




# Comparative Analysis of Analytical and Empirical Methods for Estimating the Longitudinal Dispersion Coefficient in Open-Channel Flows



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**Received:** 09-21-2024

**Revised:** 03-16-2025

**Accepted:** 04-08-2025

**Citation:** M. J. Mawat, "Comparative analysis of analytical and empirical methods for estimating the longitudinal dispersion coefficient in open-channel flows," *J. Civ. Hydraul. Eng.*, vol. 3, no. 2, pp. 91–99, 2025. <https://doi.org/10.56578/jche030203>.



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**Abstract:** The accurate estimation of the longitudinal dispersion coefficient is crucial for predicting solute transport in natural water bodies. In this study, an analytical (integral) method based on first principles is compared with Fischer's widely used empirical approach, which is implemented in hydraulic modeling software such as the Hydrologic Engineering Center-River Analysis System (HEC-RAS). The primary objective is to evaluate the accuracy, applicability, and limitations of both methods under varying hydraulic conditions. A key advantage of the analytical approach is its ability to estimate the dispersion coefficient using velocity data alone, eliminating the need for high-cost tracer experiments that rely on solute concentration measurements. The determination index suggests an acceptable level of agreement between the two methods; however, the empirical approach systematically overestimates dispersion coefficients. Furthermore, a clear inverse relationship is observed between the slope of the channel and the magnitude of the dispersion coefficient, which is attributed to the increasing influence of shear velocity on the diffusion process. As slope values increase, solute separation time decreases, and concentration gradients become steeper. Conversely, at lower slopes, solute dispersion occurs over a broader time frame, resulting in lower concentration peaks. These findings indicate that while Fischer's method provides a robust empirical framework, it should be supplemented with field measurements to improve reliability. In contrast, the analytical method offers a more theoretically grounded alternative that may enhance predictive accuracy in solute transport modeling. The implications of these results extend to water quality management, contaminant transport studies, and hydraulic engineering applications, where the selection of an appropriate dispersion estimation method significantly influences predictive outcomes.

**Keywords:** Longitudinal dispersion coefficient; Empirical estimation; Hydrologic Engineering Center-River Analysis System (HEC-RAS); Analytical method; Solute transport; Shear velocity; Open-channel flow

## 1 Introduction

The study of solute transport in open channels is critical for understanding the behavior of pollutants and nutrients in natural and engineered water systems [1]. Due to the inherent coupling of hydrodynamic and ecological equations, water quality models are typically defined by significant complexity and challenging mathematical issues [2]. A one dimensional (1D) advection dispersion equation is widely used in water quality modelling in rivers [3]. The concentration change can be succinctly expressed in one dimension, according to the principle of conservation of mass. One key parameter in this context is the longitudinal dispersion coefficient ( $D$ ), which quantifies the spreading of a solute along the direction of flow due to the combined effects of advection and diffusion. Over the years, various empirical and theoretical approaches have been developed to estimate this coefficient, reflecting the complex interplay of factors such as channel geometry, flow velocity, and turbulence. These methods range from empirical approaches like Fischer's method, widely adopted in hydraulic modeling tools such as HEC-RAS, to more theoretically grounded analytical methods that involve integral calculus. Each method offers unique advantages and limitations, depending on the specific conditions of the flow and the channel geometry.

Field tracer investigations may be costly and labor-intensive, particularly for long rivers [2], and the estimated dispersion coefficient is applicable just to the specific stream segment analyzed and the hydraulic conditions present

during the tracer experiment. In these cases, theoretical analysis and numerical modeling become important [4]. Solving the 1D St. Venant equations is still the most common way to make computer models [1]. An exemplary analytical solution for estimating the concentrations of dissolved oxygen (DO) in river systems is represented by the Streeter–Phelps equation, established in 1925 [5]. H.B. Fischer typically developed the analytical method for estimating the dispersion coefficient in rivers, which is known as the Fischer solution for the longitudinal dispersion coefficient. Antonopoulos et al. [6] estimated the dispersion coefficient in a segment of the Axios River using the analytical Fischer technique under varying hydrological and hydrodynamic circumstances. Ma and Daggupaty [7] presented a method to analytically resolve the turbulent diffusion equation concerning the concentration of a passive contaminant released from an elevated continuous source into the atmosphere. The diffusion equation model derived from this approach is applicable for practical predictions of contaminant concentration in a turbulent atmosphere. Jung et al. [8] have analytically developed the time-averaged and oscillatory solutions of the one-dimensional vertical (1DV) advection-diffusion equation for suspended sediment in a tidal sea zone with finite water depth. Shin et al. [9] contrasted velocity-based dispersion coefficients, determined from theoretical equations applied to velocity data, with concentration-based dispersion coefficients acquired by the routing approach applied to actual concentration curves. A comparison examination of prior theoretical and empirical equations was conducted in Gubashi [10] to assess their efficacy in forecasting the dispersion coefficient in open channels. Pannone et al. [11] presented a first-order analytical solution for the actual depth-averaged concentration of tracers in shallow river flows under conditions of elevated Peclet numbers, defined as the ratio of section-averaged velocity multiplied by channel width to the turbulent diffusion coefficient. Choi et al. [12] suggested a basic analytical solution of an advection-dispersion-deposition equation to obtain the deposition coefficient. The study [13] demonstrates the derivation of an accurate analytical solution for the transient-state spatial moments of the cross-sectional average tracer concentration in extensive open channel flows, utilizing the depth-averaged advection-diffusion equation. Pannone [14] presented an extensive analytical solution for the spatial variance and coefficient of variation of the depth-averaged concentration resulting from instantaneous, cross-sectionally uniform solute sources in pseudo rectangular open channel flows.

This study examines the dispersion and corresponding concentration of pollutants in the longitudinal direction of the Shatt Al-Arab River, utilizing the integral equation that was derived by the study [15]. The integration process and concentration determination are done by Python code assistance when the required data (velocity profile, depth, width, etc.) are collected from the hydrodynamic model (HEC-RAS model) of the river. Longitudinal dispersion coefficient could be calculated from the velocity data without using the concentration data obtained from high-cost tracer tests. Also, the paper aims to compare this method with the empirical Fischer method, focusing on their applicability and accuracy in predicting solute transport, thereby providing a deeper understanding of their respective roles in environmental modeling and water resource management.

## 2 Materials and Methods

### 2.1 Theory of Dispersion and Analytical Equations

The conventional approach to the study of one-dimensional mass transport in rivers relies on the advection-dispersion equation, which, in a broader context, includes variable channel attributes and solute modification as stated in [16, 17]:

$$\frac{\partial C}{\partial t} = \frac{1}{A} \frac{\partial}{\partial x} \left( AD \frac{\partial C}{\partial x} \right) - \frac{1}{A} \frac{\partial (AUC)}{\partial x} + R_i + S_i \quad (1)$$

In this context,  $C$  represents the mean cross-sectional concentration (mg/l),  $A$  denotes the cross-sectional area ( $\text{m}^2$ ),  $D$  signifies the longitudinal dispersion coefficient ( $\text{m}^2/\text{s}$ ),  $U$  indicates the mean cross-sectional velocity (m/s),  $x$  refers to the distance (m),  $t$  is the time (s), and  $S_i$  is an expression for inflow/outflow (mg/l·s). The expression  $R_i (= dC/dt)$  denotes the rate of concentration fluctuation resulting from solute transformations (mg/l·s).

If a slug is released into a river at  $t = 0$  and  $x = 0$ , the initial condition and boundary conditions are initial condition:  $C(x, 0) = 0$ , Boundary condition:  $\int C(x, t) dx = M$ ,  $C(x, \infty) = 0$ , where  $M$  is the mass of the solute (kg) initially deposited at  $x = 0$ . The solution of Eq. (1) for instantaneous point solute source at  $(x)$  reads:

$$C(x, t) = \frac{M}{A\sqrt{4\pi D_x t}} \exp \left[ -\frac{(x - Ut)^2}{4D_x t} \right] \quad (2)$$

Eq. (2) is applicable for the conditions of a  $R_i$  that is neglected, time independent, and  $U$  and  $A$  are constant, with no inflow-outflow ( $S_i = 0$ ) and an instantaneous point source at the upstream boundary condition.

The integral equation that has been provided to estimate the longitudinal dispersion coefficient involves integrating over the depth of the flow, considering the distribution of flow velocities and the eddy diffusivity. Fischer [15] introduced this integral equation to account for the effects of velocity variations and turbulent diffusion across the depth of a river. He presented the integral equation to calculate the longitudinal dispersion coefficient, derived from

the integration of the mass conservation equation throughout the depth, using boundary conditions of zero mass flux at the bed and sea surface [6, 18]:

$$D = -\frac{1}{A} \int_0^B q'(y) \left\{ \int_0^y \frac{1}{E_y d(y)} \left[ \int_0^y q'(y) dy \right] dy \right\} dy \quad (3)$$

where,  $B$  is the width of channel (m),  $d(y)$  is the depth of local flow (m),  $q'(y)$  is the depth integrated velocity at point  $y$  as:

$$q'(y) = \int_0^{d(y)} u'(y, z) dz \quad (4)$$

$u'(y, z)$  is the velocity deviation (m/s) from the cross-sectional mean velocity, as illustrated in Figure 1.

$$u'(y, z) = u(y, z) - U \quad (5)$$

where,  $u(y, z)$  is the velocity at each point of cross-section and  $U$  is the cross-sectional average velocity (m/s):

$$U = \frac{1}{H} \int_0^H u(y, z) dz \quad (6)$$

$E_y$  is the transverse turbulent diffusion coefficient:

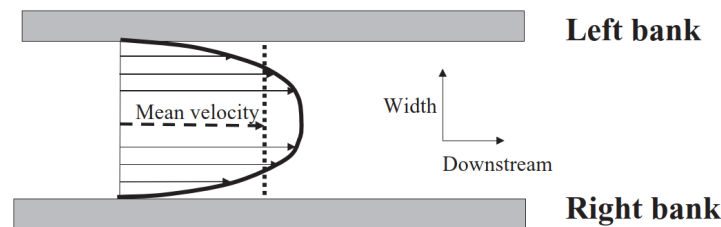
$$E_V = kH u^* \quad (7)$$

$z$  is the vertical distance,  $y$  is the transverse distance,  $k$  is the van Karman constant,  $u^*$  is the bed shear velocity:

$$u^* = \sqrt{gRs} \quad (8)$$

With  $g$  being the gravity acceleration ( $m/s^2$ ),  $R$  being the hydraulic radius (m) and  $S$  being the gradient of energy line.

Under particular circumstances, the integral approach for determining the longitudinal dispersion coefficient is useful. It is best appropriate for fully developed flow conditions in both natural and synthetic channels, as it guarantees steady and uniform flow, hence maintaining consistent velocity throughout sections. The method doesn't work for smooth flows because it assumes a turbulent flow regime where mixing is controlled by eddy diffusion instead of molecular diffusion. While transverse and vertical dispersion should be far quicker than longitudinal dispersion to maintain well-mixed conditions, accuracy depends on the river or canal being used, allowing dispersion to completely occur. Without abrupt variations in width, depth, or roughness, the channel geometry should be well-defined as that of a rectangular, trapezoidal, or stable natural cross-section. This approach is most suited for situations involving single source of contaminant release since several sources call for different considerations. It also supposes that the solute is conservative; that is, it does not deteriorate, react, or settle, therefore unsuitable for compounds experiencing notable adsorption or degradation. Moreover, external elements like wind, tides, or groundwater inflows are not taken into consideration; thus, the approach is most accurate in inland rivers, streams, and controlled channels where these forces are few.



**Figure 1.** Velocity deviation from the cross-sectional mean velocity

## 2.2 Velocity Profile in Transvers Direction

Eq. (3) demonstrates that obtaining the longitudinal dispersion coefficient requires the distribution of the depth-averaged streamwise velocity in the transverse direction. The velocity distribution equation can be obtained from

several approaches, such as the studies [18–20]. In this work, a simplified method where the velocity distribution in transverse direction is modeled as a parabolic profile across the width  $B$ .

$$u(y) = u_{\max} \left( 1 - \left( \frac{2y}{B} \right)^2 \right) \quad (9)$$

where,  $u_{\max}$  is the maximum velocity at the centreline of the river. The boundary conditions used to derive the velocity expression include a zero velocity gradient at the river's centerline and no-slip conditions at the banks.

### 2.3 Empirical Dispersion Coefficient in HEC-RAS Model

There are a variety of hypotheses that have led to the development of several empirical equations for calculating the dispersion coefficient. The most important and well-known of these is the Fischer method [16], which is used by the HEC-RAS software in its calculations [21]. This equation is based on hydraulic and geometric quantities.

$$D = m0.011 \frac{U^2 B^2}{H u_*} \quad (10)$$

where,  $m$  is the user assigned multiplier, set it equal to 1 to obtain Fischer's original form.

### 2.4 Study Area and Data Sets

The Tigris and Euphrates Rivers converge at the Al-Qurnah area in southern Iraq, creating the Shatt Al-Arab River (Figure 2). The Shatt Al-Arab River extends about 192 kilometers, running southeast through Basrah City before flowing into the Arabian Gulf [22, 23]. The seven creeks of the province—Jubyla, Muftya, Robot, Khandek, Ashar, Khora, and Saraji—are linked to the Shatt Al-Arab River and are affected by tidal phenomena [24]. The width of the Shatt Al Arab River varies along its course, measuring 250-300 meters near the confluence of the Euphrates and Tigris, 600 meters around Basrah City Center, and 2000 meters at the estuary [25]. For the remaining 95 kilometers of its path, adjacent to Umm Al Rasas Island, the river delineates the boundary between Iraq and Iran [26, 27]. Besides transportation, the Shatt Al Arab River plays a crucial role in supplying water for home purposes, agriculture, and industrial processes [28]. Numerous tributaries of the Shatt Al-Arab River, such as Al-Sweeb, Ezz, Garmat Ali, Karkkeh, and Karun Rivers, contributed to its flow along its length. The contributions from the Tigris and Euphrates Rivers, along with their low flow rates, have diminished due to the policies of neighboring governments, leading to a substantial increase in Total Dissolved Solids (TDS) in the Shatt Al Arab River, attributed to salinity intrusion from the Arabian Gulf [29–31]. The Shatt Al-Arab River is of considerable significance in a region that is characterized by aridity and a hot, humid environment, as it enables agricultural productivity [32]. The water system of the Shatt Al Arab River is now under increasing strain, both in terms of water quantity and quality [26].



Figure 2. Shatt Al Arab River

The discharge measurements collected from May to November 2020 were utilized to gather the required data. Velocity and the geometric data (depth and width) were obtained from hydrodynamic model output. Using this data of velocity distribution over the cross-section, the concentration and dispersion coefficient are estimated throughout using Eqs. (2) and (3).

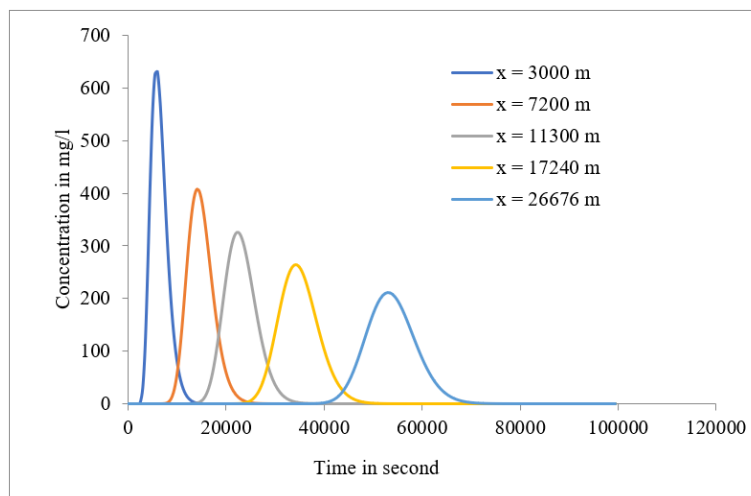
### 3 Results and Discussion

Analytical solution is used to describe the estimated values of  $D$  with Eq. (3). Table 1 presents the values of depth, width, area, velocity, shear velocity, and the estimated dispersion coefficient from available pairs of data. As shown in the table, the estimated dispersion coefficient is calculated at five stations along the river, where the positions are recorded from the upstream end. The hydraulic parameters are extracted from the HEC-RAS model. The value of  $D$  is affected by these parameters and ranges between 26.25 and 55.95  $\text{m}^2/\text{s}$ ; this range matches the results of previous works [5]. The table also shows that the highest value of  $D$  corresponds to the lowest value of  $R$ , and vice versa.

**Table 1.** The hydraulic variable of cross-sections of Shatt Al-Arab River at the multi stations

Station	$H(\text{m})$	$B(\text{m})$	$A(\text{m}^2)$	$R(\text{m})$	$U(\text{m/s})$	$u^*(\text{m/s})$	$D(\text{m}^2/\text{s})$
3000	5.01	436.15	1294.99	2.96	0.500	0.0170	55.95
7200	7.94	222.57	1054.96	4.7	0.533	0.0215	26.25
11300	9.18	331.45	1239.55	3.72	0.483	0.0191	37.26
17240	8.61	501.67	1662.82	3.31	0.533	0.0180	44.67
26670	10.29	478.6	2076.36	4.33	0.522	0.0206	29.79

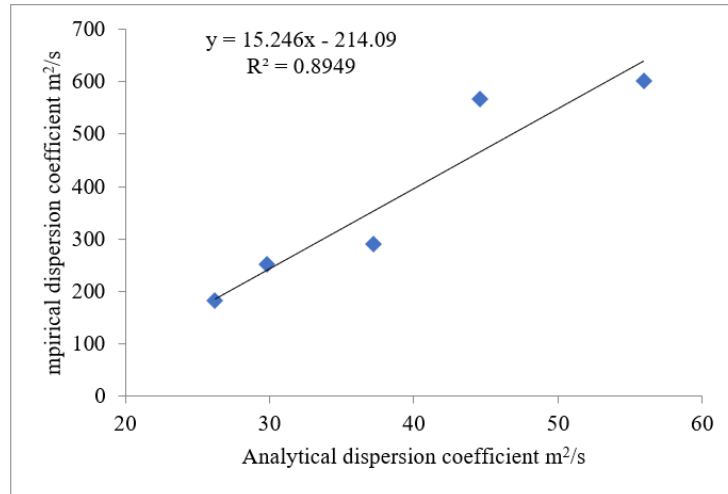
Figure 3 presents the concentrations as a function of time at the five stations (measured from upstream end). The concentrations have been computed using the analytical solution of the convection-dispersion Eq. (2). The mass of the solute (1 kg) was initially deposited at the upstream end, and the Python code was run through (30 hours = 108000 seconds). The figure depicts the duration required to achieve the solute effect at each station, and shows that the solute concentration first affects the nearest position to upstream (3000 m) after 2300 seconds, reaches its maximum concentration after 5400 seconds, and then drops to zero after 16000 seconds. The maximum concentrations are calculated at 5400, 14000, 22900, 34500, and 53600 sec for 3000, 7200, 11300, 17240, and 26676 m, respectively.



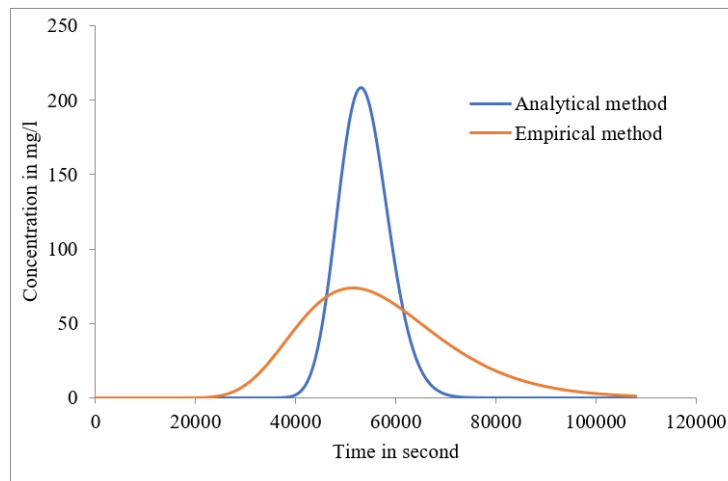
**Figure 3.** Concentration as function of time at several stations (Distances measured from upstream end)

The dispersion coefficient calculated by the HEC-RAS model is compared with the analytical method, and the results are plotted in Figure 4. The figure shows an acceptable agreement in the determination index, but the scales of axes reveal an overestimation in the empirical method's values.

In Figure 5, the concentrations are calculated based on Eq. (2), at station 26676 m from the upstream end. The values of  $D$  substituted in the equation are calculated by analytical and empirical methods. Figure 5 clearly shows the notable effect of large values of the empirical dispersion coefficient on the solute concentration. Where the maximum concentration is decreased and the period of separated concentration is increased. A notable discrepancy exists between analytical and empirical results, often arising from shear stress and the challenges in accurately depicting velocity across stream cross-sections in empirical methods, leading to a decline in their use in recent years. The integral approach is more precise and logical, although it needs comprehensive velocity data and is computationally demanding. The empirical technique is user-friendly and suitable for rapid estimations; however, it may lack accuracy beyond the specific conditions for which the equations were formulated. The selection among them is contingent upon the available data, the necessary precision, and the intricacy of the flow circumstances.



**Figure 4.** Comparison between empirical and analytical dispersion coefficients



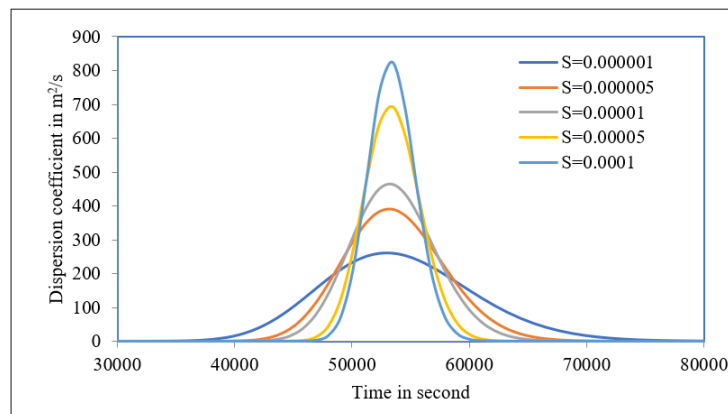
**Figure 5.** The mathematical calculated concentration values at station 26670 m based on analytical and empirical dispersion estimation

**Table 2.** Results of dispersion coefficient as a function of slope

$S$	$u^*$	$E_y$	$D$
0.000001	0.0065	0.0268	94.2
0.000005	0.0146	0.0601	42.2
0.00001	0.0206	0.0848	29.8
0.00005	0.0461	0.1897	13.3
0.0001	0.0652	0.2684	9.4
0.0005	0.1457	0.5997	4.2
0.001	0.2016	0.8298	3
0.005	0.4609	1.8971	1.3

The effect of turbulence on the dispersion coefficient can be studied through variation of the  $E_y$  value, which depends on the  $u^*$  parameter, which in turn depends on the value of the slope as it appears in Eqs. (7) and (8). So, different values for the slope were taken, and the corresponding calculations were made, and the results are listed in Table 2. The table reveals that an increase in slope values leads to a decrease in the dispersion coefficient, as the diffusion process in the water body encounters an increase in shear stress (shear velocity). Therefore, at the lowest slope, the solute concentration and its separation time exhibit a broader range and the smallest value. As the slope value increases, the separation time period decreases and the concentration rises (Figure 6). For example, at slope = 0.0001, the separation time period and maximum concentration are 73090 sec and 261 m<sup>2</sup>/s, respectively.

Conversely, at slope = 0.00001, the separation time period and maximum concentration are 40363 sec and 464 m<sup>2</sup>/s, respectively.



**Figure 6.** The effect of turbulence on the dispersion coefficient

#### 4 Conclusions

Precisely estimating the longitudinal dispersion coefficient is essential for comprehending and forecasting the movement of solutes in natural water bodies. Throughout time, several empirical and theoretical methods have been created to calculate this coefficient, which represents the intricate interaction between parameters such as channel design, flow velocity, and turbulence. Out of these options, the Fischer technique is still extensively used since it takes into account the velocity distribution, channel features, and turbulence effects in a complete manner. Fischer’s technique offers a dependable foundation for simulating the dispersion of solutes in rivers and streams. Although Fischer’s approach is resilient, it is important to use it with awareness of its constraints and in combination with on-site measurements wherever feasible. On the other hand, the analytical (integral) technique provides a more basic approach to determining the longitudinal dispersion coefficient. This is done by directly integrating the velocity profile and turbulent diffusion effects over the cross-section of the channel. This approach takes into consideration the spatial variations in velocity and turbulence, resulting in a more intricate depiction of the dispersion phenomenon. Although the analytical technique requires more processing power, it provides more precision in situations when the flow conditions are intricate or where the empirical assumptions of alternative methods may not be valid. The integral technique enables the calculation of the dispersion coefficient based on fundamental principles, making it a potent tool for theoretical investigations and for verifying empirical approaches in controlled environments. The integral approach is more precise and logical, although it needs comprehensive velocity data and is computationally demanding. The empirical technique is user-friendly and suitable for rapid estimations; however, it may lack accuracy beyond the specific conditions for which the equations were formulated. The selection among them is contingent upon the available data, the necessary precision, and the intricacy of the flow circumstances.

#### Data Availability

Not applicable.

#### Conflicts of Interest

The author declare no conflict of interest.

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