Journal of Civil and Hydraulic Engineering
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Stability Analysis of Steel Columns with Fixed-Free Ends under Axial Compression: Uniform and Non-Uniform Square Hollow Sections

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Received: 04-20-2024 **Revised:** 06-14-2024 **Accepted:** 06-21-2024

Citation: E. M. M. Fonseca, "Stability analysis of steel columns with fixed-free ends under axial compression: Uniform and non-uniform square hollow sections," *J. Civ. Hydraul. Eng.*, vol. 2, no. 2, pp. 109–119, 2024. https://doi.org/10.56578/jche020204.

Abstract: This study investigates the stability of steel columns subjected to axial compression, focusing on square hollow sections (SHS) with both uniform and non-uniform cross-sections. The stability of fixed-free end SHS columns with uniform cross-sections was initially verified using analytical equations. To obtain the critical load and design buckling resistance for each SHS column, Finite Element Analysis (FEA) was employed. The results indicate that while analytical equations can validate the stability of uniform SHS columns, they are insufficient for columns with non-uniform cross-sections. Consequently, the FEA emerges as a robust alternative for analyzing columns with varying cross-sections along their length. This study highlights the necessity of numerical methods for verifying the stability of structurally complex columns, such as those with perforations for mechanical and electrical applications. The finite element model was validated and applied to non-uniform cross-section columns, providing insights into the stability of these columns under practical conditions. This research aims to offer an alternative analytical approach for structural engineering applications where column stability is critical, especially for non-uniform cross-sectional designs that facilitate handling processes in various engineering scenarios.

Keywords: Steel columns; Finite Element Analysis (FEA); Stability; Square hollow sections (SHS)

1 Introduction

Columns are structural elements in many applications subjected to axial compression. Accordingly, the stability of columns is an important topic to be studied. Euler was a pioneer in the study of buckling in uniform columns [1]. Subsequently, other authors contributed to this field [1–4]. Among others, some researchers carried out the analysis of different cross-section columns using FEA and experimental tests, comparing solutions with the critical buckling load [2, 5, 6]. The problem of stability is related to column buckling, which happens with a minimum lateral displacement under an axial compressive load [7]. The failure modes involve yielding, crushing, and buckling failure, even for short and long columns [7]. However, in the literature, it is difficult to find solutions to the buckling problem of a non-uniform column, where the cross-sections can vary along the length.

In engineering, with all the constant technological evolution and the research for new innovative solutions, the continuous review and improvement of new analyses and methods is needed. In metallic structures, with an emphasis on buckling in closed tubular or hollow sections, it is crucial to understand the challenges inherent when subjected to compressive loads. Therefore, it is important to focus on new solutions and incorporate normative references or other methodologies to develop more efficient and safe structures [8, 9].

Buckling represents a complex and critical phenomenon of instability, which can compromise the integrity of a structure if not properly considered. The investigation of the buckling of columns became the most systematic and influential during the second half of the 20th century [1]. To analyze this phenomenon, it is necessary to understand the factors that influence its behavior. Thus, its geometric configuration becomes an important factor, as the relationship between the height and the radius of rotation of the cross-section, expressed by slenderness, becomes fundamental in the analysis of cross-sections with the ability to resist acting loads. Given that the nature of the material can also significantly influence structural behavior, there is a need to obtain more sophisticated approaches by international standards, which has led to further revisions of conventional design methods. Regarding the buckling phenomenon, the Eurocode considers specific guidelines for the evaluation and design of this type of section. The Eurocode incorporates geometric characteristics of sections and the influence of initial geometric imperfections, among other factors that can interfere with the overall stability of the structure [9].

In addition to these analytical approaches, it is essential to employ advanced methods, such as numerical techniques like FEA, and even experimental methods. These procedures allow for a deeper understanding and realistic simulation of structural behavior in complex situations. Through this practical method, it is imperative to consider real cases in these studies. From the construction of bridges and buildings to the maritime and offshore environment, buckling analysis is present in constituent hollow sections, directly influencing the design and its safety. However, through new solutions to minimize buckling in columns with hollow cross-sections, new progress is being made in their application. For these new solutions, the growth in the use of high-strength materials to improve the efficiency and behavior of structural elements against buckling phenomena is notable [9–11].

In metallic structures, the increasing availability of steels with increasingly higher strengths has driven the design and construction of increasingly slender structures. This type of structure is associated with benefits such as greater economy and/or architectural quality. However, the word slenderness also denotes a susceptibility to the occurrence of instability or buckling phenomena. The analysis and design of these structures are conditioned by the need to consider such phenomena, whose nature is non-linear [8].

In this way, buckling can be identified by a certain value of compression load, which is exceeded and manifests itself in a structural disturbance of the column. In this way, a sudden lateral deformation known as "bending" is observed, and the load that produces it is called the Euler critical load. Collapse occurs due to a loss of stiffness, even when the column is still in an elastic regime [10]. This phenomenon is characterized by the occurrence of large transverse deformations in elements subjected to compression efforts for any laterally restricted section [10].

This work aims to achieve different results in the analysis of the buckling phenomenon in closed hollow sections of steel subjected to compression. Based on standards using analytical equations, it will be possible to subsequently implement and develop different numerical analyses and compare the solutions. The finite element method will be used, as a numerical methodology, to determine the buckling resistance. Both methods will be extended to a set of different parametric models applicable to the project. Circular hollow sections (CHS) and rectangular/square hollow sections (RHS/SHS) are types of hollow sections used in construction such as columns, beams, tension members, and truss members, where they have gained preference due to their aesthetics and advantages compared with open sections [11]. Because they have thin walls, local buckling can occur before or after the material yields and becomes a critical design factor [11]. In many applications, the cross-sections can vary through the length of the column, to provide the economy of the material. Non-uniform cross-sections along the length of the column have less weight than those with uniform thickness, but the stability needs to be verified carefully [12].

By achieving the main objective of this study, different specific objectives will be achieved:

- Define the buckling phenomenon;
- Identify the set of specifications applicable to buckling design in hollow fixed-free end steel columns in compression;
- Assess analytical models for determining buckling resistance in several SHS sections;
- Implement numerical algorithms to determine the buckling resistance in the same sections of SHS;
- Implement the numerical algorithm tested to verify stability in SHS columns with non-uniform cross-sections along the length;
- Discussion of results.

2 Stability of Columns, Analytical Equations

The elastic buckling theory combined with empirical factors is transformed into recommendations [13], described in Eurocode 3, part 1-1 [14]. An element subjected to a compressive load may fail or collapse when it reaches the ultimate strength or yield stress of the material, or when it becomes unstable through the phenomenon of lateral deformation due to buckling or bending. When one of these phenomena occurs, the load value at which this happens is called the critical load [9].

Columns in compression have a limit on their resistance, which is equal to the yield limit of the material multiplied by the cross-section area [15], designed by plastic load N_{pl}, represented in Eq. (1).

Shorter, slender, or very stocky columns fail to yield, where the critical buckling stress may be greater than the yield stress of the material. In this region, the material no longer behaves elastically [13, 15]. The buckling of these regions is called inelastic buckling [13, 15], and the calculation of N_{pl} indicates its resistance to yield [16], Eq. (1).

$$N_{pl} = Af_y \tag{1}$$

where, f_y is the yield strength, and A is the cross-section area of the column.

Long, slender columns will tend to buckle and fail under much lower loads by elastic buckling when the critical buckling stress is reached [13, 15]. When this stress is in the elastic range of the material, it is called elastic buckling [13, 15], and the elastic buckling or Euler critical load Ncr provides a measure of the slenderness of a compressive member [16], Eqs. (2)-(5). In practice, members in compression present slenderness between these two extremes and fail due to a combination of yielding and buckling [15].

The elastic buckling, or Euler critical load N_{cr} is the smallest or ideal value reached when the column is being loaded, calculated according to Eq. (2). This equation allows the calculation of the critical load for an ideal column pinned at both ends.

$$N_{cr} = \frac{\pi^2}{L^2} EI \tag{2}$$

In Eq. (2), L is the length of the column, E is the Young's Modulus, and I the smallest moment of inertia.

For other applied boundary conditions, it needs the inclusion of a different effective length $Le = kL$. The effective length is defined as the column length between two points of zero moment. The value of this factor is derived from different boundary conditions, where for fixed-free end columns $k = 2$, Eq. (3).

$$N_{cr} = \frac{\pi^2}{4L^2} EI \tag{3}$$

Figure 1 represents the boundary conditions imposed in the studied fixed-free end columns, according to the previous equation. The Euler theory applied to column buckling is based on assumptions: the columns are straight before the load application on the longitudinal axis, the columns have a uniform cross-section through the length, and the material is isotropic [17].

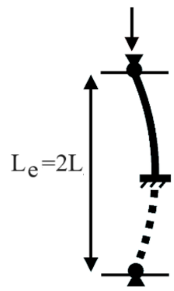

Figure 1. Boundary conditions in fixed-free end columns

In summary, buckling loads are sensitive to material stiffness, column length, cross-section dimensions and shape, boundary conditions, load, and initial eccentricity.

The properties of steel S235 adopted in this work are shown in Table 1 [14].

Table 1. Properties of steel grade S235

Property	Symbol	Value
Young Modulus of Elasticity	E	210 GPa
Poisson's Ratio	υ	0.3
Yield strength	f_y	235 MPa
Ultimate tensile stress	f_u	360 MPa

The classification of the cross-sections of structural elements by class allows us to understand how the resistance and rotation capacity of a column cross-section can be affected by plasticity or local buckling phenomena [14]. The classification of a cross-section depends on the width-to-thickness ratio, of each part of the element subject to compression.

According to Eurocode 3 part 1-1 [14], there are four classes of steel cross-sections that are designed as:

• Class 1 (Plastic cross-section) - Sections in which a plastic hinge can be formed, with the rotation capacity necessary for plastic formation, without reducing its resistance;

• Class 2 (Compact cross-section) - Sections that can reach the plastic-resistant moment, but whose rotation capacity is limited by local buckling;

• Class 3 (Semi-compact cross-section) - Sections in which the stresses in the extreme fibers, calculated based on the elastic distribution of stresses, can reach the value of the yield stress, but where local buckling can prevent the plastic resistant moment from being reached;

• Class 4 (Slender cross-section) - Sections in which local buckling occurs before the yield stress is reached in one or more parts of the transversal cross-section.

3 Simplified Equation from Eurocode 3 Part 1-1 Applied to Columns Subject to Axial Compression

The critical buckling load does not deliver enough information about when collapse due to column instability will occur. This depends on other factors such as initial geometrical imperfections, eccentricities of loading, and nonlinear material behavior [17]. According to Eurocode 3, part 1-1 [14] the buckling resistance of members in compression is determined using the following simplified equations. $N_{b,Rd}$ is the design buckling resistance of the compressive member given by Eq. (5).

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \tag{4}$$

where, χ is the reduction factor for the relevant buckling mode, and γ_{M1} the partial member resistance factor to instability is equal to 1 for buildings.

The reduction factor for axial compression members is:

$$\chi = \frac{1}{\Phi\sqrt{\Phi^2 - \bar{\lambda}^2}} \quad \text{but } \chi \leq 1 \tag{5}$$

where,

$$\Phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right] \tag{6}$$

And the non-dimensional slenderness is obtained according to Eq. (7), for class 1, 2, or 3 cross-sections.

$$\bar{\lambda} = \sqrt{\frac{A f_y}{N_{cr}}} \tag{7}$$

α is an imperfection factor, corresponding to the appropriate buckling curve according to a cross-section [14]. For hollow sections and material S235, depending on hot or cold finishing, the constant value is equal to 0.21 or 0.49, respectively. For slenderness $\bar{\lambda} \leq 0.2$ or for $\frac{N_{Ed}}{N_{cr}} \leq 0.04$, the buckling effects can be ignored.

4 Analytical and Simplified Results Applied to Fixed-Free End SHS Columns

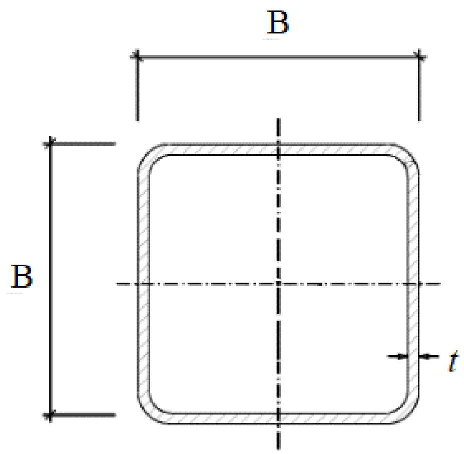

Figure 2. Geometry SHS (B=50, 60, 80, t=1.5) mm

Several SHS sections were considered in columns in compression with fixed-free end columns with three different lengths, see Figure 2.

Table 2 shows all the SHS sections under study and the analytical calculations of their stability. The cross-sections in the study are classified as class 1.

It is observed that the plastic load increases with the SHS cross-section dimensions. Always, the plastic load is lower than the Euler critical load, with an exception for section B80 with a column length equal to L=1500 mm. It means that the damage to the column occurs due to the plastic load, except for column B80 with L=1500 mm. Buckling analysis strongly depends on the column length and cross-section dimension. In all the columns studied, the design buckling resistance decreases as the slenderness of the column increases, always presenting a safety factor according to the load that allows damage to the column.

Table 2. Analytical results in fixed-free end SHS columns with a uniform cross-section

Cross-Section	L, mm	Eq. 1 N_{pl}, kN	Eq. 3 N_{cr}, kN	Eq. 4 $N_{b,Rd}$, kN
B50	500		236.7	62.4
50 mm	1000	68.4	59.2	41.9
$t = 1.5$ mm	1500		26.3	22.5
B60	500		415.2	77.6
60 mm	1000	82.5	103.8	61.0
$t = 1.5$ mm	1500		46.1	37.1
B80	500		1003.0	107.4
80 mm	1000	110.7	250.7	95.6
$t = 1.5$ mm	1500		111.4	73.9

5 Numerical Method Applied to Fixed-Free End SHS Columns in Compression

Buckling is a significant failure condition for many types of structures. Linear numerical solutions can suit such structures if loads and boundary conditions are carefully evaluated [7]. However, for the common instability in structures, a full nonlinear analysis is essential [7]. This type of analysis is very sensitive to assuming eccentricity and the applied boundary conditions, where an applied and careful numerical methodology is required that will agree with the structural occurrence.

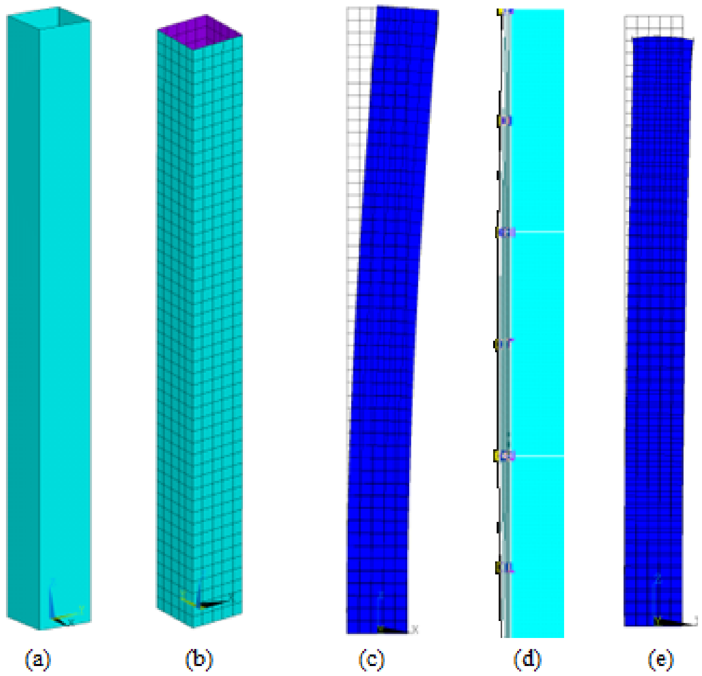

Figure 3. Example of a deformed fixed-free end SHS column with uniform cross-section: a) model, b) mesh, c) Euler critical load, d) geometric imperfections, e) design buckling resistance

The studied SHS sections in compression (Table 2) were carried out using numerical models. The main purpose of the numerical models was to investigate the shape and calculate the critical stability load of the column, as shown in Subgraph (a) of Figure 3. The typical element chosen was SHELL281 with 8 nodes and 6 degrees of freedom per node (3 translations and 3 rotations), Subgraph (b) of Figure 3. This element is suitable for analyzing thin or moderately thick shell elements, for linear, high rotation, and/or large nonlinear deformation applications [18].

Shell elements allow the study of elastic and inelastic analyses, considering geometric imperfections. Shell finite elements of steel hollow section SHS columns were created and validated with analytical and simplified results.

Due to the variety of parameters, a study mesh was initially applied. Some studies of convergence were performed to reach the most favorable mesh size, where several trial-and-errors reached the minimum with the analytical solution. In the end, in all studied cases, the mesh size considered was equal to 10 mm for the element length, Subgraph (b) of Figure 3. The material properties used in all simulations are presented in Table 1. The boundary conditions to represent the fixed-free end, as shown in Figure 1, correspond to all degrees of freedom fixed in the column end. In this condition, three rotations and three translations are prescribed.

To obtain Euler critical load results, the effect of prestress on the linear elastic buckling behavior of columns is used using ANSYS® (version 2022 R2) [18]. Prestress effects ensure the calculation of the stress stiffness matrix. This is achieved by first performing a structural analysis on an ideal, loaded structure, and then calculating the stress field to continue a modal analysis that results in the buckling mode. Eigenvalue buckling analysis calculates the theoretical buckling strength (the bifurcation point) of a perfect linear elastic structure [7]. The eigenvalue solver uses a unit load to determine the required buckling load on the column, Subgraph (c) of Figure 3.

To find the design buckling resistance, a nonlinear inelastic buckling analysis was used in ANSYS® (version 2022 R2) [18], including geometric imperfections to update the geometry and nonlinear properties of the material, Subgraph (d) of Figure 3. The shape of the global imperfections may be derived from the column elastic buckling [14]. The longitudinal curvature that columns normally present is a result of imperfections from the manufacturing, handling, and transport processes. According to some authors, initial imperfections can be characterized by a harmonic function [19, 20]:

$$u(x) = \left(\frac{L}{1000}\right) \sin\left(\frac{\pi x}{L}\right) \tag{8}$$

where, $L/1000$ is the maximum measured amplitude of the geometric column imperfection. For the middle length of the column $x = L/2$ the maximum value of the harmonic function occurs.

To update the geometry, the UPGEOM command from ANSYS® (version 2022 R2) [18] was used. This command adds displacements from the previous analysis and updates the geometry of the finite element model to the deformed configuration [17].

To implement it, it is necessary to calculate a small $FACT$ factor, obtained with the lateral displacement $u(x)$ of linear elastic buckling, which is multiplied by the displacements being added to the coordinates.

$$FACT = \left(\frac{L}{1000}\right) / u(x) \tag{9}$$

For this simulation, the applied compressive load is increased incrementally until the start of buckling in the column, Subgraph (e) of Figure 3. Nonlinear buckling analysis involves the application of Newton-Raphson, to solve an iterative and numerical system of equations with strong nonlinearity [21]. Nonlinear buckling is a more realistic simulation than linear buckling, providing a lower buckling load. Nonlinear buckling analysis involves the application of Newton-Raphson, to solve an iterative and numerical system of equations with strong nonlinearity [21]. Nonlinear buckling is a more realistic simulation than linear buckling, providing a lower buckling load.

6 Numerical Method Validation with Analytical and Simplified Equations

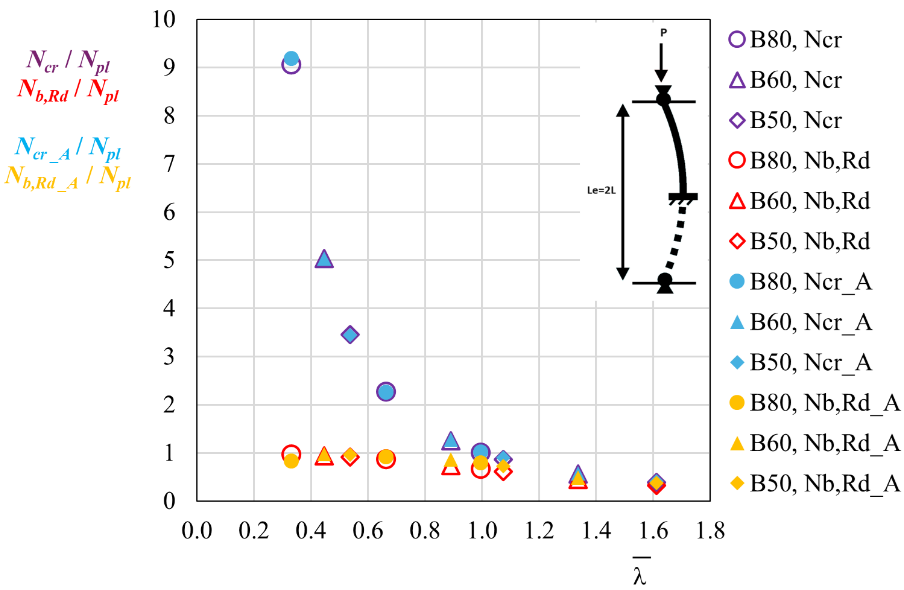

Figure 4. Comparison of results in fixed-free end SHS columns

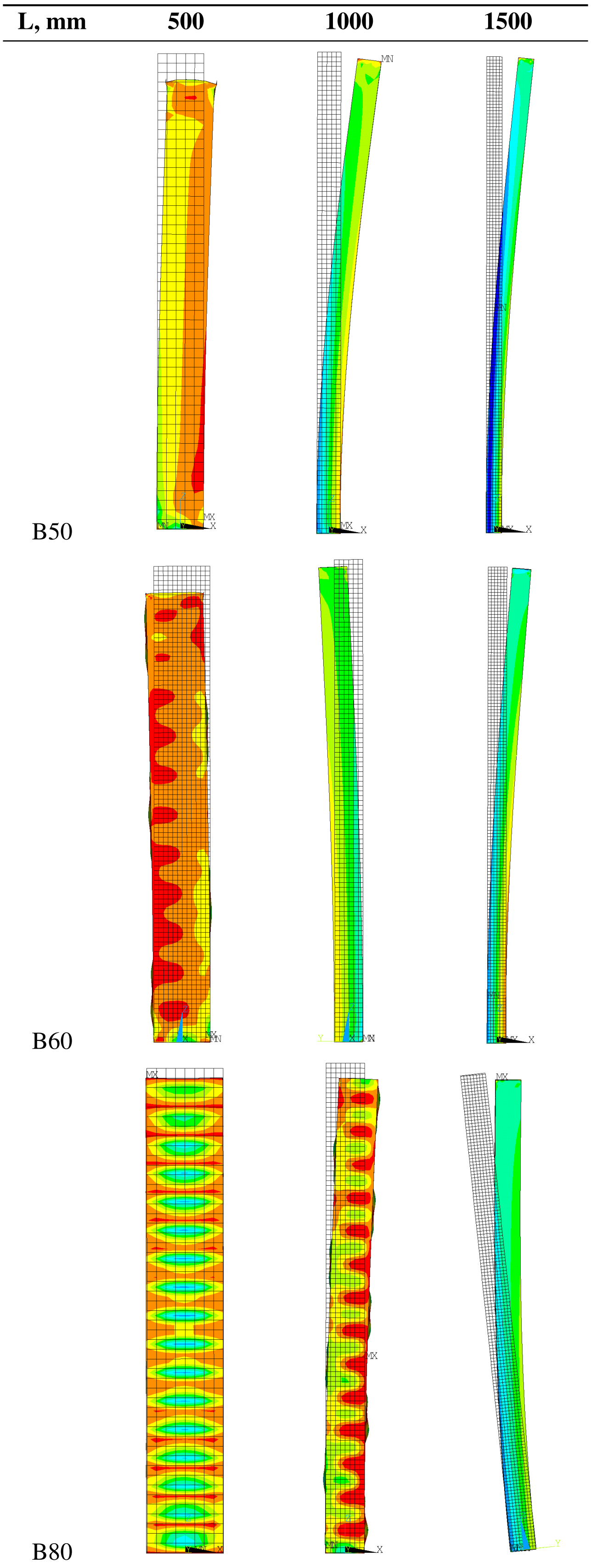

Figure 5. Numerical results of design buckling in column with uniform cross-section

The critical buckling, plastic, and resistance loads were determined using ANSYS® (version 2022 R2) [18]. The following graphics represent the comparison between the numerical, analytical, and simplified results in Figure 4.

The comparison of results was obtained with the analytical variation of N_{cr}/N_{pl}, $N_{b,Rd}/N_{pl}$ with $\bar{\lambda}$ and the numerical results from ANSYS® (version 2022 R2) [18], namely N_{cr_A}/N_{pl}, $N_{b,Rd\_A}/N_{pl}$ related to N_{pl}.

The results indicate that the Euler critical load varies with the length of the columns studied. The ratio of Euler critical load to yield load corresponds to the largest section, B80. The ratio of Euler critical load to yield load corresponds to the largest section B80 to B50. Also, columns with shorter lengths have higher loads. The great columns in the study have buckling effects, due to their slenderness $\bar{\lambda} > 0.2$. The large columns in the study present buckling effects, due to the slenderness $\bar{\lambda} > 0.2$.

The numerical results agree with the analytical ones using the simplified equations. The design buckling resistance related to resistance to yield of the columns is lower compared with the Euler critical load related to resistance to yield, namely until the slenderness is equal to 1.0. The columns will fail to achieve this slenderness value.

The results presented indicate that the design buckling resistance also varies with the length and the cross-section size of the columns studied, showing a large difference when the slender column is small has a larger cross-section dimension. In all studied columns, when the buckling length of the column decreases, the stability of the buckling increases. According to the cross-section size, for small dimensions, the instability of the column increases.

Figure 5 shows the numerical results of the design buckling resistance in the SHS columns of the study, which represents the applied compressive load when starting the buckling. For this value, the maximum yield stress is reached.

It is observed that for longer column lengths (L=1500 mm) the design buckling resistance approaches the Euler critical load.

For small lengths (L=500 mm) local plasticization occurs where the maximum load is close to the plastic load. This behavior was shown in the results presented in Figure 5.

The SHS column with B80 and a length of L=1000 mm presents local plastic behavior. This local buckling has the consequence of reducing the load-carrying capacity of columns due to the decrease in strength and stiffness of the locally buckled elements.

7 Numerical Proposal to Study Fixed-Free End Steel SHS Columns in Compression, with Non-Uniform Cross-Sections

In engineering structures, there are columns with non-uniform cross-sections where there may be different holes for different applications. The use of steel columns with non-uniform cross-sections is very common [22]. This type of geometry makes structural elements more unstable under buckling.

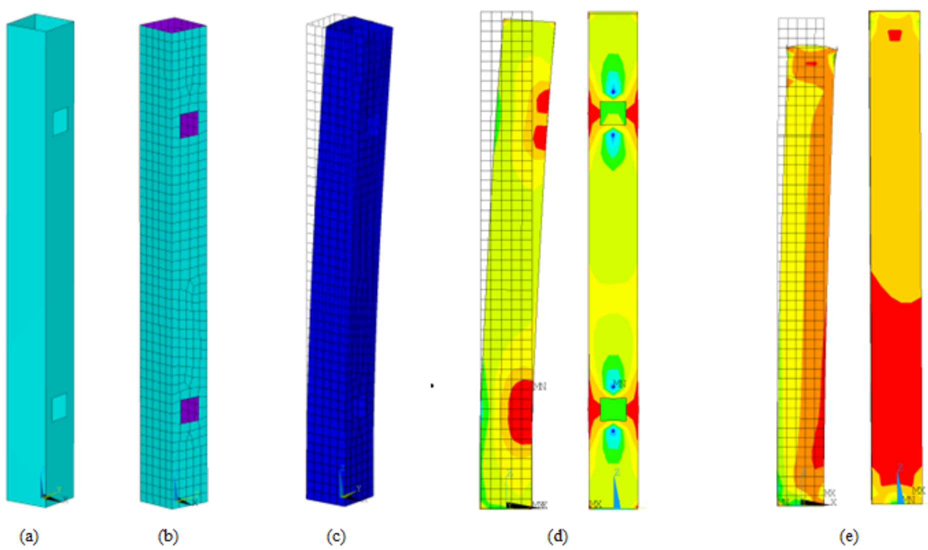

Figure 6. Example of a fixed-free end SHS column with non-uniform cross-section: a) model, b) mesh, c) deformed and Euler critical load, d) two views of the deformed and design buckling resistance, e) comparison with two views of the deformed and design buckling in column with uniform cross-section

Konstantakopoulos et al. [22] present a method for the study of non-uniform steel members with or without imperfections, where the governing equation of the problem is solved by the Galerkin method using the eigen shapes of the member. In its research, he proves the accuracy of its method using different cross-sections from IPE and HEB profiles.

In this work, after validation, the developed numerical shell models were used to obtain results for SHS columns with non-uniform cross-sections. Two different holes were introduced through the length of the columns, with dimensions of 25 × 25 mm, and a distance from the bottom and top equal to 100 mm. Figure 6 represents the results obtained for the fixed-free end SHS column (B50 and L=500 mm).

It is observed that the cavities along the entire length of the column reach the maximum resistances neighboring them. In columns with uniform cross-sections, the maximum resistance is constant, with a predominance close to the fixed end.

Table 3. Numerical results in fixed-free end SHS columns with uniform and non-uniform cross-section

Cross-Section	L	**Uniform Cross-Section** N_{cr_A}	N_{b,Rd_A}	**Non-Uniform Ccross-Section** $N_{cr\_A^*}$	N_{b,RdA_A^*}	**Eq. 10** R_{cr*}	**Eq. 11** $R_{b,Rd*}$
	mm	kN	kN	kN	kN	%	%
B50	500	235.3	65.8	227.9	46.8	3.1	28.9
	1000	61.7	50.0	56.5	37.1	8.4	25.8
	1500	28.1	25.8	25.6	22.2	8.8	14.0
B60	500	415.3	80.2	394.8	61.0	4.9	24.0
	1000	105.9	71.3	96.1	53.6	9.2	24.7
	1500	48.0	40.1	44.9	38.4	6.4	4.2
B80	500	1016.3	91.8	998.8	70.4	1.7	23.3
	1000	250.2	100.8	248.1	72.3	0.8	28.3
	1500	112.7	88.0	116.7	73.28	3.5	16.7

Table 3 represents all other columns of the study with a non-uniform cross-section. The same holes were always imposed on all the studied columns. Results from ANSYS® (version 2022 R2), namely N_{cr_A}, N_{b,Rd_A}, and $N_{cr\_A^*}$, N_{b,RdA_A^*}, respectively, for columns with a uniform and non-uniform cross-section, were obtained numerically. According to these values, a relation was obtained to verify the reduction of the Euler critical load R_{cr*} and the design buckling resistance $R_{b,Rd*}$, Eqs. (10) and (11), respectively.

$$R_{cr*} = \left(\frac{Ncr_A - Ncr_{A^*}}{Ncr\_A} \right) \times 100 \tag{10}$$

$$R_{b,Rd*} = \left(\frac{N_{b,Rd\_A} - N_{b,Rd\_A*}}{Nb, Rd\_A} \right) \times 100 \tag{11}$$

The result showed that there was a reduction in the Euler critical load and the design buckling resistance in all columns studied. In columns with non-uniform cross-sections, when the buckling length of the column decreases, the stability of the buckling increases, as shown for columns with uniform cross-sections. In general, the reduction in column resistance is greater for columns with shorter lengths, being more significant when comparing the value of the design buckling resistance obtained, rather than the Euler critical load.

The reduction in buckling resistance is not constant or linear when comparing all results obtained in the studied columns. The resistance to buckling must be calculated depending on the required column dimensioning.

8 Conclusions

According to the main objectives of this work, the following conclusions were reached:

• The buckling phenomenon was studied;

• Different uniform and non-uniform cross-sections were selected to study the buckling phenomena in hollow fixed-free end steel columns in compression;

• Analytical calculations were produced to determine the critical load and the buckling resistance in several uniform SHS cross-sections;

• Numerical algorithms were implemented to determine the critical load and the buckling resistance in the same uniform SHS cross-sections;

• The validated numerical algorithm was used to verify the stability of non-uniform SHS columns, which is the highlight of this manuscript.

The main conclusions obtained from all the results showed that the use of the numerical model agrees with the results from the analytical and simplified equations. The numerical shell models simulate the conditions in a way close to the analytical proposal, concluding that the finite element method can reproduce the same response. Accordingly, the validated finite element model was subsequently used on other columns with different cross-sections. These results were used to evaluate the application of numerical models in SHS where there is variation in length and simplified or analytical methods are not available. In engineering, such columns have been used due to their lightweight, providing material economy, and facilitating handling in electrical and mechanical applications.

Buckling failure cases can be observed in various circumstances: in continuously welded rail tracks, dynamic train loads, as well as track and wheel defects, and thin-walled tubes. The cross-section size and material properties play a significant role in determining the resistance to buckling.

Following their validation, in future work, the developed shell numerical models can be used to carry out parametric studies to generate extensive benchmarks in structural performance for other steel hollow cross-section columns. It is intended to study other different cross-section characteristics, member slendernesses, boundary conditions, and material properties.

Data Availability

Data supporting these findings are available within the article or upon request.

Conflicts of Interest

The author declares no conflict of interest.

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Nomenclature

A	Cross-section area
E	Young Modulus of elasticity
f_y	Yield strength
f_u	Ultimate tensile stress
I	Moment of inertia
Le	Effective length
L	Length of the column
N_{Ed}	Design value of the compression load
$N_{b,Rd}$	Design buckling resistance of the compressive load
N_{cr}	Euler critical load
N_{pl}	Yield load
k	Length factor

Greek symbols

α	Imperfection factor
χ	Reduction factor
γ_{M1}	Partial member resistance factor to instability
$\bar{\lambda}$	Non-dimensional slenderness
υ	Poisson´s Ratio