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Multi-Attribute Green Supplier Decision-Making Using Picture Fuzzy Rough Schweizer-Sklar Aggregation Operators

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Abstract: For reducing uncertainty in data gathered from real-world scenarios, the picture fuzzy rough set (PFRS) framework is a reliable resource. This article presents new aggregation operators (AOs) based on the Schweizer-Sklar t-conorm (SS-TC) and Schweizer-Sklar t-norm (SS-TN). They present the PFRS framework with SS, which aims to handle the intricacies in contexts where decision-making is marked by ambiguity and uncertainty. In the context of Green Supply Chain Management (GSCM), where supply chain procedures incorporate sustainability considerations, this framework is especially pertinent. GSCM places a strong emphasis on minimizing environmental impacts by employing techniques such as effective resource management and sustainable sourcing. The adaptability and versatility required to assess and optimize these inexperienced practices are significantly improved with the aid of our expert PFRS framework. Businesses can keep operational efficiency and align their supply chain operations with environmental desires with the aid of using this framework. By considering both the blessings and disadvantages of environmental sustainability, using PFRS in GSCM enhances decision-making and promotes environmental sustainability. To handle picture fuzzy rough values (PFRVs), these operators include picture fuzzy rough weighted averaging (PFRSSWA) and picture fuzzy rough weighted geometric (PFRSSWG) operators. We investigate these recently created AOs' basic characteristics and use them to solve multi-attribute group decision-making (MAGDM) issues under the framework of picture fuzzy (PF) data. Our results demonstrate how the outcomes in SS-TN and SS-TC vary with varying parameter values. We also contrast these outcomes with the ones obtained from pre-existing AOs. In addition, we provide a graphic representation of all observations and findings to show how flexible and successful the suggested operators are at handling MAGDM problems.

Keywords: Picture fuzzy set; Picture fuzzy rough set; Rough set; Aggregation operators; MAGDM

1 Introduction

The growing emphasis on sustainability within business operations has positioned GSCM as a relevant element in strategic planning. GSCM includes environmental concerns into conventional supply chain techniques, to lower the ecological footprint to each degree, from sourcing uncooked materials to handing over completed merchandise. By adopting GSCM practices, agencies can minimize their environmental impact and price financial savings, and realize and beautify their aggressive part. For example, the fashion enterprise's shift closer to sourcing organic cotton instead of conventional cotton reduces the use of harmful pesticides and conserves water, exemplifying how sustainable sourcing practices are imperative to GSCM. This method lets companies align their operations with broader environmental goals, even by keeping products excellent and supply chain performance [1, 2]. Furthermore, GSCM fosters the development of sustainable business models by prompting corporations to reevaluate their production and distribution techniques. Businesses that embrace GSCM frequently experience an improvement in brand popularity and customer loyalty, as purchasers increasingly favor eco-friendly merchandise [3]. Strategic sourcing of materials with a decreased environmental effect is an important element of this approach, aiding organizations in navigating the complexities of modern delivery chains [4, 5]. Beyond the environmental benefits, GSCM also contributes to operational efficiencies, which include waste discounts and decreased electricity consumption, leading to full-size fee financial savings [6, 7]. Consequently, organizations that combine GSCM into their operations are better prepared to satisfy the rising demand for sustainable merchandise and practices within the global market [3].

To characterize an object's membership grade (MG), Zadeh [8] introduced the idea of the fuzzy set (FS). Atanassov [9] subsequently pointed out the shortcomings of FSs and suggested an addition called the intuitionistic fuzzy set (IFS), which offers details on both the membership guide (MG) and non-membership guide (NMG) in addition to ambiguous issues. Because IFSs are so effective at characterizing uncertain data and information, academics have shown a great deal of interest in them. Xu [10] and Yager [11] introduced intuitionistic fuzzy simple weighted average operators. When a decision-maker believes that the degree of uncertainty is 0.1, the non-membership degree is 0.3, and the membership degree is 0.2, this is an example of an evaluation value that cannot be represented by IFSs. Cuong et al. [12] proposed the idea of a picture fuzzy set (PFS), which has three components, including MG, neutral grade, and NMG, to address this circumstance. Koam et al. [13] and Sarfraz and Božanić [14] developed the concepts of the Maclaurin symmetric mean AOs on different fuzzy extensions. Numerous picture fuzzy AOs have been put forth to effectively aggregate picture fuzzy information by aggregating PFNs [15–18]. Cuong and Pham [19] introduced fuzzy logic operators for PFS. Tian et al. [20] extended the PF on Schweizer-Sklar AOs. Jana et al. [21] developed the PFS on Dombi aggregation operators and their application of MADM. Ullah [22] introduced the PF on the Maclaurin symmetric mean operator and their application to MADM. Wei [23] developed the PF aggregation operator and its application to the MADM. Tian et al. [24] gave the concept of weighted PF aggregation operators and their applications in MADM. The authors [25, 26] expanded the PF on the Maclaurin symmetric mean AOs based on Frank norm and conorm.

Pawlak [27] initially pioneered the paradigm of rough set (RS) by taking the crisp set's lower and upper approximations as an additional tool to decrease ambiguity from evidence. Numerous researchers in a variety of fields have employed RS because it is a very helpful technique for reducing ambiguity and uncertainty. For instance, Grzymala talked about using the RS in data mining, Dinçer et al. [28] talked about using the RS in neural networks, and so on. Moreover, Sahu et al. [29] created a connection between the FS and RS to present the fuzzy rough set (FRS) concept. The FRS was a novel concept in which the FS was established using the object's lower and upper approximations for description. Li et al. [30] used the concepts of PF rough (PFR) on renewable-friendly smart grid technologies. These authors [31–33] used the concepts of PFSs with RS theory and developed a class of AOs with decision analytic applications.

Yasin et al. [34] developed the Schweizer-Sklar AOs using supply chain management. Wei et al. [35] used the Schweizer-Sklar operators in the applications of green supplier selection. Ma et al. [36] developed the theory of the Schweizer-Sklar AOs with the application of the agricultural system. Zeng et al. [37] expanded the theory of fuzzy formation with the application of green supplier selection. Liu et al. [38] gave the concepts of Dombi AOs and their application in green supply chain management. Some t-norms and t-conorms put forward by Aczél and Alsina [39], Garg [40], Frank [41], and others have remained present. Particularly, in fuzzy frameworks, these t-norms and t-conorms have been crucial to information handling. The t-norm and t-conorm have very important generalizations in the form of SS-TN and SS-TC [42]. Since SS-TN and SS-TC involve a variable parameter, they offer decision-makers the flexibility to select the parameter's value, making them very flexible. Because of their versatility, researchers have made extensive use of SS-TN and SS-TC. To solve the MAGDM problem, for example, Wang and Liu [43] developed a methodology based on the Maclaurin symmetric mean operator, Biswas and Deb [44] developed AOs based on Pythagorean FS.

Decision-makers (DMs) frequently face conditions where figuring out which criteria to accept or reject and how to prioritize them will become tough. This includes navigating numerous levels of uncertainty and ambiguity whilst deciding on, evaluating, and assessing parameters. Additionally, DMs require a decision-making framework that contains positive and negative remarks, even while ensuring objectivity in evaluating options based on these parameters. After an intensive evaluation of the present literature, it will become evident that there is an opening for a mathematical framework that addresses these complexities. In response to this recognized want, a new mathematical framework, the PFRS with Schweizer-Sklar AOs, has evolved. This framework offers more desirable flexibility and adaptability, effectively coping with the subsequent conditions collectively. The following is a list of this paper's contributions:

- The PFRSSWA and PFRSSWG operators are two of the new AOs we suggested for the PFs; some relevant attributes are covered.
- Look into the boundedness, monotonicity, and idempotency of the proposed operators.
- Using the PFRSSWA or PFRSSWG operator as a foundation, we created a novel PFRVs MAGDM technique.
- By addressing matters concerning investment choices, we evaluated the appropriateness of our suggested aggregation function-based MAGDM technique.

Four additional sections make up the further study. Section 2 provides the foundational information of the articles for easier comprehension. Section 3 develops the PFRSSWA and PFRSSWG operators. Section 4 discusses the application of the created AOs to the MAGDM. In addition, a practical case study legitimizes the developed decision-making strategy, and detailed comparative and sensitivity analyses are also provided. The article is summed up in Section 5.

2 Preliminaries

The definitions of the common terms are provided in this section. This section defines a few basic binary relations: PFS, PFRS, PFRV, SS-TN, and SS-TC.

Definition 2.1 [19]: The PFS γ on the universe X is distinct as:

$$\gamma = \{\zeta, (\Lambda_\zeta, \boldsymbol{\beta}_\zeta, \vartheta_\zeta) : \zeta \in X\}$$

The PFS contains MG Λ, Hesitancy guide (HG) $\boldsymbol{\beta}$, and NMG $\boldsymbol{\beta}$ for all $\Lambda_\zeta, \boldsymbol{\beta}_\zeta, \vartheta_\zeta, X \to [0,1], 0 \le \Lambda_\zeta + \boldsymbol{\beta}_\zeta + \vartheta_\zeta \le 1$. Consider three PFVs, $\gamma = (\Lambda_\zeta, \boldsymbol{\beta}_\zeta, \vartheta_\zeta)$ and $\gamma_\sigma = (\Lambda_{\zeta\sigma}, \boldsymbol{\beta}_{\zeta\sigma}, \vartheta_{\zeta\sigma})$ for $\sigma = 1, 2$. The followings are some basic properties of PFVs.

- $\gamma_1 \cup \gamma_2 = (\max(\Lambda_{\zeta 1}, \Lambda_{\zeta 2}), \min(\boldsymbol{\beta}_{\zeta 1}, \boldsymbol{\beta}_{\zeta 2}), \min(\vartheta_{\zeta_1}, \vartheta_{\zeta 2}))$;
- $\gamma_1 \cap \gamma_2 = (\min(\Lambda_{\zeta 1}, \Lambda_{\zeta 2}), \max(\boldsymbol{\beta}_{\zeta 1}, \boldsymbol{\beta}_{\zeta 2}), \max(\vartheta_{\zeta 1}, \vartheta_{\zeta 2}))$;
- $\gamma_1 \oplus \gamma_2 = (\sqrt{\Lambda_{\zeta 1} + \Lambda_{\zeta 2} - \Lambda_{\zeta 1}\Lambda_{\zeta 2}}, \boldsymbol{\beta}_{\zeta 1}\boldsymbol{\beta}_{\zeta 2}, \vartheta_{\zeta 1}\vartheta_{\zeta 2})$;
- $\gamma_1 \otimes \gamma_2 = (\sqrt{\Lambda_{\zeta 1}\Lambda_{\zeta 2}}, \boldsymbol{\beta}_{\zeta 1} + \boldsymbol{\beta}_{\zeta 2} - \boldsymbol{\beta}_{\zeta 1}\boldsymbol{\beta}_{\zeta 2}, \vartheta_{\zeta 1} + \vartheta_{\zeta 2} - \vartheta_{\zeta 1}\vartheta_{\zeta 2})$;
- $\gamma^c = (\Lambda_\zeta, \boldsymbol{\beta}_\zeta, \vartheta_\zeta)$ where γ^c is the complement of the PFV γ;
- $\sigma\gamma = \left(\sqrt{1 - (1 - \Lambda_{\zeta Z})^\sigma}, \boldsymbol{\beta}_Z^\alpha, \vartheta_Z^\sigma\right)$ for $\sigma > 0$;
- $\gamma^\sigma = \left(\sqrt{\Lambda_{\zeta Z}}, 1 - (1 - \boldsymbol{\beta}_Z)^\sigma, 1 - (1 - \vartheta_Z)^\sigma\right)$, used for $\sigma > 0$.

The binary relation forms the foundation of the PFRS [32]. This is the definition of the binary relation Φ.

Definition 2.2 [27]: Consider $\Phi \in X \times X$ to be the dualistic relation on the set X. Before Φ is:

- Reflexive $(\mathcal{M}, \mathcal{M}) \forall \mathcal{M} \in \gamma$;
- Symmetric $(\mathcal{M}, b) = (b, \mathcal{M}) \in \Phi \forall \mathcal{M}, b \in \gamma$;
- Transitive $(b, \mathcal{J}) \in \Phi$ and $(\mathcal{J}, \mathcal{M}) \in \Phi$ then $(b, \mathcal{M}) \in \Phi \forall \mathcal{M},\ b, \mathcal{J} \in \gamma$.

Definition 2.3 [27]: Consider γ the universal set and Φ the relation. Now we assume a mapping $\Phi^* : \gamma \to \mathcal{A}(\gamma)$ as:

$$\Phi^*(\alpha) = \{\mathcal{M} \in \gamma : (\alpha, \mathcal{M}) \in \Phi\}, for\ \alpha \in \gamma$$

where, $\Phi^*(\alpha)$ is referred to as the crisp space of the approximation Φ and (γ, Φ) is designated as the successor neighborhood of an element a concerning Φ. The definitions of the LA and UA are provided below for any set $\alpha \subseteq \gamma$.

$$\Phi^{\mathrm{LA}}(\alpha) = \{\mathcal{M} \in \Phi \mid \Phi^*(\mathcal{M}) \subseteq \alpha\},$$
$$\Phi^{\mathrm{UA}}(\alpha) = \{\mathcal{M} \in \Phi \mid \Phi^*(\mathcal{M}) \cap \alpha \neq \emptyset\}.$$

The set $(\Phi^{\mathrm{LA}}(\alpha), \Phi^{\mathrm{UA}}(\alpha))$ is supposed to be an RS.

Definition 2.4 [28]: Consider γ to be the collective set and Φ to be the relation from $PFS(\gamma \times \gamma)$. Formerly,

Φ is called reflexive if $\Lambda_\Phi(\mathcal{M}, \mathcal{M}) = 1$ and $\boldsymbol{\beta}_\Phi(\mathcal{M}, \mathcal{M}) = 0 \forall \mathcal{M} \in \gamma$.

Φ termed as symmetric if $\forall (\mathcal{M}, b) \in \gamma \times \gamma$ then $\Lambda_\Phi(b, \mathcal{M}) = \Lambda_\Phi(\mathcal{M}, b) \forall \mathcal{M}, b \in \gamma$ and $\boldsymbol{\beta}_\Phi(b, \mathcal{M}) = \boldsymbol{\beta}_\Phi(\mathcal{M},)$.

Φ said to be transitive if $\forall \mathcal{M}, b, \mathcal{J} \in \gamma$ if $(b, \mathcal{J}) \in \Phi$ and $(\mathcal{J}, \mathcal{M}) \in \Phi$ then $\Lambda_\Phi(b, \mathcal{M}) \ge \mathrm{V}[\Lambda_\Phi(b, \mathcal{J}) \wedge \Lambda_\Phi(\mathcal{J}, \mathcal{M})]$ and $\boldsymbol{\beta}_\Phi(\lfloor, \mathcal{M}) \ge \Lambda[\boldsymbol{\beta}_\Phi(b, \mathcal{J}) \wedge \boldsymbol{\beta}_\Phi(\mathcal{J}, \mathcal{M})]$.

Definition 2.5 [16]: Consider γ to be the universal set and $\Phi(\gamma \times \gamma)$ be the PF relation, then the approximation's PF space is defined as (γ, Φ). The following defines the PFLA and PFUA for any set $\alpha \subseteq PFS(\gamma)$.

$$\Phi^{\mathrm{PFUA}}(\alpha) = \left\{\mathcal{M}, \Lambda_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}), \boldsymbol{\beta}_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}), \vartheta_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}) \mid \mathcal{M} \in \gamma\right\}$$
$$\Phi^{\mathrm{PFLA}}(\alpha) = \left\{\mathcal{M}, \Lambda_{\Phi^{\mathrm{PFLA}}(\alpha)}(\mathcal{M}), \boldsymbol{\beta}_{\Phi^{\mathrm{PFLA}}(\alpha)}(\mathcal{M}), \vartheta_{\Phi^{\mathrm{PFLA}}(\alpha)}(\mathcal{M}) \mid \mathcal{M} \in \gamma\right\}$$

where,

$$\Lambda_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}) = \bigvee_{l \in \gamma} \left[\Lambda_{\Phi(\mathcal{M})}(\mathcal{M}, \iota) \vee \Lambda_{\alpha(\mathcal{M})}\right]$$

$$\boldsymbol{\beta}_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}) = \bigwedge_{l \in \gamma} \left[\boldsymbol{\beta}_{\Phi(\mathcal{M})}(\mathcal{M}, \iota) \wedge \boldsymbol{\beta}_{\alpha(\mathcal{M})}\right]$$

$$\vartheta_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}) = \bigwedge_{l \in \gamma} \left[\vartheta_{\Phi(\mathcal{M})}(\mathcal{M}, \iota) \wedge \vartheta_{\alpha(\mathcal{M})}\right]$$

$$\Lambda_{\Phi^{\mathrm{PFLA}}(\alpha)}(\mathcal{M}) = \bigwedge_{l \in \gamma} \left[\Lambda_{\Phi(\mathcal{M})}(\mathcal{M}, \iota) \wedge \Lambda_{\alpha(\mathcal{M})}\right]$$

$$\boldsymbol{\beta}_{\Phi^{\mathrm{PFLA}}(\alpha)}(\mathcal{M}) = \bigvee_{l \in \gamma} \left[\boldsymbol{\beta}_{\Phi(\mathcal{M})}(\mathcal{M}, \iota) \vee \boldsymbol{\beta}_{\alpha(\mathcal{M})}\right]$$

$$\vartheta_{\Phi^{\mathrm{PFLA}}(\alpha)}(\mathcal{M}) = \bigvee_{l \in \mathcal{V}} \left[\vartheta_{\Phi(\mathcal{M})}(\mathcal{M}, \iota) \vee \vartheta_{\alpha(\mathcal{M})}\right]$$

Such that $0 \leq \Lambda_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}) + \boldsymbol{\beta}_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}) + \vartheta_{\Phi^{\mathrm{PFUA}}(\alpha)}(\mathcal{M}) \leq 1$. Moreover, the ordered pair $\left(\Phi^{\mathrm{PFLA}}(\alpha), \Phi^{\mathrm{PFUA}}(\alpha)\right)$ is considered a PFRS.

Definition 2.6 [22]: Consider $\mathcal{R}_1 = ((\underbrace{\Lambda_1^{\iota}}, \underbrace{\beta_1^{\iota}}, \underbrace{\vartheta_1^{\iota}}), (\overbrace{\Lambda_1^{\mu}}, \overbrace{\beta_1^{\mu}}, \overbrace{\vartheta_1^{\mu}}))$ be an PFV. Then,

$$Sco\,(\mathcal{R}_1) = \frac{(\Lambda_1^{\iota} + \Lambda_1^{\mu} - \beta_1^{\iota} - \beta_1^{\mu} - \vartheta_1^{\iota} - \vartheta_1^{\mu})}{6} \tag{1}$$

The score value of $\mathcal{R}_1$.

Definition 2.7 [22]: Consider $\mathcal{R}_1 = ((\underbrace{\Lambda_1^{\iota}}, \underbrace{\beta_1^{\iota}}, \underbrace{\vartheta_1^{\iota}}), (\overbrace{\Lambda_1^{\mu}}, \overbrace{\beta_1^{\mu}}, \overbrace{\vartheta_1^{\mu}}))$ be an PFV. Then,

$$Acc\,(\mathcal{R}_1) = \frac{(\Lambda_1^{\iota} + \Lambda_1^{\mu} + \beta_1^{\iota} + \beta_1^{\mu} + \vartheta_1^{\iota} + \vartheta_1^{\mu})}{6} \tag{2}$$

The degrees of accuracy of $\mathcal{R}_1$.
If $Sco\,(\mathcal{R}_1) < Sco\,(\mathcal{R}_2)$, then $\mathcal{R}_1$ has less partiality than $\mathcal{R}_2$.
If $Sco\,(\mathcal{R}_1) = Sco\,(\mathcal{R}_2)$, then $\mathcal{R}_1$ and $\mathcal{R}_2$ are the same.
If $Acc\ (\mathcal{R}_1) < Acc\ (\mathcal{R}_2)$, then $\mathcal{R}_1$ has less partiality than $\mathcal{R}_2$.
If $Acc\ (\mathcal{R}_1) = Acc\ (\mathcal{R}_2)$, then $\mathcal{R}_1$ and $\mathcal{R}_2$ are the same.
Definition 2.8 [42]: The Schweizer-Sklar t-norm and t-conorm are defined as:

$$\Psi_{SS}(o, \varsigma) = \left(o^{\Delta} + \varsigma^{\Delta} - 1\right)^{1/\Delta} \tag{3}$$

$$\Psi^*{}_{SS}(o, \varsigma) = 1 - \left((1 - o)^{\Delta} + (1 - \varsigma)^{\Delta} - 1\right)^{1/\Delta} \tag{4}$$

3 Aggregation Operators for PFRVs Based on Schweizer-Sklar t-Norm and t-Conorm

This section uses the SS-TN and SS-TC to develop a family of geometric AOs and weighted averages. Before developing these AOs, a few PFRV operational laws are defined using the SS-TN and SS-TC. The following defines a few fundamental PFRV operating laws.

Definition 3.1: Consider $\mathcal{R}_{\sigma} = ((\underbrace{\Lambda_{\sigma}^{\iota}}, \underbrace{\beta_{\sigma}^{\iota}}, \underbrace{\vartheta_{\sigma}^{\iota}}), (\overbrace{\Lambda_{\sigma}^{\mu}}, \overbrace{\beta_{\sigma}^{\mu}}, \overbrace{\vartheta_{\sigma}^{\mu}})); \sigma = 1, 2$ be two PFRVs. Then,

$$\mathcal{R}_1 \oplus \mathcal{R}_2 = \begin{pmatrix} (1 - ((1 - \underbrace{\Lambda_1^{l}})^{\Delta} + (1 - \underbrace{\Lambda_2^{l}})^{\Delta} - 1)^{1/\Delta}, ((\underbrace{\boldsymbol{\beta}_1^{l}})^{\Delta} + (\underbrace{\boldsymbol{\beta}_2^{l}})^{\Delta} - 1)^{1/\Delta}, ((\underbrace{\vartheta_1^{l}})^{\Delta} + (\underbrace{\vartheta_2^{l}})^{\Delta} - 1)^{1/\Delta}) \\ (((\underbrace{\boldsymbol{\beta}_1^{l}})^{\Delta} + (\underbrace{\boldsymbol{\beta}_2^{l}})^{\Delta} - 1)^{1/\Delta}, ((\overbrace{\boldsymbol{\beta}_1^{\mu}})^{\Delta} + (\overbrace{\boldsymbol{\beta}_2^{\mu}})^{\Delta} - 1)^{1/\Delta}, ((\overbrace{\vartheta_1^{\mu}})^{\Delta} + (\overbrace{\vartheta_2^{\mu}})^{\Delta} - 1)^{1/\Delta})) \end{pmatrix} \tag{5}$$

$$\mathcal{R}_1 \otimes \mathcal{R}_2 = \begin{pmatrix} ((\overbrace{\boldsymbol{\beta}_1^{\mathrm{u}}})^{\Delta} + (\overbrace{\boldsymbol{\beta}_2^{\mathrm{u}}})^{\Delta} - 1)^{1/\Delta}, ((\overbrace{\boldsymbol{\beta}_1^{\mathrm{u}}})^{\Delta} + (\overbrace{\boldsymbol{\beta}_2^{\mathrm{u}}})^{\Delta} - 1)^{1/\Delta}, 1 - \left((1 - \vartheta_1^{\iota})^{\Delta} + (1 - \vartheta_2^{\iota})^{\Delta} - 1\right)^{1/\Delta}) \\ (((\overbrace{\Lambda_1^{\mathrm{u}}})^{\Delta} + (\overbrace{\Lambda_2^{\mathrm{u}}})^{\Delta} - 1)^{1/\Delta}, 1 - ((1 - \overbrace{\boldsymbol{\beta}_1^{\mathrm{u}}})^{\Delta} + (1 - \overbrace{\boldsymbol{\beta}_2^{\mathrm{u}}})^{\Delta} - 1)^{1/\Delta}, 1 - ((1 - \overbrace{\vartheta_1^{\mathrm{u}}}) + (1 - \overbrace{\vartheta_2^{\mathrm{u}}})^{\Delta} - 1)^{1/\Delta}) \end{pmatrix} \tag{6}$$

$$\eta\mathcal{R}_1 = \begin{pmatrix} (1 - (\eta(1 - \underbrace{\Lambda_1^{\iota}})^{\Delta} - (\eta - 1))^{1/\Delta}, (\eta(\underbrace{\boldsymbol{\beta}_1^{\iota}})^{\Delta} - (\eta - 1))^{1/\Delta}, \left(\eta\,(\vartheta_1^{\iota})^{\Delta} - (\eta - 1)\right)^{1/\Delta}) \\ (1 - (\eta(1 - \overbrace{\Lambda_1^{\mu}})^{\Delta} - (\eta - 1))^{1/\Delta}, (\eta(\overbrace{\boldsymbol{\beta}_1^{\mu}})^{\Delta} - (\eta - 1))^{1/\Delta}, (\eta(\overbrace{\vartheta_1^{\mu}})^{\Delta} - (\eta - 1))^{1/\Delta}) \end{pmatrix} \tag{7}$$

$$\mathcal{R}_1^{\eta} \quad = \begin{pmatrix} ((\eta(\underbrace{\Lambda_1^{\iota}})^{\Delta} - (\eta - 1))^{1/\Delta}, 1 - (\eta(1 - \underbrace{\boldsymbol{\beta}_1^{\iota}})^{\Delta} - (\eta - 1))^{1/\Delta}, 1 - (\eta(1 - \underbrace{\vartheta_1^{\iota}})^{\Delta} - (\eta - 1))^{1/\Delta}) \\ (\eta(\overbrace{\Lambda_1^{\mu}})^{\Delta} - (\eta - 1))^{1/\Delta}, 1 - (\eta(1 - \overbrace{\boldsymbol{\beta}_1^{\mu}})^{\Delta} - (\eta - 1))^{1/\Delta}, 1 - \left(\eta(1 - \overbrace{\vartheta_1^{\mu}})^{\Delta} - (\eta - 1)\right)^{1/\Delta} \end{pmatrix} \tag{8}$$

3.1 Proposed Schweizer-Sklar Aggregation Operators PFRVs

Definition 3.1.1: Consider $\mathcal{R}_\sigma = ((\underbrace{\Lambda^\iota_\sigma}, \underbrace{\beta^\iota_\sigma}, \underbrace{\vartheta^\iota_\sigma}), (\overbrace{\Lambda^\mu_\sigma}, \overbrace{\beta^\mu_\sigma}, \overbrace{\vartheta^\mu_\sigma})), \sigma = 1, 2, \cdots, \kappa$ are κ PFRVs and α_σ is the weight of the $\sigma\ th$ PFRV such that $\sum_{\sigma=1}^{\kappa} \alpha_\sigma = 1$ (Keep in mind that, unless otherwise specified, the weight of the $\sigma\ th$ PFRV is represented by the notation α_σ). Next, the definition of the PFRSSWA operators:

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa) = \bigoplus_{\sigma=1}^{\kappa} \alpha_\sigma \mathcal{R}_\sigma$$
$$= \begin{pmatrix} (1-(\sum_{\sigma=1}^{\kappa} \alpha_\sigma (1-\underbrace{\Lambda^\iota_\sigma})^\Delta)^{\frac{1}{\Delta}}, ((\overbrace{\boldsymbol{\beta}^\mu_1})^\Delta + (\overbrace{\boldsymbol{\beta}^\mu_2})^\Delta - 1)^{1/\Delta}, ((\overbrace{\vartheta^\mu_1})^\Delta + (\overbrace{\vartheta^\mu_2})^\Delta - 1)^{1/\Delta}) \\ (1-(\sum_{\sigma=1}^{\kappa} \alpha_\sigma (1-\overbrace{\Lambda^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\vartheta^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}) \end{pmatrix} \tag{9}$$

The following lists and validates a few fundamental characteristics of the PFRSSWA operator defined in Eq. (2).

Theorem 3.1.1: Consider $\mathcal{R}_\sigma = ((\underbrace{\Lambda^\iota_\sigma}, \underbrace{\beta^\iota_\sigma}, \underbrace{\vartheta^\iota_\sigma}), (\overbrace{\Lambda^\mu_\sigma}, \overbrace{\beta^\mu_\sigma}, \overbrace{\vartheta^\mu_\sigma})), \sigma = 1, 2, \cdots, \kappa$ are κ PFRVs. The PFRSSWA operator then aggregates the data to obtain the PFRV in the manner described below.

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa) =$$
$$\begin{pmatrix} (1-(\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\vartheta^\mu_\sigma})^\Delta\right)^{\frac{1}{\Delta}}) \\ \left(1-\left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\kappa} \alpha_\sigma (\overbrace{\vartheta^\mu_\sigma})^\Delta\right)^{\frac{1}{\Delta}}\right) \end{pmatrix} \tag{10}$$

Proof: We use the following method of mathematical induction to prove Theorem 3.1.1.
For κ=2

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2) =$$
$$\begin{pmatrix} (1-(\sum_{\sigma=1}^{2} \alpha_\sigma (1-\overbrace{\Lambda^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{2} \alpha_\sigma (\underbrace{\boldsymbol{\beta}^\iota_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{2} \alpha_\sigma (\underbrace{\vartheta^\iota_\sigma})^\Delta)^{\frac{1}{\Delta}}) \\ (1-(\sum_{\sigma=1}^{2} \alpha_\sigma (1-\overbrace{\Lambda^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{2} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{2} \alpha_\sigma (\overbrace{\vartheta^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}) \end{pmatrix} \tag{11}$$

which is PFRV.
Assume Eq. (2) is true for $\kappa = \varphi$

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\varphi) =$$
$$\begin{pmatrix} ((\sum_{\sigma=1}^{\varphi} \alpha_\sigma (1-\underbrace{\Lambda^\iota_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\underbrace{\boldsymbol{\beta}^\iota_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\underbrace{\vartheta^\iota_\sigma})^\Delta)^{\frac{1}{\Delta}}) \\ ((\sum_{\sigma=1}^{\varphi} \alpha_\sigma (1-\overbrace{\Lambda^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\overbrace{\vartheta^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}) \end{pmatrix} \tag{12}$$

We show that Eq. (2) is true for $\kappa = \varphi + 1$.

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\varphi, \mathcal{R}_{\varphi+1})$$
$$= \begin{pmatrix} \left(\begin{matrix} (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (1-\underbrace{\Lambda^\iota_\sigma})^\Delta + \alpha_{\sigma+1}(1-\underbrace{\Lambda^\iota_{\sigma+1}})^\Delta)^{\frac{1}{\Delta}}, \\ (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\underbrace{\beta^\iota_\sigma})^\Delta + \alpha_{\sigma+1}(\underbrace{\boldsymbol{\beta}^\iota_{\sigma+1}})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\underbrace{\vartheta^{\iota\partial}_\sigma})^\Delta + \alpha_{\sigma+1}(\underbrace{\vartheta^\iota_{\sigma+1}})^\Delta)^{\frac{1}{\Delta}} \end{matrix} \right), \\ 1-(\sum_{\sigma=1}^{\varphi} \alpha_\sigma (1-\underbrace{\Lambda^\mu_\sigma})^\Delta + \alpha_{\sigma+1}(1-\underbrace{\Lambda^\mu_{\sigma+1}})^\Delta)^{\frac{1}{\Delta}}, \\ (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\underbrace{\beta^\mu_\sigma})^\Delta + \alpha_{\sigma+1}(\underbrace{\boldsymbol{\beta}^\mu_{\sigma+1}})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi} \alpha_\sigma (\underbrace{\vartheta^\mu_\sigma})^\Delta + \alpha_{\sigma+1}(\underbrace{\vartheta^\mu_{\sigma+1}})^\Delta)^{\frac{1}{\Delta}} \end{pmatrix} \tag{13}$$

Then,

$$PFRSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_{\varphi+1}) =$$
$$\begin{pmatrix} (1-(\sum_{\sigma=1}^{\varphi+1} \alpha_\sigma (1-\underbrace{\Lambda^l_\sigma})^\Delta)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^{\varphi+1} \alpha_\sigma \left(\underbrace{\boldsymbol{\beta}^z_\sigma}\right)^\Delta\right)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi+1} \alpha_\sigma (\underbrace{\vartheta^l_\sigma})^\Delta)^{\frac{1}{\Delta}}) \\ (1-(\sum_{\sigma=1}^{\varphi+1} \alpha_\sigma (1-\overbrace{\Lambda^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi+1} \alpha_\sigma (\overbrace{\boldsymbol{\beta}^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^{\varphi+1} \alpha_\sigma (\overbrace{\vartheta^\mu_\sigma})^\Delta)^{\frac{1}{\Delta}}) \end{pmatrix} \tag{14}$$

This concludes Theorem 3.1.1's proof.

The PFRSSWA operator's idempotency is demonstrated by Theorem 3.1.2.

Theorem 3.1.2: (Idempotency) Consider $\mathcal{R}_\sigma = ((\underbrace{\Lambda_\sigma^\iota}, \underbrace{\beta_\sigma^\iota}, \underbrace{\vartheta_\sigma^\iota}), (\overbrace{\Lambda_\sigma^\mu}, \overbrace{\beta_\sigma^\mu}, \overbrace{\vartheta_\sigma^\mu})), \sigma = 1, 2, \cdots, \kappa$ are κ PFRVs and $\mathcal{R}_\sigma = ((\Lambda_\sigma^\iota, \boldsymbol{\beta}_\sigma^\iota, \vartheta_\sigma^\iota), (\Lambda_\sigma^\mu, \boldsymbol{\beta}_\sigma^\mu, \vartheta_\sigma^\mu)) = ((\Lambda^\iota, \boldsymbol{\beta}^\iota, \vartheta^\iota), (\Lambda^\mu, \boldsymbol{\beta}^\mu, \vartheta^\mu)) = \mathcal{R}, \forall \sigma = 1, 2, \cdots, \kappa$. Then,

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa) = ((\Lambda^\iota, \boldsymbol{\beta}^\iota, \vartheta^\iota), (\Lambda^\mu, \boldsymbol{\beta}^\mu, \vartheta^\mu)) = \mathcal{R}$$

Proof: Since $\mathcal{R}_\sigma = ((\underbrace{\Lambda_\sigma^\iota}, \underbrace{\beta_\sigma^\iota}, \underbrace{\vartheta_\sigma^\iota}), (\overbrace{\Lambda_\sigma^\mu}, \overbrace{\beta_\sigma^\mu}, \overbrace{\vartheta_\sigma^\mu})) = ((\Lambda^\iota, \boldsymbol{\beta}^\iota, \vartheta^\iota), (\Lambda^\mu, \boldsymbol{\beta}^\mu, \vartheta^\mu))$, then

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa) = (\mathcal{R}, \mathcal{R}, \mathcal{R})$$

$$\begin{pmatrix} (1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \underbrace{\Lambda_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^\kappa \alpha_\sigma (\underbrace{\beta_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}, (\sum_{\sigma=1}^\kappa \alpha_\sigma (\underbrace{\vartheta_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}) \\ \left(1 - \left(\sum_{\sigma=1}^\kappa \alpha_\sigma \left(1 - \overbrace{\Lambda_\sigma^z}\right)^\Delta\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^\kappa \alpha_\sigma \left(\overbrace{\boldsymbol{\beta}_\sigma^\mu}\right)^\Delta\right)^{\frac{1}{\Delta}}, \left(\sum_{\sigma=1}^\kappa \alpha_\sigma \left(\overbrace{\vartheta_\sigma^\mu}\right)^\Delta\right)^{\frac{1}{\Delta}}\right) \end{pmatrix} \tag{15}$$

Since $\mathcal{R}_\sigma = \mathcal{R} \; \forall \; \sigma$,

$$\sum_{\sigma=1}^\kappa \alpha_\sigma = 1$$

$$= \begin{pmatrix} \left(1 - \left(\left(1 - \underbrace{\Lambda^\iota}\right)^\Delta\right)^{\frac{1}{\Delta}}, \left(\left(\underbrace{\boldsymbol{\beta}^\iota}\right)^\Delta\right)^{\frac{1}{\Delta}}, \left(\left(\underbrace{\vartheta^\iota}\right)^\Delta\right)^{\frac{1}{\Delta}}\right) \\ \left(1 - \left(\left(1 - \overbrace{\Lambda^\mu}\right)^\Delta\right)^{\frac{1}{\Delta}}, \left(\left(\overbrace{\boldsymbol{\beta}^\mu}\right)^\Delta\right)^{\frac{1}{\Delta}}, \left(\left(\overbrace{\vartheta^\mu}\right)^\Delta\right)^{\frac{1}{\Delta}}\right) \end{pmatrix} \tag{16}$$

$$= (\Lambda^\iota, \boldsymbol{\beta}^\iota, \vartheta^\iota), (\Lambda^\mu, \boldsymbol{\beta}^\mu, \vartheta^\mu)) = \mathcal{R}$$

This completes the proof.

Theorem 3.1.3: (Boundedness) Consider $\mathcal{R}_\sigma = ((\underbrace{\Lambda_\sigma^\iota}, \underbrace{\beta_\sigma^\iota}, \underbrace{\vartheta_\sigma^\iota}), (\overbrace{\Lambda_\sigma^\mu}, \overbrace{\beta_\sigma^\mu}, \overbrace{\vartheta_\sigma^\mu})), \sigma = 1, 2, \cdots, \kappa$ are κ PFRVs and $\mathcal{R}_\sigma^s$ and $\mathcal{R}_\sigma^g$ are, in order of PFRV, the smallest and largest. Next,

$$\mathcal{R}_\sigma^s \leq PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_{Z_k}) \leq \mathcal{R}_\sigma^g$$

Theorem 3.1.4: (Monotonicity) Consider $\mathcal{R}_\sigma = ((\underbrace{\Lambda_\sigma^\iota}, \underbrace{\beta_\sigma^\iota}, \underbrace{\vartheta_\sigma^\iota}), (\overbrace{\Lambda_\sigma^\mu}, \overbrace{\beta_\sigma^\mu}, \overbrace{\vartheta_\sigma^\mu})), \sigma = 1, 2, \cdots, \kappa$ and $\mathcal{R}_\sigma^v = ((\Lambda_\sigma^{v\iota}, \boldsymbol{\beta}_\sigma^{v\iota}, \vartheta_\sigma^{v\iota}) (\Lambda_\sigma^{v\mu}, \boldsymbol{\beta}_\sigma^{v\mu}, \vartheta_\sigma^{v\mu}))$ are two sets of PFRVs and $\mathcal{R}_\sigma \leq \mathcal{R}_\sigma^v$. Then,

$$PFRSSWA(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa) \leq PFRSSWA(\mathcal{R}_1^v, \mathcal{R}_2^v, \cdots, \mathcal{R}_\kappa^v) \, PFRAAWG(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa)$$

A weighted average of PFRVs is used in Eq. (2) to define the PFRSSWA operator. The geometric mean of PFRVs is then used to define the PFRSSWG operator. PFRSSWG operator is defined by Eq. (3) in the following.

Definition 3.1.2: Consider $\mathcal{R}_\sigma = ((\underbrace{\Lambda_\sigma^\iota}, \underbrace{\beta_\sigma^\iota}, \underbrace{\vartheta_\sigma^\iota}), (\overbrace{\Lambda_\sigma^\mu}, \overbrace{\beta_\sigma^\mu}, \overbrace{\vartheta_\sigma^\mu})), \sigma = 1, 2, \cdots, \kappa$ are κ PFRVs and α_σ denotes the weight of $\sigma \; th$ the PFRV. Then,

$$PFRAAWG(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa) = \bigotimes_{\sigma=1}^\kappa \mathcal{R}_\sigma^{\alpha_\sigma}$$

$$= \begin{pmatrix} (\sum_{\sigma=1}^\kappa \alpha_\sigma (\underbrace{\Lambda_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \underbrace{\beta_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \underbrace{\vartheta_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}) \\ ((\sum_{\sigma=1}^\kappa \alpha_\sigma (\overbrace{\Lambda_\sigma^\mu})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \overbrace{\beta_\sigma^\mu})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \overbrace{\vartheta_\sigma^\mu})^\Delta)^{\frac{1}{\Delta}}) \end{pmatrix} \tag{17}$$

Below are a few of the PFRSSWG operators' fundamental characteristics. The aforementioned proofs of these properties are deemed insufficient due to their simplicity.

Theorem 3.1.5: Consider $\mathcal{R}_\sigma = ((\underbrace{\Lambda_\sigma^\iota}, \underbrace{\beta_\sigma^\iota}, \underbrace{\vartheta_\sigma^\iota}), (\overbrace{\Lambda_\sigma^\mu}, \overbrace{\beta_\sigma^\mu}, \overbrace{\vartheta_\sigma^\mu})), , \sigma = 1, 2, \cdots, \kappa$ are κ PFRVs. Then, PFRV is obtained after the aggregation from the PFRSSWG operator and

$$PFRAAWG(\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_\kappa)$$

$$= \begin{pmatrix} (\sum_{\sigma=1}^\kappa \alpha_\sigma (\underbrace{\Lambda_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \underbrace{\beta_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \underbrace{\vartheta_\sigma^\iota})^\Delta)^{\frac{1}{\Delta}}) \\ ((\sum_{\sigma=1}^\kappa \alpha_\sigma (\overbrace{\Lambda_\sigma^\mu})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \overbrace{\beta_\sigma^\mu})^\Delta)^{\frac{1}{\Delta}}, 1 - (\sum_{\sigma=1}^\kappa \alpha_\sigma (1 - \overbrace{\vartheta_\sigma^\mu})^\Delta)^{\frac{1}{\Delta}}) \end{pmatrix} \tag{18}$$

Theorem 3.1.6: (Idempotency) Consider $\mathcal{R}_{\sigma}=((\underbrace{\Lambda^{\iota}_{\sigma}},\underbrace{\beta^{\iota}_{\sigma}},\underbrace{\vartheta^{\iota}_{\sigma}}),(\overbrace{\Lambda^{\mu}_{\sigma}},\overbrace{\beta^{\mu}_{\sigma}},\overbrace{\vartheta^{\mu}_{\sigma}})),\sigma=1,2,\cdots,\kappa$ are κ PFRVs and $\mathcal{R}_{\sigma}=((\underbrace{\Lambda^{\iota}_{\sigma}},\underbrace{\beta^{\iota}_{\sigma}},\underbrace{\vartheta^{\iota}_{\sigma}}),(\overbrace{\Lambda^{\mu}_{\sigma}},\overbrace{\beta^{\mu}_{\sigma}},\overbrace{\vartheta^{\mu}_{\sigma}}))=((\Lambda^{\iota},\boldsymbol{\beta}^{\iota},\vartheta^{\iota}),(\Lambda^{\mu},\boldsymbol{\beta}^{\mu},\vartheta^{\mu}))=\mathcal{R},\forall\sigma=1,2,\cdots,\kappa$. Then,

$$PFRSSWG(\mathcal{R}_1,\mathcal{R}_2,\cdots,\mathcal{R}_{\kappa})=((\Lambda^{\iota},\boldsymbol{\beta}^{\iota},\vartheta^{\iota}),(\Lambda^{\mu},\boldsymbol{\beta}^{\mu},\vartheta^{\mu}))=\mathcal{R}$$

Theorem 3.1.7: (Boundedness) Consider $\mathcal{R}_{\sigma}=((\underbrace{\Lambda^{\iota}_{\sigma}},\underbrace{\beta^{\iota}_{\sigma}},\underbrace{\vartheta^{\iota}_{\sigma}}),(\overbrace{\Lambda^{\mu}_{\sigma}},\overbrace{\beta^{\mu}_{\sigma}},\overbrace{\vartheta^{\mu}_{\sigma}})),,\sigma=1,2,\cdots,\kappa$ are κ PFRVs and in order of PFRV, the smallest and largest. Next,

$$\mathcal{R}^{s}_{\sigma}\leq PFRSSWG(\mathcal{R}_1,\mathcal{R}_2,\cdots,\mathcal{R}_{\kappa})\leq\mathcal{R}^{g}_{\sigma}$$

Theorem 3.1.8: (Monotonicity) Consider $\mathcal{R}_{\sigma}=((\underbrace{\Lambda^{\iota}_{\sigma}},\underbrace{\beta^{\iota}_{\sigma}},\underbrace{\vartheta^{\iota}_{\sigma}}),(\overbrace{\Lambda^{\mu}_{\sigma}},\overbrace{\beta^{\mu}_{\sigma}},\overbrace{\vartheta^{\mu}_{\sigma}})),,\sigma=1,2,\cdots,\kappa$ and $\mathcal{R}^{v}_{\sigma}=\left(\left(\Lambda^{vl}_{\sigma},\boldsymbol{\beta}^{vl}_{\sigma},\vartheta^{vl}_{\sigma}\right),(\Lambda^{v\mu}_{\sigma},\boldsymbol{\beta}^{v\mu}_{\sigma},\vartheta^{v\mu}_{\sigma})\right)$ are two sets of PFRVs and $\mathcal{R}_{\sigma}\leq\mathcal{R}^{v}_{\sigma}$. Then,

$$PFRSSWG(\mathcal{R}_1,\mathcal{R}_2,\cdots,\mathcal{R}_{\kappa})\leq PFRSSWG(\mathcal{R}^{v}_1,\mathcal{R}^{v}_2,\cdots,\mathcal{R}^{v}_{\kappa})$$

4 A MAGDM Based on Proposed Operators

The MAGDM is a crucial process for selecting an option from the set of necessary alternatives, according to the opinions of certain experts corresponding to attributes. A group of specialists evaluates each option according to a set of agreed-upon standards and offers a PFRV opinion for each option that matches each attribute. The information from the experts is combined in the decision matrices using their weights. The resulting aggregated data is then combined using the weights assigned to the attributes to determine the overall aggregated value for each alternative. In a variety of fields, the MAGDM technique is highly beneficial. It's widely used in many fields, including engineering, mathematics, business, and economics. Consider $\{\Xi_1,\Xi_2,\cdots,\Xi_r\}$ be alternatives that are assessed by $\{\mathcal{Z}_1,\mathcal{Z}_2,\cdots,\mathcal{Z}_h\}$ experts based on the attributes $\{\mathcal{F}_1,\mathcal{F}_2,\cdots,\mathcal{F}_r\}$ with weights $\mathcal{W}_{\varphi}\in[0,1]$ $\varphi=1,2,\cdots,h$ so that $\sum_{\varphi=1}^{h}\mathcal{W}_{\varphi}=1$. With the aid of the MAGDM, we wish to determine which option among $\{\Xi_1,\Xi_2,\cdots,\Xi_r\}$ is the best fit. The following lists the steps for choosing an alternative:

Step 1. In the form of PFRVs, the experts express their opinions, so that $((\underbrace{\Lambda^{\iota}_{\sigma}},\underbrace{\beta^{\iota}_{\sigma}},\underbrace{\vartheta^{\iota}_{\sigma}}),(\overbrace{\Lambda^{\mu}_{\sigma}},\overbrace{\beta^{\mu}_{\sigma}},\overbrace{\vartheta^{\mu}_{\sigma}}))$. There are typically two distinct categories of attributes. If a charge type attribute is present in the attribute list, we take the normalization of the complement of that attribute so that $((\underbrace{\Lambda^{\iota}_{\sigma}},\underbrace{\beta^{\iota}_{\sigma}},\underbrace{\vartheta^{\iota}_{\sigma}}),(\overbrace{\Lambda^{\mu}_{\sigma}},\overbrace{\beta^{\mu}_{\sigma}},\overbrace{\vartheta^{\mu}_{\sigma}}))$.

Step 2. After normalization, the data in the matrices is prepared for aggregation. The PFRVSSWA/PFRSSWG operator is employed to separately aggregate the attributes. The outcome is the aggregated decision matrix, which contains the PFRVs for each alternative that corresponds to each attribute.

Step 3. For each alternative, we aggregate the attributes collectively using the PFRSSWA/PFRSSWG operator. After aggregation in this step, PFRVs represent the total aggregated value that we obtain.

Step 4. To determine each alternative's score, use the score function Definition 2.6.

Step 5. Finally, we use the score values for each alternative to help us rank the options.

Figure 1 represents the flowchart depiction of the proposed MAGDM strategy.

Example 4.1: Finding the best source of organic cotton that fits a clothing company's budget, quality requirements, and sustainability objectives is the aim of this decision-making process. The organization is dedicated to minimizing its impact on the environment, guaranteeing superior product quality, and upholding a dependable supply chain. The company looks for a supplier that can support its larger mission of encouraging ethical and sustainable practices in the fashion industry, in addition to providing high-quality organic cotton. To this end, it evaluates potential suppliers based on critical attributes like cost, environmental impact, quality, and supply chain reliability. In the end, this choice will affect the business's capacity to provide consumers with environmentally friendly products while preserving operational effectiveness and profitability. Assume a clothing company that wishes to procure organic cotton for its manufacturing process. However, the business decides to assess the suitability of several sources before choosing one. The business gives three specialists the task of assessing possible suppliers. With weights $(0.35,0.33,0.32)^{T}$, we have $\mathcal{Z}_{\sqcup}$ $(\sqcup=1,2,3)$.. Four possible sources of organic cotton $\Xi_{\sqcup}$ $(\sqcup=1,2,3,4)$ that need to be evaluated are as follows:

(1) Cost $(\Xi_{\mathbf{1}})$: The cost of purchasing organic cotton from the supplier is an important consideration because it has a direct impact on the production costs of the business.

(2) Environmental Impact $(\Xi_{\mathbf{2}})$: Water use, pesticide use, and total carbon footprint are all important aspects of the supplier's sustainability practices that must be taken into account for the company's environmental objectives to be met.

(3) Quality (Ξ_3): The strength and softness of the organic cotton fibers affect the final product's durability and appeal to consumers.

(4) Supply Chain Reliability (Ξ_4): To ensure seamless operations, suppliers must be dependable and consistent in meeting demand and delivery dates.

Based on the characteristics with weights $(0.21, 0.29, 0.23, 0.27)^T$, these factors are assessed.

(1) Prior Performance History ($\mathcal{F}_1$): Every supplier is evaluated according to its historical performance, which is integral to forecasting future dependability.

(2) Anticipated Time Frame ($\mathcal{F}_2$): Long-term planning relies heavily on the anticipated time frame for preserving an uninterrupted supply of organic cotton.

(3) Estimated Rate of Change ($\mathcal{F}_3$): Future performance may be affected by expected changes in supplier costs or practices.

(4) Comparative Analysis ($\mathcal{F}_4$): Every supplier is evaluated against other businesses that are effectively participating in the organic cotton industry.

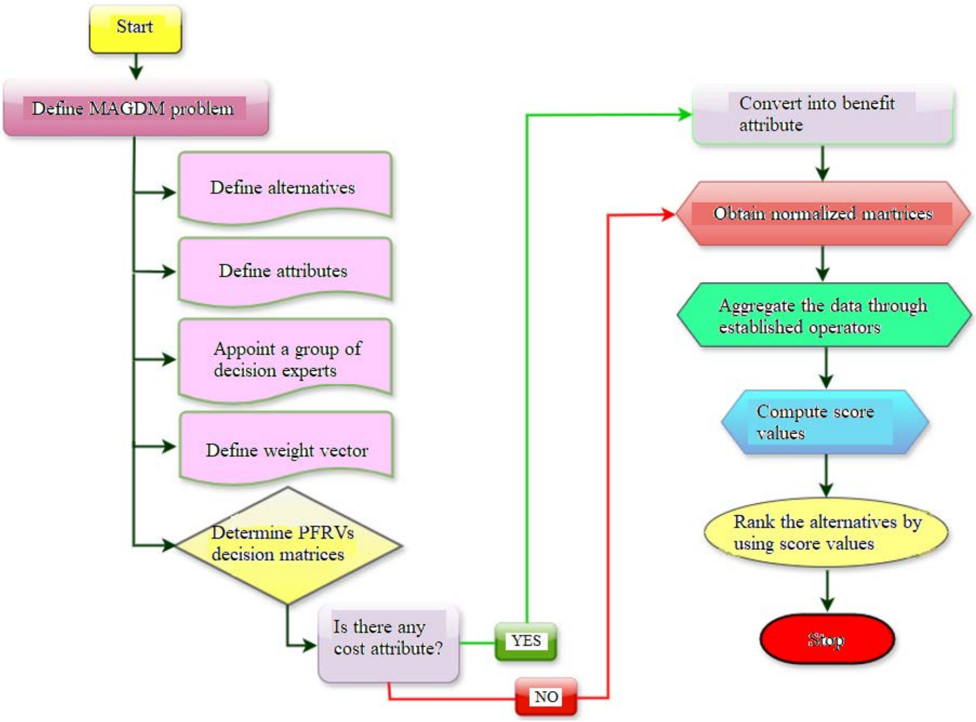

Figure 1. Framework of the proposed MAGDM scheme

The experts assess each potential supplier in light of these characteristics. PFRVs are the assessments that the experts offer. The data collected from the experts is displayed in the following tables.

Table 1. PFRVs provided by expert $\mathcal{Z}_1$

	Ξ_1						Ξ_2					
	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
$\mathcal{F}_1$	0.45	0.56	0.51	0.62	0.59	0.64	0.53	0.37	0.53	0.43	0.63	0.53
$\mathcal{F}_2$	0.53	0.52	0.56	0.72	0.45	0.53	0.42	0.45	0.46	0.34	0.54	0.65
$\mathcal{F}_3$	0.52	0.44	0.43	0.56	0.53	0.56	0.46	0.56	0.54	0.44	0.66	0.56
$\mathcal{F}_4$	0.65	0.34	0.35	0.72	0.82	0.68	0.53	0.43	0.63	0.45	0.72	0.67
	Ξ_3						Ξ_4					
	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
$\mathcal{F}_1$	0.54	0.45	0.45	0.45	0.35	0.56	0.34	0.56	0.45	0.63	0.73	0.54
$\mathcal{F}_2$	0.47	0.56	0.44	0.43	0.62	0.53	0.45	0.43	0.34	0.35	0.53	0.64
$\mathcal{F}_3$	0.56	0.66	0.36	0.37	0.49	0.58	0.46	0.34	0.46	0.44	0.54	0.66
$\mathcal{F}_4$	0.73	0.45	0.48	0.42	0.54	0.63	0.46	0.52	0.48	0.56	0.63	0.72

Table 2. PFRVs provided by expert $\mathcal{Z}_2$

	Ξ_1						Ξ_2					
	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}
$\mathcal{F}_1$	0.45	0.56	0.45	0.63	0.67	0.69	0.56	0.36	0.56	0.45	0.64	0.73
$\mathcal{F}_2$	0.43	0.43	0.53	0.65	0.54	0.56	0.45	0.45	0.55	0.54	0.72	0.67
$\mathcal{F}_3$	0.54	0.32	0.52	0.64	0.54	0.63	0.45	0.44	0.54	0.56	0.73	0.56
$\mathcal{F}_4$	0.44	0.32	0.53	0.73	0.56	0.68	0.36	0.52	0.43	0.58	0.66	0.58
	Ξ_3						Ξ_4					
	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}
$\mathcal{F}_1$	0.56	0.56	0.56	0.62	0.72	0.68	0.45	0.38	0.36	0.53	0.58	0.62
$\mathcal{F}_2$	0.45	0.46	0.63	0.75	0.67	0.67	0.52	0.36	0.45	0.53	0.68	0.57
$\mathcal{F}_3$	0.44	0.44	0.45	0.64	0.77	0.59	0.53	0.34	0.33	0.56	0.67	0.78
$\mathcal{F}_4$	0.36	0.45	0.36	0.73	0.74	0.65	0.44	0.37	0.45	0.56	0.73	0.67

Table 3. PFRVs provided by expert $\mathcal{Z}_3$

	Ξ_1						Ξ_2					
	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}
$\mathcal{F}_1$	0.44	0.45	0.46	0.63	0.56	0.73	0.46	0.48	0.43	0.53	0.56	0.63
$\mathcal{F}_2$	0.38	0.46	0.56	0.64	0.73	0.64	0.57	0.36	0.44	0.56	0.44	0.64
$\mathcal{F}_3$	0.43	0.38	0.54	0.56	0.77	0.73	0.45	0.38	0.45	0.43	0.47	0.74
$\mathcal{F}_4$	0.36	0.35	0.55	0.65	0.56	0.65	0.46	0.37	0.56	0.33	0.56	0.65
	Ξ_3						Ξ_4					
	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}
$\mathcal{F}_1$	0.44	0.56	0.36	0.67	0.55	0.72	0.54	0.56	0.52	0.63	0.72	0.83
$\mathcal{F}_2$	0.56	0.44	0.34	0.76	0.67	0.65	0.56	0.38	0.34	0.72	0.75	0.78
$\mathcal{F}_3$	0.62	0.39	0.46	0.67	0.55	0.65	0.39	0.45	0.45	0.75	0.65	0.59
$\mathcal{F}_4$	0.43	0.38	0.56	0.56	0.56	0.56	0.35	0.44	0.44	0.69	0.67	0.69

Table 4. The values aggregated individually by the PFRSSWA operator

	Ξ_1						Ξ_2					
	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}
$\mathcal{F}_1$	0.4475	0.5275	0.4763	0.6262	0.6164	0.6776	0.5267	0.3897	0.5213	0.4597	0.6180	0.9414
$\mathcal{F}_2$	0.4626	0.4679	0.5490	0.6696	0.5757	0.5648	0.4767	0.4248	0.4940	0.4519	0.6050	0.9588
$\mathcal{F}_3$	0.5088	0.3751	0.4963	0.5868	0.6119	0.6181	0.4538	0.4596	0.5203	0.4763	0.6543	0.9317
$\mathcal{F}_4$	0.5261	0.3347	0.4849	0.7025	0.6802	0.6720	0.4597	0.4427	0.5503	0.4516	0.6629	0.9478
	Ξ_3						Ξ_4					
	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}	Λ^{ι}	β^{ι}	ϑ^{ι}	Λ^{μ}	β^{μ}	ϑ^{μ}
$\mathcal{F}_1$	0.5269	0.5134	0.4822	0.5496	0.5841	0.6329	0.4457	0.4766	0.4424	0.5862	0.6778	0.6127
$\mathcal{F}_2$	0.4890	0.4867	0.5147	0.5729	0.6524	0.6025	0.5079	0.3893	0.3893	0.4728	0.6542	0.6313
$\mathcal{F}_3$	0.5394	0.4832	0.4240	0.5078	0.6371	0.5993	0.4754	0.3640	0.4169	0.5339	0.6225	0.6704
$\mathcal{F}_4$	0.5662	0.4309	0.4681	0.5343	0.6351	0.6167	0.4293	0.4355	0.4592	0.5853	0.6808	0.6920

Step 1. If the cost attribute exists, we always normalize the decision matrices. No cost type attribute is present in Table 1, Table 2 and Table 3. Normalization is, therefore, unnecessary. Therefore, we use the PFRSSWA and PFRSSWG operators to individually aggregate the attributes for each possible influence.

Step 2. The individual aggregated values of the attributes from the PFRSSWA and PFRSSWG operators are shown in Table 4 and Table 5, respectively.

Step 3. Table 4 and Table 5 present the different total standards for every attribute. Currently, we have to combine the attribute values for every possible supplier by using the PFRSSWG and PFRSSWA operators. The standards of the attributes that are attained from the PFRSSWG and PFRSSWA operators are shown in Table 6 and Table 7 in the subsequent section.

Step 4. The combined aggregated values from PFRSSWA and PFRSSWG operators are displayed in Table 6 and Table 7. We compute the score standards given in Table 8 in the following manner using the data from Table 6 and Table 7.

Table 5. The values aggregated individually by the PFRSSWG operator

	Ξ_1						Ξ_2					
	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
$\mathcal{F}_1$	0.6263	0.6075	0.6833	0.4475	0.5366	0.4733	0.4661	0.6134	0.6427	0.5206	0.4002	0.8829
$\mathcal{F}_2$	0.6753	0.5305	0.5722	0.4478	0.4741	0.5481	0.4909	0.5548	0.6553	0.4625	0.4309	0.8632
$\mathcal{F}_3$	0.5930	0.5719	0.6360	0.5011	0.3864	0.4872	0.4907	0.6102	0.6154	0.4537	0.4807	0.8854
$\mathcal{F}_4$	0.7078	0.6178	0.6727	0.4709	0.3353	0.4528	0.4889	0.6460	0.6334	0.4381	0.4571	0.8896
	Ξ_3						Ξ_4					
	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
$\mathcal{F}_1$	0.5843	0.4899	0.6525	0.5176	0.5246	0.4577	0.5959	0.6554	0.6633	0.4212	0.5062	0.4263
$\mathcal{F}_2$	0.6690	0.6498	0.6197	0.4812	0.4977	0.4616	0.5493	0.6202	0.6589	0.5002	0.3934	0.3773
$\mathcal{F}_3$	0.5784	0.5755	0.6027	0.5180	0.5355	0.4156	0.5884	0.6077	0.6966	0.4635	0.3739	0.4017
$\mathcal{F}_4$	0.6029	0.6000	0.6221	0.4649	0.4346	0.4429	0.5980	0.6723	0.6942	0.4215	0.4524	0.4581

Table 6. The total values acquired by the PFRSSWA operator

Ξ_1						Ξ_2					
Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
0.6138	0.3177	0.6193	0.4677	0.7107	0.4707	0.6024	0.3304	0.6360	0.3531	0.7195	0.7259
Ξ_3						Ξ_4					
Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
0.6441	0.3642	0.5926	0.4044	0.7117	0.4580	0.5951	0.3181	0.5675	0.4107	0.1365	0.4924

Table 7. The total values acquired by the PFRSSWG operator

Ξ_1						Ξ_2					
Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
0.7303	0.4373	0.7269	0.3597	0.5701	0.3702	0.6086	0.4570	0.7184	0.3562	0.5849	0.6709
Ξ_3						Ξ_4					
Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ	Λ^ι	β^ι	ϑ^ι	Λ^μ	β^μ	ϑ^μ
0.6966	0.4301	0.7101	0.3795	0.6231	0.3384	0.6812	0.4714	0.7505	0.3436	0.5645	0.3198

Table 8. Score values found by PFRSSWA and PFRSSWG operators

Operator	**Score Values**
$PFRSSWA$	$\Xi_1 = 0.563, \Xi_2 = -0.206, \Xi_3 = 0.153, \Xi_4 = -0.114$
$PFRSSWG$	$\Xi_1 = 0.620, \Xi_2 = -0.160, \Xi_3 = 0.167, \Xi_4 = -0.208$

Step 5. The potential suppliers are ranked, and the best source of organic cotton for the clothing company is evaluated based on the score standards found in Table 8. Table 9 presents the resulting ranking.

Table 9. Ranking of possible influences

Operator	**Score Values**
$PFRSSWA$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
$PFRSSWG$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$

Based on the evaluation of PFRSSWA and PFRSSWG operators, the supplier with the highest level of suitability is Ξ_1, respectively. Compared to the PFRSSWA operator, the PFRSSWG operator is thought to be more dependable because it is founded on the geometric average of the data.

4.1 Sensitivity Analysis

The flexibility and significance of the SSTrM and SS-TC operators in handling fuzzy information stems from their composition with the parameter Δ. Parameter $\Delta = 3$. It is utilized in our example. Adjustments to the parameter, however, might affect how the possible suppliers are ranked. In light of this, we investigate any potential variance in the outcomes from the two operators. The ranking of the results from the PFRSSWA and PFRSSWG operators varies, as Table 10 below illustrates, depending on whether the parameter Δ.

Table 10. Analysis of PFRSSWA and PFRSSWG operators' sensitivity

Δ	PFRSSWA	PFRSSWG
-3	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
-5	$E_1 \succ E_3 \succ E_2 \succ E_4$	$E_1 \succ E_3 \succ E_2 \succ E_4$
-10	$E_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
-20	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$	$E_1 \succ E_3 \succ E_2 \succ E_4$
-30	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$	$E_1 \succ E_3 \succ E_2 \succ E_4$
-50	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
-70	$E_1 \succ E_3 \succ E_2 \succ E_4$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
-90	$E_1 \succ E_3 \succ E_2 \succ E_4$	$E_1 \succ E_3 \succ E_2 \succ E_4$
-100	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$

Table 10 indicates that, at $\Delta = 3$, Ξ_1 is the most appropriate supplier identified by the PFRSSWA and PFRSSWG operators. That being said, we varied the Δ values and saw the ranking for the PFRSSWG and PFRSSWA operators at various Δ values. Notably, for every value Δ, we came up with the same ranking.

4.2 Comparative Analysis

An exemplary explanation is offered for evaluating the efficacy of the proposed approach with positive redeveloped strategies before imparting a top-level view of the work. In this context, an appropriate degree is hired to determine whether or not the assessment procedure is successful, i.e., whether the implemented widespread variables are found in the contemporary techniques or no longer. It is predicted that the counseled method is more appealing in terms of computing simplicity and logical inference. Ullah [22] introduced the PF on the Maclaurin symmetric mean operator and their application to MADM. Wei [15] developed the PF aggregation operator and its application to the MADM. Tian et al. [24] gave the concept of weighted PF aggregation operators and their applications in MADM. Dinçer et al. [28] used the concepts of PFR on renewable-friendly smart grid technologies.

Table 11. Comparison of different operators

Operator	Score Values
$PFRSSWA$	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
$PFRSSWG$	$\Xi_1 > \Xi_3 > \Xi_2 > \Xi_4$
$PFMSM$ [22]	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
$PFHA$ [15]	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
$PFHWA$ [15]	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$
$WPFA$ [24]	$\Xi_1 > \Xi_3 > \Xi_2 > \Xi_4$
$WPFG$ [24]	$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$

It is evident from Table 11 that Ξ_1 is the most desirable alternative diagnosed by using all the averaging and geometric operators.

5 Conclusions

We have presented the PFRSSWA and PFRSSWG operators in this article. These operators use SS-TN and SS-TC to process data that is presented as PFRVs. A number of these AO's basic properties have been investigated, and we have used them to address MAGDM issues. To determine their significance, the analysis involved comparing the results with previously developed methods while analyzing the results across a range of parameter values in SS-TN and SS-TC. The PFRSSWA and PFRSSWG operators that were developed have proven to be highly significant because SS-TN and SS-TC provide adaptive and flexible operational laws for managing fuzzy data. These frameworks improve the accuracy and applicability of the results by enabling decision-makers to choose parameter values that are specific to their needs. A useful tool for handling difficult decision-making problems is the PFRS framework, which serves as a link between PFS and RS. We obtain more dependable and optimized results than other methods by first aggregating data through PFRS and then refining it with PFRSSWA and PFRSSWG operators. The PFRSSWA and PFRSSWG operators further aggregate the information after PFRS has completed its initial stage of information aggregation in the form of PFRVs. As such, the outcomes are additional dependable and optimized than with the other operators and contexts. We hope to add the established strategy to the context specified in the future.

Data Availability

Not applicable.

Conflict of Interests

The authors declare that they have no conflicts of interest.

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