2+(0.5)^2=1.42 \succ 1$, and $(0.6)^4+(0.9)^4+(0.5)^4=0.83 \prec 1(t=4)$. Thus, T-SpFS model is more flexible as compared to PcFS and SpFS models.

Existing FS and their extension theories fail to capture partial unfamiliarity and temporal changes in data, making them inefficient for handling periodic information. To overcome these issues, Ramot et al. [8] introduced the complex fuzzy set (CFS), extending FS by incorporating complex-valued membership degrees. This advancement enhances the processing of multidimensional data with greater precision and flexibility. As a result, CFS provides a more efficient framework for handling dynamic and evolving information. Alkouri and Salleh [9] present a complex intuitionistic fuzzy set (CIFS), in which each number or element can be written as $\left(xe^{i2\pi a}, ye^{i2\pi b}\right)$ under conditions $0 \leq x+y \leq 1$ and $0 \leq \frac{a}{2\pi}+\frac{b}{2\pi} \leq 1$. A CIFS extends IFS by incorporating complex-valued MD and NMD, allowing for richer uncertainty modeling in pattern recognition, management science, and decision-making. Due to real and imaginary parts, CIFS enhances the representation of uncertainty and vagueness in complex systems. Ullah et al. [10] familiarized complex Pythagorean fuzzy set (CPyFS) with conditions: $0 \leq x^2+y^2 \leq 1$ and $0 \leq \left(\frac{a}{2\pi}\right)^2+\left(\frac{b}{2\pi}\right)^2 \leq 1$. Akram et al. [11] familiarized complex Picture fuzzy set (CPcFS) with conditions: $0 \leq x+y+z \leq 1$ and $0 \leq \frac{a}{2\pi}+\frac{b}{2\pi}+\frac{c}{2\pi} \leq 1$, an advanced mathematical tool that extends PcFSs by incorporating complex-valued membership functions. This approach enables a more nuanced representation of uncertainty in MAGDM problems. Naeem et al. [12] familiarized complex Spherical fuzzy set (CSpFS) under conditions: $0 \leq x^2+y^2+z^2 \leq 1$ and $0 \leq \left(\frac{a}{2\pi}\right)^2+\left(\frac{b}{2\pi}\right)^2+\left(\frac{c}{2\pi}\right)^2 \leq 1$. Ali et al. [13] familiarized CTSpFS, with $0 \leq x^t+y^t+z^t \leq 1$ and $0 \leq \left(\frac{a}{2\pi}\right)^t+\left(\frac{b}{2\pi}\right)^t+\left(\frac{c}{2\pi}\right)^t \leq 1$. Thus, the CTSpFSs is a more advanced and flexible tool than the CPcFS and CSpFS, as it effectively addresses their limitations in handling uncertainty and imprecision. By incorporating a refined membership structure, CTSpFS enhances decision-making processes in complex environments. Its ability to manage higher degrees of imprecision and uncertainty makes it a superior choice for real-world applications.

1.1 Exploring Aggregation Operators: A Comprehensive Literary Review

Aggregation operators play a fundamental and essential role in Multi-Criteria Group Decision Making by integrating multiple inputs into a single output, improving the decision-making process with various information. Several researchers such as Wang and Liu [14, 15], Zhang et al. [16], Al-Kenani et al. [17], Rahman [18] and Rahman et al. [19, 20], have developed various approaches based on IF-information. Their studies have presented innovative approaches, improving the accuracy and applicability of decision models. These contributions continue to shape the development of aggregation techniques in complex decision-making processes. Garg [21, 22], Rahman and Ali [23], Rahman [24], and Rahman et al. [25] presented advanced methods using PyF information, presenting a diverse array of novel methodologies. Their contributions highlight significant applications in many areas, demonstrating the effectiveness of PyF-based techniques. These studies collectively enhance the understanding and application of PyF-driven approaches. Munir et al. [26], Zeng et al. [27], and Liu et al. [28] presented new approaches based on T-SFNs, demonstrating their effectiveness in tackling complex decision-making problems. These methods have been successfully applied, offering efficient and reliable solutions. Their contributions significantly enhance decision-making processes by improving accuracy and computational efficiency. Garg and Reni [29], Rani and Garg [30], and Rahman [31] introduced various aggregation operators utilizing complex intuitionistic fuzzy information, applying them effectively to decision-making problems. Their contributions enhanced the accuracy and applicability of fuzzy decision models in complex scenarios. Rahman et al. [32], Hezam et al. [33], Jin et al. [34], and Liu et al. [35] have introduced various approaches for handling CPyF information. Their contributions highlight innovative methods that enhance the understanding and application of these frameworks. Later, Ali et al. [36] and Rahman et al. [37] introduced various methods using complex Fermatean fuzzy information under complex Fermatean fuzzy numbers, enhancing decision-making processes in uncertain environments. Their work extended fuzzy logic applications, improving accuracy and flexibility in multi-criteria analysis. These advancements have contributed significantly to fields like artificial intelligence and data science. Akram et al. [38] introduced various operators and an enhanced VIKOR method utilizing complex spherical fuzzy information, improving decision-making accuracy. Their approach effectively handles uncertainty and ambiguity in multi-criteria decision analysis.

1.2 Contributions and Motivation of the Novel Research

This research paper advances the field by presenting novel approaches that enhance the applicability, accuracy, and precision of CTSpFS in the fuzzy decision-making process.

i) Basic Operations based on Einstein Norms: The research work introduces basic operational laws within a complex T-spherical fuzzy environment.

ii) Novel Methods based on Induccing Variables: This research presents induced operators, such as the I-CTSpFEOWAA operator and I-CTSpFEHAA operator.

iii) Decision-Making Algorithm: This research work familiarizes an algorithm for decision-making process.

iv) Ensuring Reliability and Enhancing Visibility: This research paper confirms its contributions by showcasing an example that demonstrates the consistency, efficiency, and steadfastness of the novel approaches in improving decision-making algorithms.

1.3 Study Framework

The paper is planned as: Section 2 reviews existing studies relevant to the research, establishing the foundation for the work. Section 3 contains critical operational laws grounded in Einstein information, while Section 4 contains some novel aggregation operators, such as I-CTSpFEOWAA and I-CTSpFEHAA operators. Section 5 explores the application of these newly developed techniques, and Section 6 demonstrates their effectiveness with a detailed example. Section 7 presents the sensitivity analysis of the novel proposed model, focusing on the parameter "t". This section evaluates how variations in "t" influence the model's performance, providing insights into its stability and robustness. Section 8 presents a detailed comparative analysis of the proposed novel model against existing methodologies. Section 9 outlines the limitations of the proposed model. It highlights scenarios where the model fails, particularly when any one of these values reaches its upper limit. Section 10 summarizes the contributions and broader implications of the research.

2 Preliminaries

In this segment, we present main definitions that form the basis of our research, such as CPcFS, CSpFS, and CTSpFS. These ideas provide a structured framework essential for understanding and advancing our new work. Their precise definitions ensure clarity and consistency throughout the study.

Definition 1 [11]: Let $\boldsymbol{Z}$ be a univeristy set and $\boldsymbol{P}$ be a CPcFS, then $\boldsymbol{P}$ can be defined as:

$$P = \left\{ \left\langle z, xe^{i2\pi a}, ye^{i2\pi b}, ze^{i2\pi c} \right\rangle \mid z \in \mathbb{Z} \right\}$$

with conditions: x + y + z ≤ 1 and $\frac{a}{2\pi} + \frac{b}{2\pi} + \frac{c}{2\pi} \leq 1$ where $x, y, z \in [0,1]$ and $a, b, z \in [0, 2\pi]$.

Definition 2 [12]: Let $\boldsymbol{Z}$ be a univeristy set and $\boldsymbol{S}$ be a CSpFS, then $\boldsymbol{S}$ can be defined as:

$$S = \left\{ \left\langle z, xe^{i2\pi a}, ye^{i2\pi b}, ze^{i2\pi c} \right\rangle \mid z \in \mathbb{Z} \right\}$$

where, $x, y, z \in [0,1], a, b, z \in [0, 2\pi]$ with condition: such as $x^2 + y^2 + z^2 \leq 1$ and $\left(\frac{a}{2\pi}\right)^2 + \left(\frac{b}{2\pi}\right)^2 + \left(\frac{c}{2\pi}\right)^2 \leq 1$.

Definition 3 [13]: Let $\boldsymbol{Z}$ be a univeristy set and $\boldsymbol{T}$ be a CTSpFS, then $\boldsymbol{T}$ can be defined as:

$$T = \left\{ \left\langle z, xe^{i2\pi a}, ye^{i2\pi b}, ze^{i2\pi c} \right\rangle \mid z \in \mathbb{Z} \right\}$$

where, $x, y, z \in [0,1]$ and $a, b, z \in [0, 2\pi]$ with conditions: $x^t + y^t + z^t \leq 1$ and $\left(\frac{a}{2\pi}\right)^t + \left(\frac{b}{2\pi}\right)^t + \left(\frac{c}{2\pi}\right)^t \leq 1$.

3 Einstein Operational Laws for T-Spherical Fuzzy Model

In this section, we introduced some basic laws for CTSpFS. Based on these laws, we will develop some aggregation operators under inducing variables.

Definition 4: Let $\mathcal{F} = \left(xe^{i2\pi a}, ye^{i2\pi b}, ze^{i2\pi c}\right) (1 \leq j \leq n)$ are CTSpFS, $\partial \succ 0$, then

i)

$$\mathcal{F}_1 + \mathcal{F}_2 = \begin{bmatrix} \frac{(x_1^t + x_2^t)^{\frac{1}{t}}}{(1 + x_1^t + x_2^t)^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(\left(\frac{a_1}{2\pi}\right)^t + \left(\frac{a_2}{2\pi}\right)^t \right)^{\frac{1}{t}}}{\left(1 + \left(\frac{a_1}{2\pi}\right)^t \left(\frac{a_2}{2\pi}\right)^t \right)^{\frac{1}{t}}} \right), \\ \frac{y_1 y_2}{(1 + (1 - y_1^t)(1 - y_2^t))^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(\frac{b_1}{2\pi}\right)\left(\frac{b_2}{2\pi}\right)}{(1 + (1 - b_1^t)(1 - b_2^t))^{\frac{1}{t}}} \right), \\ \frac{z_1 z_2}{(1 + (1 - z_1^t)(1 - z_2^t))^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(\frac{c_1}{2\pi}\right)\left(\frac{c_2}{2\pi}\right)}{(1 + (1 - c_1^t)(1 - c_2^t))^{\frac{1}{t}}} \right) \end{bmatrix}$$

ii)

$$
\partial\mathcal{F} = \begin{bmatrix}
\frac{\left((1+x^t)^\partial-(1-x^t)^\partial\right)^{\frac{1}{t}}}{((1+x^t)^\partial+(1-x^t)^\partial)^{\frac{1}{q}}} \exp i2\pi \left(\frac{\left(\left(1+\left(\frac{a}{2\pi}\right)^t\right)^\partial-\left(1-\left(\frac{a}{2\pi}\right)^t\right)^\partial\right)^{\frac{1}{t}}}{\left(\left(1+\left(\frac{a}{2\pi}\right)^t\right)^\partial+\left(1-\left(\frac{a}{2\pi}\right)^t\right)^\partial\right)^{\frac{1}{t}}} \right), \\
\frac{(2y^{t\partial})^{\frac{1}{t}}}{((2-y^t)^\partial+y^{t\partial})^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(2\left(\frac{b}{2\pi}\right)^{t\partial}\right)^{\frac{1}{t}}}{\left(\left(2-\left(\frac{b}{2\pi}\right)^t\right)^\partial+\left(\frac{b}{2\pi}\right)^{t\partial}\right)^{\frac{1}{t}}} \right), \\
\frac{(2z^{t\partial})^{\frac{1}{t}}}{((2-z^t)^\partial+z^{t\partial})^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(2\left(\frac{c}{2\pi}\right)^{t\partial}\right)^{\frac{1}{t}}}{\left(\left(2-\left(\frac{c}{2\pi}\right)^t\right)^\partial+\left(\frac{c}{2\pi}\right)^{t\partial}\right)^{\frac{1}{t}}} \right)
\end{bmatrix}
$$

Definition 5: Let $\mathcal{F} = \left(xe^{i2\pi a}, ye^{i2\pi b}, ze^{i2\pi c}\right)$ are complex T-spherical fuzzy number (CTSpFN), then the score function:

$$s(\mathcal{F}) = \frac{1}{3}\left[\left(1+x^t+y^t-z^t\right)+\left(1+a^t+b^t-c^t\right)\right]$$

and accuracy degree:

$$s(\mathcal{F}) = \frac{1}{3}\left[\left(1+x^t+y^t-z^t\right)+\left(1+a^t+b^t-c^t\right)\right]$$

with $s(\mathcal{F}) \in [-2,2]$ and $a(\mathcal{F}) \in [0,2]$ respectively.

Theorem 1: Let $\mathcal{F} = \left(xe^{i2\pi a}, ye^{i2\pi b}, ze^{i2\pi c}\right)$ $(1 \leq j \leq 3)$ are CTSpFNs, then

i) $\mathcal{F}_1 + \mathcal{F}_2 = \mathcal{F}_2 + \mathcal{F}_1$

ii) $\mathcal{F}_1 \times \mathcal{F}_2 = \mathcal{F}_2 \times \mathcal{F}_1$

iii) $(\mathcal{F}_1 + \mathcal{F}_2) + \mathcal{F}_3 = \mathcal{F}_1 + (\mathcal{F}_2 + \mathcal{F}_3)$

iv) $(\mathcal{F}_1 \times \mathcal{F}_2) \times \mathcal{F}_3 = \mathcal{F}_1 \times (\mathcal{F}_2 \times \mathcal{F}_3)$

v) $\mathcal{F}_1 \times (\mathcal{F}_2 + \mathcal{F}_3) = (\mathcal{F}_1 \times \mathcal{F}_2) + (\mathcal{F}_1 \times \mathcal{F}_3)$

vi) $(\mathcal{F}_1 + \mathcal{F}_2) \times \mathcal{F}_3 = (\mathcal{F}_1 \times \mathcal{F}_3) + (\mathcal{F}_2 \times \mathcal{F}_3)$

Proof: Proof is easy, so it is omitted here.

4 Induced Complex T-Spherical Fuzzy Einstein Operators

In this section, we present two complex T-spherical fuzzy Einstein operators using inducing variable. Aggregation operators are precise and accurate tools used to combine multiple data points, values, or information into a single representative value. We present induced complex T-spherical fuzzy operators using Einstein t-norm and t-conorm, such as: I-CTSpFEOWAA operator, and I-CTSpFEHAA operator. We present different properties of the proposed operators to make them suitable for specific applications. Some of the common properties are monotonicity, idempotency and boundedness.

Definition 6: Let $\langle \Im_j, \mathcal{F}_j \rangle$ $(1 \leq j \leq n)$ be a finite family 2-tuple of CTSpFNs, with their corresponding weighted vector as: $v = (v_1, v_2, \ldots, v_n)$ such that $(0 \leq v_j \leq 1)$ and $\sum_{j=1}^{n} v_j = 1$. And $\Im_j \in \langle \Im_j, \mathcal{F}_j \rangle$ be the ordered pair of CTFOWA having the j[th] largest value is known as the order-inducing variable, such as $\Im_j \in \langle \Im_j, \mathcal{F}_j \rangle$ and $\mathcal{F}_j$ as the CTF argument. Then the I-CTSpFEOWAA operator can be presented mathematically as:

$$\text{I - CTFEOWAA}_v \langle (\Im_1, \mathcal{F}_1), (\Im_2, \mathcal{F}_2), \ldots, (\Im_n, \mathcal{F}_n) \rangle$$

$$
= \begin{bmatrix}
\frac{\left(\prod_{j=1}^n (1+x_{\propto(j)}^{\ t})^{\nu_j} - \prod_{j=1}^n (1-x_{\propto(j)}^{\ t})^{\nu_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^n (1+x_{\propto(j)}^{\ t})^{\nu_j} + \prod_{j=1}^n (1-x_{\propto(j)}^{\ t})^{\nu_j}\right)^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(\prod_{j=1}^n \left(1+\left(\frac{a_{\propto(j)}}{2\pi}\right)^t\right)^{v_j} - \prod_{j=1}^n \left(1-\left(\frac{a_{\propto(j)}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}}{\left(\left(\prod_{j=1}^n \left(1+\left(\frac{a_{\propto(j)}}{2\pi}\right)^t\right)^{v_j} + \prod_{j=1}^n \left(1-\left(\frac{a_{\propto(j)}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}} \right), \\
\frac{\left(2\prod_{j=1}^n (y_{\propto(j)}^t)^{\nu_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^n (2-y_{\propto(j)}^t)^{\nu_j} + \prod_{j=1}^n (y_{\propto(j)}^t)^{\nu_j}\right)^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(2\prod_{j=1}^n \left(\left(\frac{b_{\propto(j)}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}}{\left(\left(\prod_{j=1}^n \left(2-\left(\frac{b_{\propto(j)}}{2\pi}\right)^t\right)^{v_j} + \prod_{j=1}^n \left(\left(\frac{b_{\propto(j)}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}} \right), \\
\frac{\left(2\prod_{j=1}^n (z_{\propto(j)}^{\ t})^{\nu_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^n (2-z_{\propto(j)}^{\ t})^{\nu_j} + \prod_{j=1}^n (z_{\propto(j)}^{\ t})^{\nu_j}\right)^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(2\prod_{j=1}^n \left(\left(\frac{c_{\propto(j)}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^n \left(2-\left(\frac{c_{\propto(j)}}{2\pi}\right)^t\right)^{v_j} + \prod_{j=1}^n \left(\left(\frac{c_{\propto(j)}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}} \right)
\end{bmatrix}
$$

Example 1: Here, we have an example, for this, we have the following four induced-complex T-spherical fuzzy numbers (I-CTSpFNs), such as:

$$\langle \Im_1, \mathcal{F}_1 \rangle = \left\langle 0.7, \left(0.8e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.6)}\right)\right\rangle$$
$$\langle \Im_2, \mathcal{F}_2 \rangle = \left\langle 0.9, \left(0.7e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.6)}\right)\right\rangle$$
$$\langle \Im_3, \mathcal{F}_3 \rangle = \left\langle 0.6, \left(0.4e^{i2\pi(0.8)}, 0.8e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$$
$$\langle \Im_4, \mathcal{F}_4 \rangle = \left\langle 0.6, \left(0.4e^{i2\pi(0.8)}, 0.8e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$$

And $v = (0.10, 0.20, 0.30, 0.40)$ be their weighted vector, with $t = 4$. According to the inducing variable, we order the value above as follows:

$$\langle \Im_2, \mathcal{F}_2 \rangle = \left\langle 0.9, \left(0.7e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.6)}\right)\right\rangle$$
$$\langle \Im_4, \mathcal{F}_4 \rangle = \left\langle 0.8, \left(0.5e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.5)}\right)\right\rangle$$
$$\langle \Im_1, \mathcal{F}_1 \rangle = \left\langle 0.7, \left(0.8e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.6)}\right)\right\rangle$$
$$\langle \Im_3, \mathcal{F}_3 \rangle = \left\langle 0.6, \left(0.4e^{i2\pi(0.8)}, 0.8e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$$

Next, we need to ordering the values as:

$$\left\langle \Im_{\propto(1)}, \mathcal{F}_{\propto(1)} \right\rangle = \left\langle 0.9, \left(0.7e^{i2\pi(0.6)}, 0.6e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.6)}\right)\right\rangle$$
$$\left\langle \Im_{\propto(2)}, \mathcal{F}_{\propto(2)} \right\rangle = \left\langle 0.8, \left(0.5e^{i2\pi(0.7)}, 0.7e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.5)}\right)\right\rangle$$
$$\left\langle \Im_{\propto(3)}, \mathcal{F}_{\propto(3)} \right\rangle = \left\langle 0.7, \left(0.8e^{i2\pi(0.6)}, 0.5e^{i2\pi(0.7)}, 0.6e^{i2\pi(0.6)}\right)\right\rangle$$
$$\left\langle \Im_{\propto(4)}, \mathcal{F}_{\propto(4)} \right\rangle = \left\langle 0.6, \left(0.4e^{i2\pi(0.8)}, 0.8e^{i2\pi(0.4)}, 0.6e^{i2\pi(0.5)}\right)\right\rangle$$

Now, we need to calculate the required values as:

$$\prod_{j=1}^{4}(1+x_j^t)^{\nu_j} = (1+(0.7)^4)^{0.10}(1+(0.5)^4)^{0.20}(1+(0.8)^4)^{0.30}(1+(0.4)^4)^{0.40} = 1.15$$
$$\prod_{j=1}^{4}(1-x_j^t)^{\nu_j} = (1-(0.7)^4)^{0.10}(1-(0.5)^4)^{0.20}(1-(0.8)^4)^{0.30}(1-(0.4)^4)^{0.40} = 0.81$$
$$\prod_{j=1}^{4}\left(1+\left(\frac{a_j}{2\pi}\right)^t\right)^{\nu_j} = \left(1+\left(\frac{0.6}{2\pi}\right)^4\right)^{0.10}\left(1+\left(\frac{0.7}{2\pi}\right)^4\right)^{0.20}\left(1+\left(\frac{0.6}{2\pi}\right)^4\right)^{0.30}\left(1+\left(\frac{0.8}{2\pi}\right)^4\right)^{0.40} = 1.25$$
$$\prod_{j=1}^{4}\left(1-\left(\frac{a_j}{2\pi}\right)^t\right)^{\nu_j} = \left(1-\left(\frac{0.6}{2\pi}\right)^4\right)^{0.10}\left(1-\left(\frac{0.7}{2\pi}\right)^4\right)^{0.20}\left(1-\left(\frac{0.6}{2\pi}\right)^4\right)^{0.30}\left(1-\left(\frac{0.8}{2\pi}\right)^4\right)^{0.40} = 0.72$$
$$\prod_{j=1}^{4}(y_j^t)^{\nu_j} = ((0.6)^4)^{0.10}((0.7)^4)^{0.20}((0.5)^4)^{0.30}((0.8)^4)^{0.40} = 0.18$$
$$\prod_{j=1}^{4}(2-y_j^t)^{\nu_j} = (2-(0.6)^4)^{0.10}(2-(0.7)^4)^{0.20}(2-(0.5)^4)^{0.30}(2-(0.8)^4)^{0.40} = 1.75$$
$$\prod_{j=1}^{4}\left(\left(\frac{b_j}{2\pi}\right)^t\right)^{\nu_j} = \left(\left(\frac{0.4}{2\pi}\right)^4\right)^{0.10}\left(\left(\frac{0.6}{2\pi}\right)^4\right)^{0.20}\left(\left(\frac{0.7}{2\pi}\right)^4\right)^{0.30}\left(\left(\frac{0.4}{2\pi}\right)^4\right)^{0.40} = 0.06$$
$$\prod_{j=1}^{4}\left(2-\left(\frac{b_j}{2\pi}\right)^t\right)^{\nu_j} = \left(2-\left(\frac{0.4}{2\pi}\right)^4\right)^{0.10}\left(2-\left(\frac{0.6}{2\pi}\right)^4\right)^{0.20}\left(2-\left(\frac{0.7}{2\pi}\right)^4\right)^{0.30}\left(2-\left(\frac{0.4}{2\pi}\right)^4\right)^{0.40} = 1.88$$
$$\prod_{j=1}^{4}(z_j^t)^{\nu_j} = ((0.6)^4)^{0.10}((0.5)^4)^{0.20}((0.6)^4)^{0.30}((0.6)^4)^{0.40} = 0.11$$

$$\prod_{j=1}^{4}(2 - z_j^t)^{\nu_j} = (2-(0.6)^4)^{0.10}(2-(0.5)^4)^{0.20}(2-(0.6)^4)^{0.30}(2-(0.6)^4)^{0.40} = 1.88$$

$$\prod_{j=1}^{4}\left(\left(\frac{c_j}{2\pi}\right)^t\right)^{\nu_j} = \left(\left(\frac{0.6}{2\pi}\right)^4\right)^{0.10}\left(\left(\frac{0.5}{2\pi}\right)^4\right)^{0.20}\left(\left(\frac{0.6}{2\pi}\right)^4\right)^{0.30}\left(\left(\frac{0.5}{2\pi}\right)^4\right)^{0.40} = 0.08$$

$$\prod_{j=1}^{4}\left(2-\left(\frac{c_j}{2\pi}\right)^t\right)^{\nu_j} = \left(2-\left(\frac{0.6}{2\pi}\right)^4\right)^{0.10}\left(2-\left(\frac{0.5}{2\pi}\right)^4\right)^{0.20}\left(2-\left(\frac{0.6}{2\pi}\right)^4\right)^{0.30}\left(2-\left(\frac{0.5}{2\pi}\right)^4\right)^{0.40} = 1.91$$

Using the I-CTSpFEOWA aggregation operator, we proceed as follows:

$$\text{I-CTSpFEOWAA}_v(\mathcal{F}_1, \mathcal{F}_2, \mathcal{F}_3, \mathcal{F}_4)$$
$$= \left[\frac{(1.15-0.81)^{\frac{1}{4}}}{(1.15+0.81)^{\frac{1}{4}}}e^{i2\pi\left(\frac{(1.25-0.72)^{\frac{1}{4}}}{(1.25+0.72)^{\frac{1}{4}}}\right)}, \frac{(2\times(0.18))^{\frac{1}{4}}}{(1.75+0.18)^{\frac{1}{4}}}e^{i2\pi\left(\frac{(2\times(0.06))^{\frac{1}{4}}}{(1.88+0.06)^{\frac{1}{4}}}\right)}, \frac{(2\times(0.11))^{\frac{1}{4}}}{(1.88+0.11)^{\frac{1}{4}}}e^{i2\pi\left(\frac{(2\times(0.08))^{\frac{1}{4}}}{(1.91+0.08)^{\frac{1}{4}}}\right)}\right]$$
$$= \left[\frac{(0.34)^{\frac{1}{4}}}{(1.96)^{\frac{1}{4}}}e^{i2\pi\left(\frac{(0.53)^{\frac{1}{4}}}{(1.97)^{\frac{1}{4}}}\right)}, \frac{(0.36)^{\frac{1}{4}}}{(1.93)^{\frac{1}{4}}}e^{i2\pi\left(\frac{(0.12)^{\frac{1}{4}}}{(1.94)^{\frac{1}{4}}}\right)}, \frac{(0.22)^{\frac{1}{4}}}{(1.99)^{\frac{1}{4}}}e^{i2\pi\left(\frac{(0.16)^{\frac{1}{4}}}{(1.99)^{\frac{1}{4}}}\right)}\right]$$
$$= \left[0.64e^{i2\pi(0.71)}, 0.66e^{i2\pi(0.51)}, 0.67e^{i2\pi(0.53)}\right]$$

Definition 7: Let $\langle \Im_j, \mathcal{F}_j \rangle$ $(1 \leq j \leq n)$ be a group of complex T-spherical fuzzy numbers (CTFNs), and $\vartheta = (\vartheta_1, \vartheta_2, \ldots, \vartheta_n)$ be their associated vector with $\vartheta_j \in [0,1]$ and $\sum_{j=1}^{n} \vartheta_j = 1$. Let $v = (v_1, v_2, \ldots, v_n)$ be the weighted vectors with $v_j \in [0,1]$ and $\sum_{j=1}^{n} v_j = 1$. And $\dot{\mathcal{F}}_{\propto(j)} = n\vartheta_j\mathcal{F}_j$, such that $\dot{\mathcal{F}}_{\propto(j)}$ be the maximum value, and $\boldsymbol{n}$ is known as the balancing coefficient, which shows a vigorous role to balance the equation. If $\vartheta = (\vartheta_1, \vartheta_2, \ldots, \vartheta_n)$ tends to $\left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$, then $(n\vartheta_1\mathcal{F}_1, n\vartheta_2\mathcal{F}_2, \ldots, n\vartheta_n\mathcal{F}_n)$ tends to $(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_n)$. Also, $\mathfrak{I}_j \in \langle \mathfrak{I}_j, \mathcal{F}_j \rangle$ the ordered pair of CTFOWA corresponding to the j-th largest value can be identified. The I-CTSpFEHAA operator is then defined as:

$$\text{I - CTSpFEHAA}_{\vartheta,v}\langle(\Im_1, \mathcal{F}_1), (\Im_2, \mathcal{F}_2), \ldots, (\Im_n, \mathcal{F}_n)\rangle$$
$$= \left[\begin{array}{l} \frac{\left(\prod_{j=1}^{n}\left(1+\left(x_{\dot{\mathcal{F}}_{\infty(j)}}\right)^t\right)^{\nu_j} - \prod_{j=1}^{n}\left(1-\left(x_{\dot{\mathcal{F}}_{\infty(j)}}\right)^t\right)^{\nu_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^{n}\left(1+\left(x_{\dot{\mathcal{F}}_{\infty(j)}}\right)^t\right)^{\nu_j} + \prod_{j=1}^{n}\left(1-\left(x_{\dot{\mathcal{F}}_{\infty(j)}}\right)^t\right)^{\nu_j}\right)^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(\prod_{j=1}^{n}\left(1+\left(\frac{a_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^t\right)^{v_j} - \prod_{j=1}^{n}\left(1-\left(\frac{a_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^{n}\left(1+\left(\frac{a_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^t\right)^{v_j} + \prod_{j=1}^{n}\left(1-\left(\frac{a_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^t\right)^{v_j}\right)^{\frac{1}{t}}}\right), \\ \frac{\left(2\prod_{j=1}^{n}\left(y_{\dot{\mathcal{F}}_{\infty(j)}}\right)^{t\nu_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^{n}\left(2-\left(y_{\dot{\mathcal{F}}_{\infty(j)}}\right)^t\right)^{\nu_j} + \prod_{j=1}^{n}\left(y_{\dot{\mathcal{F}}_{\infty(j)}}\right)^{t\nu_j}\right)^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(2\prod_{j=1}^{n}\left(\frac{b_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^{tv_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^{n}\left(2-\left(\frac{b_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^t\right)^{v_j} + \prod_{j=1}^{n}\left(\frac{b_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^{tv_j}\right)^{\frac{1}{t}}}\right), \\ \frac{\left(2\prod_{j=1}^{n}\left(z_{\dot{\mathcal{F}}_{\infty(j)}}\right)^{t\nu_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^{n}\left(2-\left(z_{\dot{\mathcal{F}}_{\infty(j)}}\right)^t\right)^{\nu_j} + \prod_{j=1}^{n}\left(z_{\dot{\mathcal{F}}_{\infty(j)}}\right)^{t\nu_j}\right)^{\frac{1}{t}}} \exp i2\pi \left(\frac{\left(2\prod_{j=1}^{n}\left(\frac{c_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^{tv_j}\right)^{\frac{1}{t}}}{\left(\prod_{j=1}^{n}\left(2-\left(\frac{c_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^t\right)^{v_j}\right) + \prod_{j=1}^{n}\left(\frac{c_{\dot{\mathcal{F}}_{\propto(j)}}}{2\pi}\right)^{tv_j}\right)^{\frac{1}{t}}}\right) \end{array}\right]$$

5 An Application of the Proposed Approaches

Decision-making is the process of selecting the best solution from multiple alternatives, a vital aspect of human life and critical in fields like business and governance. Trusting one's judgment while remaining open to advice is essential for sound decisions. In complex scenarios, such as multi-criteria group decision-making (MCGDM), complex T-Spherical fuzzy sets provide an effective framework for evaluating and ranking alternatives based on multidimensional, interrelated criteria. By leveraging operators like the I-CTSpFEOWAA and I-CTSpFEHAA, these methods aggregate complex fuzzy information to identify optimal solutions. This approach enables decision-makers to navigate uncertainty and complexity with precision and confidence.

In this Algorithm, we considered a set of $\boldsymbol{m}$ options $\mathring{A} = \left\{\mathring{A}_i, i = 1, 2, \ldots, m\right\}$, a set of $\boldsymbol{n}$ criteria $C = \{C_j, j = 1, 2, \ldots, n\}$ and $v = (v_1, v_2, \ldots, v_n)$ be their corresponding weights under conditions: such that $v_j \in [0,1]$ and $\sum_{j=1}^{n} v_j = 1$. Moreover, a set of $\boldsymbol{e}$ experts $D = \{D_1, D_2, \ldots, D_e\}$ whose weights is $\$ = (\$_1, \$_2, \ldots, \$_e)$. The main steps of the algorithm are as follows in Figure 1.

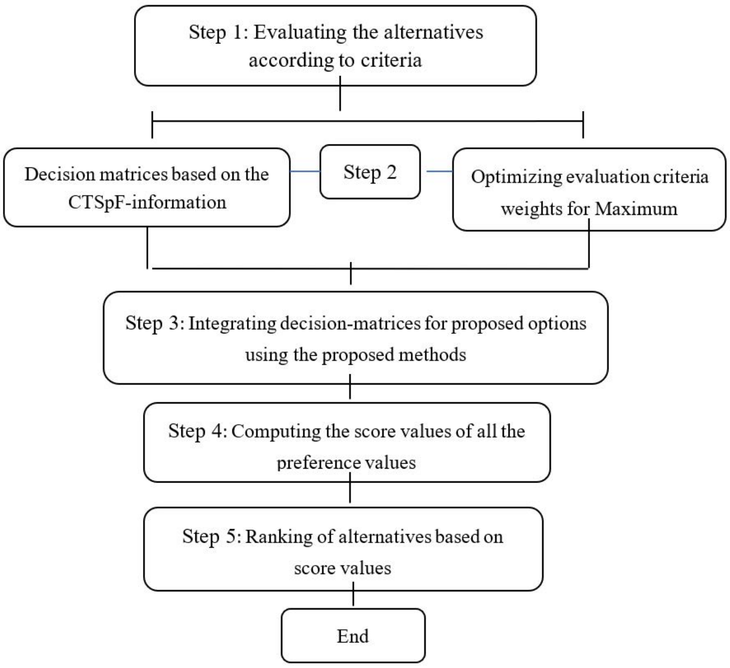

Figure 1. Flowchart for the proposed decision-making

Step 1: In the first step, we putting expert information in the form of matrices, detailing criteria and options for each decision-maker.

$$D_l(1 \leq l \leq e) = \begin{matrix} \mathring{A}_1 \\ \mathring{A}_2 \\ . \\ . \\ . \\ \mathring{A}_m \end{matrix} \begin{pmatrix} C_1 & C_2 & C_3 & . & . & C_n \\ \alpha^e_{11} & \alpha^e_{12} & \alpha^e_{13} & \cdot & \cdot & \alpha^e_{1n} \\ \alpha^e_{21} & \alpha^e_{22} & \alpha^e_{23} & . & . & \alpha^e_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \alpha^e_{m1} & \alpha^e_{m2} & \alpha^e_{m3} & . & . & \alpha^e_{mn} \end{pmatrix}$$

Step 2: Using Definition 6, to combine all individual expert matrices into a unified matrix, integrating diverse evaluations into a comprehensive structure.

Step 3: Again using Definition 6 and Definition 7, and calculate all preference values.

Step 4: Calculating the score function of all preference values.

Step 5: To choose that alternative which has the high value. This approach confirms that the ideal option is identified based on alternatives evaluation.

6 Illustrative Example

In today's world, keeping people and property safe is more important than ever. Organizations such as banks, schools, collages and businesses, must take strong security steps to protect against threats. By using advanced and innovative technology, these systems help ensure that only the right people can enter, creating a safe environment for everyone.

Shaheed Benazir Bhutto University Sheringal required the best security system to ensure safety on the main campus. To make a fair decision, the university made a committee of three experts, whose weights is $\$ = (0.4, 0.3, 0.3)$. The committee carefully reviewed four security systems in the first selection round. Their goal was to choose the most reliable and effective system to protect the university.

Card scanner system (A_1): This system utilizes keycard readers, which are installed at doors. The reader scans encrypted information stored on cards, such as bank cards, ID cards, or student cards, and unlocks the door upon successful verification.

Fingerprint scanner system (A_2): The fingerprint scanner system (FSS) employs optical sensor technology to capture an image of a fingerprint, which is then compared to stored data for authentication. Additional variants include thermal or ultrasonic scanners that operate on similar principles.

Facial recognition system (A_3): Facial recognition systems use specialized algorithms, such as convolutional neural networks (CNNs), along with machine learning and artificial intelligence to analyze human facial features.

These systems convert facial features into a "face print" (numerical code), requiring significant data storage for accurate recognition.

Iris recognition system (A_4): Iris recognition uses mathematical pattern recognition algorithms to identify a person based on the unique and stable patterns in their irises. Infrared light scans the iris, transforming the pattern into an iris code, which is then compared against a database for verification. Iris scanners offer rapid and precise identification. When selecting the best security system, several factors matter, but the experts will focus only on the four main criteria: Cost (C_1): It is a crucial factor, assessing the overall expenses, including installation, maintenance, and any recurring fees. A cost-effective system should provide high value without compromising quality. Reliability (C_2): This refers to the system's ability to function effectively, even in challenging conditions or failure scenarios. Accuracy (C_3): It evaluates how efficiently the system captures, processes, and stores data with minimal errors. A highly accurate system reduces false alarms and enhances overall security. These criteria collectively determine the system's efficiency, dependability, and affordability. A well-balanced system should excel in all these areas, providing robust protection while remaining cost-effective and highly functional. This criterion measures the system's efficiency in accurately capturing and storing data. Compatibility with existing infrastructure (C_4): A system's compatibility with the organization's physical and technological environment is essential for seamless integration and optimal performance.

Let $\upsilon = (0.4, 0.3, 0.2, 0.1)$ be the weighted vector with a given parameter $t = 4$.

Step 1: Decision matrices provided by the decision-making experts are tabulated in Table 1, Table 2 and Table 3.

Step 2: Using the I-CTSpFEOWAA operator to combine all matrices into a single matrix, where $\$ = (0.4, 0.3, 0.3)$ and $t = 4$, and get all information in Table 4.

Table 1. Data of the first expert based on inducing variable

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
A_1	$\left[0.4, \begin{bmatrix} 0.89e^{i2\pi(0.80)} \\ 0.48e^{i2\pi(0.50)} \\ 0.72e^{i2\pi(0.90)} \end{bmatrix}\right]$	$\left[0.5, \begin{bmatrix} 0.78e^{i2\pi(0.40)} \\ 0.87e^{i2\pi(0.50)} \\ 0.56e^{i2\pi(0.90)} \end{bmatrix}\right]$	$\left[0.3, \begin{bmatrix} 0.89e^{i2\pi(0.60)} \\ 0.68e^{i2\pi(0.40)} \\ 0.57e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.2, \begin{bmatrix} 0.87e^{i2\pi(0.60)} \\ 0.78e^{i2\pi(0.40)} \\ 0.67e^{i2\pi(0.30)} \end{bmatrix}\right]$
A_2	$\left[0.5, \begin{bmatrix} 0.68e^{i2\pi(0.50)} \\ 0.47e^{i2\pi(0.40)} \\ 0.86e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.68e^{i2\pi(0.60)} \\ 0.76e^{i2\pi(0.40)} \\ 0.80e^{i2\pi(0.50)} \end{bmatrix}\right]$	$\left[0.6, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.3, \begin{bmatrix} 0.56e^{i2\pi(0.80)} \\ 0.49e^{i2\pi(0.40)} \\ 0.87e^{i2\pi(0.60)} \end{bmatrix}\right]$
A_3	$\left[0.6, \begin{bmatrix} 0.88e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.30)} \\ 0.78e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.7, \begin{bmatrix} 0.78e^{i2\pi(0.40)} \\ 0.87e^{i2\pi(0.50)} \\ 0.56e^{i2\pi(0.90)} \end{bmatrix}\right]$	$\left[0.5, \begin{bmatrix} 0.89e^{i2\pi(0.50)} \\ 0.67e^{i2\pi(0.60)} \\ 0.57e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.58e^{i2\pi(0.72)} \\ 0.47e^{i2\pi(0.63)} \\ 0.79e^{i2\pi(0.44)} \end{bmatrix}\right]$
A_4	$\left[0.3, \begin{bmatrix} 0.56e^{i2\pi(0.80)} \\ 0.49e^{i2\pi(0.40)} \\ 0.87e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.2, \begin{bmatrix} 0.78e^{i2\pi(0.40)} \\ 0.87e^{i2\pi(0.50)} \\ 0.56e^{i2\pi(0.90)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.5, \begin{bmatrix} 0.89e^{i2\pi(0.50)} \\ 0.67e^{i2\pi(0.60)} \\ 0.57e^{i2\pi(0.40)} \end{bmatrix}\right]$

Table 2. Data of the second expert based on inducing variable

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
A_1	$\left[0.2, \begin{bmatrix} 0.87e^{i2\pi(0.60)} \\ 0.78e^{i2\pi(0.40)} \\ 0.67e^{i2\pi(0.30)} \end{bmatrix}\right]$	$\left[0.6, \begin{bmatrix} 0.68e^{i2\pi(0.50)} \\ 0.89e^{i2\pi(0.80)} \\ 0.48e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.3, \begin{bmatrix} 0.58e^{i2\pi(0.70)} \\ 0.68e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.90)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.89e^{i2\pi(0.60)} \\ 0.68e^{i2\pi(0.40)} \\ 0.57e^{i2\pi(0.80)} \end{bmatrix}\right]$
A_2	$\left[0.3, \begin{bmatrix} 0.56e^{i2\pi(0.80)} \\ 0.49e^{i2\pi(0.40)} \\ 0.87e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.1, \begin{bmatrix} 0.89e^{i2\pi(0.60)} \\ 0.68e^{i2\pi(0.40)} \\ 0.57e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.6, \begin{bmatrix} 0.88e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.30)} \\ 0.78e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$
A_3	$\left[0.1, \begin{bmatrix} 0.89e^{i2\pi(0.60)} \\ 0.68e^{i2\pi(0.40)} \\ 0.57e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.7, \begin{bmatrix} 0.39e^{i2\pi(0.80)} \\ 0.58e^{i2\pi(0.90)} \\ 0.79e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.6, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$
A_4	$\left[0.5, \begin{bmatrix} 0.89e^{i2\pi(0.50)} \\ 0.67e^{i2\pi(0.60)} \\ 0.57e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.7, \begin{bmatrix} 0.39e^{i2\pi(0.80)} \\ 0.58e^{i2\pi(0.90)} \\ 0.79e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.3, \begin{bmatrix} 0.62e^{i2\pi(0.80)} \\ 0.69e^{i2\pi(0.50)} \\ 0.72e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.5, \begin{bmatrix} 0.89e^{i2\pi(0.50)} \\ 0.67e^{i2\pi(0.60)} \\ 0.57e^{i2\pi(0.40)} \end{bmatrix}\right]$

Table 3. Data of the third expert based on inducing variable

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
A_1	$\left[0.5, \begin{bmatrix} 0.89e^{i2\pi(0.50)} \\ 0.67e^{i2\pi(0.60)} \\ 0.57e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.7, \begin{bmatrix} 0.39e^{i2\pi(0.80)} \\ 0.58e^{i2\pi(0.90)} \\ 0.79e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.3, \begin{bmatrix} 0.62e^{i2\pi(0.80)} \\ 0.69e^{i2\pi(0.50)} \\ 0.72e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.6, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$
A_2	$\left[0.8, \begin{bmatrix} 0.78e^{i2\pi(0.74)} \\ 0.79e^{i2\pi(0.53)} \\ 0.92e^{i2\pi(0.81)} \end{bmatrix}\right]$	$\left[0.7, \begin{bmatrix} 0.69e^{i2\pi(0.60)} \\ 0.47e^{i2\pi(0.40)} \\ 0.78e^{i2\pi(0.90)} \end{bmatrix}\right]$	$\left[0.5, \begin{bmatrix} 0.78e^{i2\pi(0.74)} \\ 0.79e^{i2\pi(0.53)} \\ 0.92e^{i2\pi(0.81)} \end{bmatrix}\right]$	$\left[0.6, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$
A_3	$\left[0.3, \begin{bmatrix} 0.62e^{i2\pi(0.80)} \\ 0.69e^{i2\pi(0.50)} \\ 0.72e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$	$\left[0.6, \begin{bmatrix} 0.67e^{i2\pi(0.80)} \\ 0.51e^{i2\pi(0.30)} \\ 0.89e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.5, \begin{bmatrix} 0.89e^{i2\pi(0.50)} \\ 0.67e^{i2\pi(0.60)} \\ 0.57e^{i2\pi(0.40)} \end{bmatrix}\right]$
A_4	$\left[0.6, \begin{bmatrix} 0.88e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.30)} \\ 0.78e^{i2\pi(0.60)} \end{bmatrix}\right]$	$\left[0.7, \begin{bmatrix} 0.78e^{i2\pi(0.40)} \\ 0.87e^{i2\pi(0.50)} \\ 0.56e^{i2\pi(0.90)} \end{bmatrix}\right]$	$\left[0.5, \begin{bmatrix} 0.89e^{i2\pi(0.50)} \\ 0.67e^{i2\pi(0.60)} \\ 0.57e^{i2\pi(0.40)} \end{bmatrix}\right]$	$\left[0.4, \begin{bmatrix} 0.58e^{i2\pi(0.50)} \\ 0.78e^{i2\pi(0.60)} \\ 0.67e^{i2\pi(0.80)} \end{bmatrix}\right]$

Table 4. Combined decision of all experts

	$\mathbf{C_1}$	$\mathbf{C_2}$	$\mathbf{C_3}$	$\mathbf{C_4}$
A_1	$\begin{bmatrix} 0.99e^{i2\pi(0.84)} \\ 0.57e^{i2\pi(0.55)} \\ 0.73e^{i2\pi(0.95)} \end{bmatrix}$	$\begin{bmatrix} 0.90e^{i2\pi(0.68)} \\ 0.72e^{i2\pi(0.42)} \\ 0.65e^{i2\pi(0.84)} \end{bmatrix}$	$\begin{bmatrix} 0.96e^{i2\pi(0.65)} \\ 0.83e^{i2\pi(0.42)} \\ 0.75e^{i2\pi(0.37)} \end{bmatrix}$	$\begin{bmatrix} 0.87e^{i2\pi(0.77)} \\ 0.76e^{i2\pi(0.55)} \\ 0.97e^{i2\pi(0.68)} \end{bmatrix}$
A_2	$\begin{bmatrix} 0.72e^{i2\pi(0.64)} \\ 0.57e^{i2\pi(0.56)} \\ 0.79e^{i2\pi(0.61)} \end{bmatrix}$	$\begin{bmatrix} 0.56e^{i2\pi(0.55)} \\ 0.69e^{i2\pi(0.61)} \\ 0.79e^{i2\pi(0.71)} \end{bmatrix}$	$\begin{bmatrix} 0.68e^{i2\pi(0.69)} \\ 0.56e^{i2\pi(0.50)} \\ 0.89e^{i2\pi(0.60)} \end{bmatrix}$	$\begin{bmatrix} 0.57e^{i2\pi(0.55)} \\ 0.68e^{i2\pi(0.61)} \\ 0.81e^{i2\pi(0.70)} \end{bmatrix}$
A_3	$\begin{bmatrix} 0.93e^{i2\pi(0.62)} \\ 0.71e^{i2\pi(0.35)} \\ 0.87e^{i2\pi(0.59)} \end{bmatrix}$	$\begin{bmatrix} 0.95e^{i2\pi(0.57)} \\ 0.71e^{i2\pi(0.66)} \\ 0.69e^{i2\pi(0.46)} \end{bmatrix}$	$\begin{bmatrix} 0.68e^{i2\pi(0.75)} \\ 0.59e^{i2\pi(0.64)} \\ 0.86e^{i2\pi(0.48)} \end{bmatrix}$	$\begin{bmatrix} 0.96e^{i2\pi(0.58)} \\ 0.72e^{i2\pi(0.67)} \\ 0.68e^{i2\pi(0.45)} \end{bmatrix}$
A_4	$\begin{bmatrix} 0.88e^{i2\pi(0.73)} \\ 0.78e^{i2\pi(0.45)} \\ 0.93e^{i2\pi(0.65)} \end{bmatrix}$	$\begin{bmatrix} 0.89e^{i2\pi(0.77)} \\ 0.75e^{i2\pi(0.55)} \\ 0.96e^{i2\pi(0.68)} \end{bmatrix}$	$\begin{bmatrix} 0.89e^{i2\pi(0.54)} \\ 0.75e^{i2\pi(0.68)} \\ 0.96e^{i2\pi(0.45)} \end{bmatrix}$	$\begin{bmatrix} 0.90e^{i2\pi(0.68)} \\ 0.72e^{i2\pi(0.42)} \\ 0.65e^{i2\pi(0.84)} \end{bmatrix}$

Step 3: Again applying the I-CTSpFEOWAA operator with weights $v = (0.1, 0.2, 0.3, 0.4)$.

$$\lambda_1 = \left(0.74e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.49)}, 0.73e^{i2\pi(0.58)}\right)$$
$$\lambda_2 = \left(0.73e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.68)}, 0.70e^{i2\pi(0.69)}\right)$$
$$\lambda_3 = \left(0.82e^{i2\pi(0.64)}, 0.69e^{i2\pi(0.51)}, 0.75e^{i2\pi(0.67)}\right)$$
$$\lambda_4 = \left(0.85e^{i2\pi(0.68)}, 0.68e^{i2\pi(0.49)}, 0.80e^{i2\pi(0.79)}\right)$$

Step 4: In this step, the score functions are calculated as follows:

$$Sc(\lambda_1) = \frac{1}{3}\left[\left(1 + (0.74)^4 + (0.72)^4 - (0.73)^4\right) + \left(1 + (0.67)^4 + (0.49)^4 - (0.58)^4\right)\right] = 0.83$$
$$Sc(\lambda_2) = \frac{1}{3}\left[\left(1 + (0.73)^4 + (0.72)^4 - (0.70)^4\right) + \left(1 + (0.67)^4 + (0.68)^4 - (0.69)^4\right)\right] = 0.81$$
$$Sc(\lambda_3) = \frac{1}{3}\left[\left(1 + (0.82)^4 + (0.69)^4 - (0.75)^4\right) + \left(1 + (0.64)^4 + (0.51)^4 - (0.67)^4\right)\right] = 0.79$$
$$Sc(\lambda_4) = \frac{1}{3}\left[\left(1 + (0.85)^4 + (0.68)^4 - (0.80)^4\right) + \left(1 + (0.68)^4 + (0.49)^4 - (0.79)^4\right)\right] = 0.73$$

Step 5: Thus, the "card scanner system" is the more suitable option.

7 Sensitivity Analysis

To check how stable our new proposed methods are, we need to do a sensitivity analysis. The Einstein norms play an essential role in determining the rankings, so it's important to see how different values affect the results.

To do this, we change the value of parameter t in the proposed aggregation operators and observe how the rankings change. By testing different values, we can find out if the rankings stay consistent or change significantly. This helps us understand how reliable our methods are. Table 5 presents the ranking results for different values of the parameter t, allowing us to compare and analyze the outcomes.

The ranking of alternatives remains unchanged even when different values are used for the given parameter. This can be seen clearly in Table 5. No matter how the parameter is adjusted, the final ranking stays the same.

Table 5. Sensitive analysis

I-CTSpFEWAA	$s(\lambda_1)$	$s(\lambda_2)$	$s(\lambda_3)$	$s(\lambda_4)$	**Best option**
$t = 1$	0.89	0.76	0.82	0.86	A_1
$t = 2$	0.90	0.73	0.80	0.85	A_1
$t = 3$	0.87	0.69	0.83	0.81	A_1
$t = 4$	0.83	0.81	0.79	0.73	A_1
$t = 5$	0.82	0.62	0.78	0.66	A_1
$t = 10$	0.73	0.56	0.67	0.58	A_1
I-CTSpFEHAA	$s(\lambda_1)$	$s(\lambda_2)$	$s(\lambda_3)$	$s(\lambda_4)$	**Best option**
$t = 1$	0.89	0.76	0.87	0.84	A_1
$t = 2$	0.86	0.74	0.85	0.83	A_1
$t = 3$	0.83	0.73	0.81	0.80	A_1
$t = 4$	0.85	0.78	0.76	0.72	A_1
$t = 5$	0.88	0.68	0.81	0.66	A_1
$t = 10$	0.71	0.51	0.64	0.55	A_1

8 Comparison and Discussion Analysis

To evaluate the effectiveness of the proposed methods, a comparison with existing works is conducted in this section using two approaches: first, by comparing the methods individually, and second, by assessing the overall performance of the different models. This comparison will help determine whether the proposed methods offer superior performance. By examining both approaches, a clear and objective evaluation of the methods is provided, leading to more precise and robust assessments. The comparison results are summarised in Table 6.

Table 6. Comparisons of the novel methods with existing methods

Operators	$s(\lambda_1)$	$s(\lambda_2)$	$s(\lambda_3)$	$s(\lambda_4)$	**Best Option**
IFEWA [15]	0.85	0.78	0.83	0.80	A_1
IFEOWA [15]	0.88	0.81	0.86	0.84	A_1
IFEWG [14]	0.87	0.73	0.84	0.80	A_1
IFEOWG [14]	0.90	0.79	0.80	0.81	A_1
PyFEWA [21]	0.87	0.75	0.82	0.78	A_1
PyFEOWA [21]	0.89	0.71	0.81	0.76	A_1
PyFEWG [22]	0.92	0.77	0.81	0.87	A_1
PyFEOWG [22]	0.89	0.76	0.83	0.80	A_1
I-CTSpFEOWAA (proposed)	0.83	0.81	0.79	0.73	A_1
I-CTSpFEHAA (proposed)	0.85	0.78	0.76	0.72	A_1

The findings highlight that operators like I-CTSpFEOWAA and I-CTSpFEHAA allow for flexible decision-making by adjusting their parameters. This adaptability makes them particularly effective for addressing intricate T-Spherical fuzzy problems. By fine-tuning these parameters, decision-makers can align the aggregation process with specific problem requirements. Therefore, our proposed operators provide a robust and versatile framework for tackling complex scenarios in fuzzy decision-making. In essence, our complex T-Spherical fuzzy Dombi operators not only effectively represent fuzzy information but also enhance the flexibility of the information aggregation process through parameter customization. This distinguishes our proposed operators from existing methods, as they cannot introduce flexibility into the data aggregation procedure. Thus, our innovative operators demonstrate advancements and reliability in the decision-making process involving complex T-Spherical fuzzy data, offering decision-makers a more adaptable and superior approach compared to currently available methods, as shown in Table 7 and Figure 2.

Table 7. Comparisons of the novel model with existing models

Models	Falsity	Uncertainty	Periodicity	2-D Information	Indeterminacy	t^{th}-Power
FSs	0	1	0	0	0	0
IFSs	1	1	0	0	1	0
PyFSs	1	1	0	0	1	0
FFSs	1	1	0	0	1	0
CFSs	0	1	1	1	0	0
CIFSs	1	1	1	1	1	0
CPyFSs	1	1	1	1	1	0
CPcFSs	1	1	1	1	1	0
CSpFSs	1	1	1	1	1	0
CTSpFSs	1	1	1	1	1	1

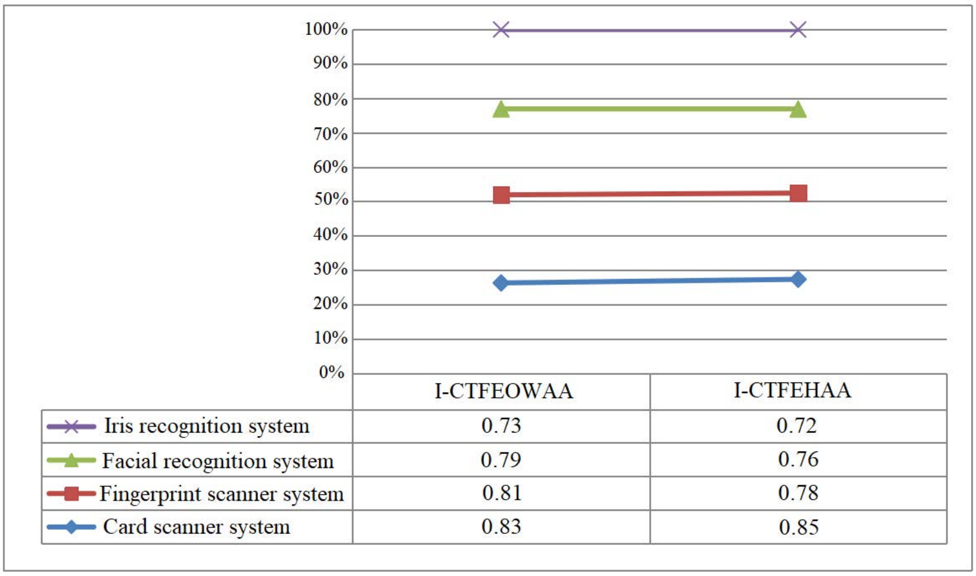

Figure 2. Ranking of all methods

9 Limitations

The CTSpFS enhances traditional fuzzy extensions, including CPcFSs and CSpFSs, offering a powerful tool for handling improbability and uncertainty in decision-making. Its effectiveness spans multiple fields, yet its applicability is limited by the constraint that the t^{th} power of membership (x), neutrality (y), and non-membership (z) grades must collectively remain below one $(0 \leq x^t + y^t + z^t \leq 1)$ and $0 \leq \left(\frac{a}{2\pi}\right)^t + \left(\frac{b}{2\pi}\right)^t + \left(\frac{c}{2\pi}\right)^t \leq 1$.

The limitation of CTSpFS stems from its strict dependence on the balance of membership (x), neutrality (y), and non-membership (z) grades. If any of these values approach their upper limits, the model loses its effectiveness. For instance, if we considered the CTSpFN as: $\left(xe^{i2\pi a}, ye^{i2\pi b}, ze^{i2\pi c}\right) = \left(1.0e^{i2\pi(0.7)}, 0.8e^{i2\pi(0.6)}, 0.7e^{i2\pi(1.0)}\right)$, then the model fails to function properly. This constraint highlights a significant weakness, restricting its adaptability in complex decision-making scenarios. As a result, the model may not always provide accurate or reliable outcomes in cases requiring greater flexibility.

10 Conclusions and Implications

CTSpFSs have been recognised as effective tools for addressing uncertainty and imprecision in decision-making processes. These sets enable the simultaneous consideration of multiple possibilities, facilitating more flexible and robust decision-making in diverse fields such as engineering, medicine, and business, where precise choices are paramount. This research has thoroughly explored CTSpFSs and their associated numbers, introducing innovative methodologies such as the I-CPoFEOWAA and I-CPoFEHAA operators. These techniques exhibit desirable properties, including idempotency, monotonicity, and boundedness, which enhance their utility in practical applications.

A novel fuzzy model incorporating Einstein operations has been demonstrated through a comprehensive illustrative example. The model's efficacy has been validated through detailed comparisons and sensitivity analyses, which confirm its high accuracy, efficiency, and reliability. These strengths underscore the model's potential for application in real-world scenarios, where decision-making often involves complex and uncertain information.

This research also lays a solid foundation for extending the model to various advanced fields. Potential areas for future work include the integration of complex Fermatean fuzzy sets, complex Hamacher operators, and complex logarithmic techniques, as well as the inclusion of complex power techniques and complex Dombi operators. These expansions offer a versatile framework for tackling intricate problems in computational and decision-making contexts. The integration of such techniques enhances the applicability of the model, providing a robust basis for the development of advanced methodologies in areas requiring sophisticated analytical approaches.

In conclusion, the proposed model has proven to be both reliable and effective, offering substantial promise for real-world applications that demand advanced computational and decision-making models.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflict of Interests

The author declares no conflicts of interest.

References

[1] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, 1965. https://doi.org/10.1016/S0019-9958(65)90241-X

[2] K. T. Atanassov, *Intuitionistic Fuzzy Sets*. Physica, Heidelberg, 1999. https://doi.org/10.1007/978-3-7908-1870-3_1

[3] R. R. Yager, "Pythagorean membership grades in multicriteria decision making," *IEEE Trans. Fuzzy Syst.*, vol. 22, no. 4, pp. 958–965, 2013. https://doi.org/10.1109/TFUZZ.2013.2278989

[4] B. C. Cuong and V. Kreinovich, "Picture fuzzy sets-A new concept for computational intelligence problems," in *2013 Third World Congress on Information and Communication Technologies (WICT 2013)*, Hanoi, Vietnam, 2013, pp. 1–6. https://doi.org/10.1109/WICT.2013.7113099

[5] S. Ashraf, S. Abdullah, T. Mahmood, F. Ghani, and T. Mahmood, "Spherical fuzzy sets and their applications in multi-attribute decision making problems," *J. Intell. Fuzzy Syst.*, vol. 36, no. 3, pp. 2829–2844, 2019. https://doi.org/10.3233/JIFS-172009

[6] F. K. Gündoğdu and C. Kahraman, "Spherical fuzzy sets and spherical fuzzy TOPSIS method," *J. Intell. Fuzzy Syst.*, vol. 36, no. 1, pp. 337–352, 2019. https://doi.org/10.3233/JIFS-181401

[7] T. Mahmood, K. Ullah, Q. Khan, and N. Jan, "An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets," *Neural Comput. & Applic.*, vol. 31, pp. 7041–7053, 2019. https://doi.org/10.1007/s00521-018-3521-2

[8] D. Ramot, R. Milo, M. Friedman, and A. Kandel, "Complex fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 2, pp. 171–186, 2002. https://doi.org/10.1109/91.995119

[9] A. M. D. J. S. Alkouri and A. R. Salleh, "Complex intuitionistic fuzzy sets," *AIP Conf. Proc.*, vol. 1482, no. 1, pp. 464–470, 2012. https://doi.org/10.1063/1.4757515

[10] K. Ullah, T. Mahmood, Z. Ali, and N. Jan, "On some distance measures of complex Pythagorean fuzzy sets and their applications in pattern recognition," *Complex Intell. Syst.*, vol. 6, pp. 15–27, 2020. https://doi.org/10.1007/s40747-019-0103-6

[11] M. Akram, A. Bashir, and H. Garg, "Decision-making model under complex picture fuzzy Hamacher aggregation operators," *Comp. Appl. Math.*, vol. 39, p. 226, 2020. https://doi.org/10.1007/s40314-020-01251-2

[12] M. Naeem, M. Qiyas, T. Botmart, S. Abdullah, and N. Khan, "Complex spherical fuzzy decision support system based on entropy measure and power operator," *J. Function Spaces*, vol. 2023, p. 9836721, 2022. https://doi.org/10.1155/2023/9836721

[13] Z. Ali, T. Mahmood, and M. S. Yang, "Complex T-spherical fuzzy aggregation operators with application to multi-attribute decision making," *Symmetry*, vol. 12, no. 8, p. 1311, 2020. https://doi.org/10.3390/sym12081311

[14] W. Z. Wang and X. W. Liu, "Intuitionistic fuzzy geometric aggregation operators based on Einstein operations," *Int. J. Intell. Syst.*, vol. 26, no. 11, pp. 1049–1075, 2011. https://doi.org/10.1002/int.20498

[15] W. Z. Wang and X. W. Liu, "Intuitionistic fuzzy information aggregation using Einstein operations," *IEEE Trans. Fuzzy Syst.*, vol. 20, no. 5, pp. 923–938, 2012. https://doi.org/10.1109/TFUZZ.2012.2189405

[16] X. Zhang, W. Pan, R. Scattolini, S. Yu, and X. Xu, "Robust tube-based model predictive control with Koopman operators," *Automatica*, vol. 137, p. 110114, 2022. https://doi.org/10.1016/j.automatica.2021.110114

[17] A. N. Al-Kenani, R. Anjum, and S. Islam, "Intuitionistic fuzzy prioritized aggregation operators based on priority degrees with application to multicriteria decision-making," *J. Function Spaces*, vol. 2023, p. 9786536, 2022. https://doi.org/10.1155/2023/9786536

[18] K. Rahman, “Some new logarithmic aggregation operators and their application to group decision making problem based on t-norm and t-conorm,” *Soft Comput.*, vol. 26, no. 6, pp. 2751–2772, 2022. https://doi.org/10.1007/s00500-022-06730-8

[19] K. Rahman, I. M. Hezam, D. Božanić, A. Puška, and M. Milovančevic, “Some logarithmic intuitionistic fuzzy Einstein aggregation operators under confidence level,” *Processes*, vol. 11, no. 4, p. 1298, 2023. https://doi.org/10.3390/pr11041298

[20] K. Rahman, B. R. Al-sinan, and A. H. Ali, “Multi-criteria group decision-making problem under intuitionistic fuzzy logarithmic aggregation operators based on T-norm and T-conorm,” *OPSEARCH*, 2024. https://doi.org/10.1007/s12597-024-00847-0

[21] H. Garg, “A new generalized Pythagorean fuzzy information aggregation using Einstein operations and its application to decision making,” *Int. J. Intell. Syst.*, vol. 31, no. 9, pp. 886–920, 2016. https://doi.org/10.1002/int.21809

[22] H. Garg, “Generalized Pythagorean fuzzy geometric aggregation operators using Einstein t-norm and t-conorm for multicriteria decision-making process,” *Int. J. Intell. Syst.*, vol. 32, no. 6, pp. 597–630, 2017. https://doi.org/10.1002/int.21860

[23] K. Rahman and A. Ali, “New approach to multiple attribute group decision-making based on pythagorean fuzzy einstein hybrid geometric operator,” *Granul. Comput.*, vol. 5, pp. 349–359, 2020. https://doi.org/10.1007/s41066-019-00166-6

[24] K. Rahman, “Approaches to some induced Einstein geometric aggregation operators based on interval-valued Pythagorean fuzzy numbers and their application,” *New Math. Nat. Comput.*, vol. 16, no. 2, pp. 211–230, 2020. https://doi.org/10.1142/s1793005720500131

[25] K. Rahman, A. Ali, and S. Abdullah, “Multiattribute group decision making based on interval-valued Pythagorean fuzzy Einstein geometric aggregation operators,” *Granul. Comput.*, vol. 5, pp. 361–372, 2020. https://doi.org/10.1007/s41066-019-00154-w

[26] M. Munir, H. Kalsoom, K. Ullah, T. Mahmood, and Y. M. Chu, “T-spherical fuzzy Einstein hybrid aggregation operators and their applications in multi-attribute decision making problems,” *Symmetry*, vol. 12, no. 3, p. 365, 2020. https://doi.org/10.3390/sym12030365

[27] S. Z. Zeng, M. Munir, T. Mahmood, and M. Naeem, “Some T-spherical fuzzy Einstein interactive aggregation operators and their application to selection of photovoltaic cells,” *Math. Probl. Eng.*, vol. 2020, no. 1, p. 1904362, 2020. https://doi.org/10.1155/2020/1904362

[28] P. D. Liu, Q. Khan, T. Mahmood, and N. Hassan, “T-spherical fuzzy power Muirhead mean operator based on novel operational laws and their application in multi-attribute group decision making,” *IEEE Access*, vol. 7, pp. 22 613–22 632, 2019. https://doi.org/10.1109/ACCESS.2019.2896107

[29] H. Garg and D. Rani, “Some generalized complex intuitionistic fuzzy aggregation operators and their application to multicriteria decision-making process,” *Arab. J. Sci. Eng.*, vol. 44, pp. 2679–2698, 2019. https://doi.org/10.1007/s13369-018-3413-x

[30] D. Rani and H. Garg, “Complex intuitionistic fuzzy power aggregation operators and their applications in multicriteria decision-making,” *Expert Syst.*, vol. 35, no. 6, p. e12325, 2018. https://doi.org/10.1111/exsy.12325

[31] K. Rahman, “New types induced complex intuitionistic fuzzy einstein geometric aggregation operators and their application to decision-making problem,” *Neural Comput. & Applic.*, vol. 36, no. 34, p. 21647–21669, 2024. https://doi.org/10.1007/s00521-024-10214-1

[32] K. Rahman, H. Khan, and S. Abdullah, “Mathematical calculation of COVID-19 disease in Pakistan by emergency response modeling based on complex Pythagorean fuzzy information,” *J. Intell. & Fuzzy Syst.*, vol. 43, no. 3, pp. 3411–3427, 2022. https://doi.org/10.3233/JIFS-212160

[33] I. M. Hezam, K. Rahman, A. Alshamrani, and D. Božanić, “Geometric aggregation operators for solving multicriteria group decision-making problems based on complex pythagorean fuzzy sets,” *Symmetry*, vol. 15, no. 4, p. 826, 2023. https://doi.org/10.3390/sym15040826

[34] H. H. Jin, A. Hussain, K. Ullah, and A. Javed, “Novel complex Pythagorean fuzzy sets under Aczel-Alsina operators and their application in multi-attribute decision making,” *Symmetry*, vol. 15, no. 1, p. 68, 2022. https://doi.org/10.3390/sym15010068

[35] P. D. Liu, Z. Ali, and T. Mahmood, “Archimedean aggregation operators based on complex Pythagorean fuzzy sets using confidence levels and their application in decision making,” *Int. J. Fuzzy Syst.*, vol. 25, no. 1, pp. 42–58, 2023. https://doi.org/10.1007/s40815-022-01391-z

[36] R. Ali, K. Rahman, and J. Muhammad, “Complex Fermatean fuzzy models and their algebraic aggregation operators in decision-making: A case study on COVID-19 vaccine selection,” *J. Oper. Strateg Anal.*, vol. 2, no. 2, pp. 119–135, 2024. https://doi.org/10.56578/josa020205

[37] K. Rahman, R. Ali, and T. Lamoudan, “Complex Fermatean fuzzy geometric aggregation operators and their

application on group decision-making problem based on Einstein T-norm and T-conorm," *Soft Comput.*, vol. 28, no. 17, pp. 9203–9224, 2024. https://doi.org/10.1007/s00500-024-09804-x

[38] M. Akram, C. Kahraman, and K. Zahid, "Group decision-making based on complex spherical fuzzy VIKOR approach," *Knowl-Based Syst.*, vol. 216, p. 106793, 2021. https://doi.org/10.1016/j.knosys.2021.106793