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Optimization of Market Risk via Maclaurin Symmetric Mean Aggregation Operators: An Application of Interval-Valued Intuitionistic Fuzzy Sets in Multi-Attribute Group Decision-Making

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Abstract: New aggregation operators (AOs) for interval-valued intuitionistic fuzzy sets (IVIFS) have been developed, offering advancements in multi-attribute group decision-making (MAGDM). IVIFS employs intervals for membership and non-membership grades, providing a robust framework to handle uncertainties inherent in real-world scenarios. This study introduces operational laws for interval-valued intuitionistic fuzzy values (IVIFVs), formulated on the Frank T-norm and T-conorm, and presents a generalization of the Maclaurin symmetric mean (MSM) operator tailored for these values. Named the interval-valued intuitionistic fuzzy Frank weighted MSM (IVIFFWMSM) and interval-valued intuitionistic fuzzy Frank MSM (IVIFFMSM), these operators incorporate new operational principles that enhance the aggregation process. The effectiveness of these operators is demonstrated through their application to a MAGDM problem, where they are compared with existing operators. This approach not only enriches the theoretical landscape of fuzzy decision-making models but also provides practical insights into the optimization of market risk.

Keywords: Interval-valued intuitionistic fuzzy set (IVIFS); Multi-attribute decision-making (MADM); Frank T-norm; Frank T-conorm; Aggregation operators (AOs); Maclaurin symmetric mean (MSM)

1 Introduction

The process of aggregating information is highly complex because it involves several different factors. MSM operators are capable of handling various informational facets. The framework known as IVIFS handles information in the form of intervals with the assistance of MG and NMG. A number of authors have attempted to compile the data by presenting methods that generalize the MSM of IFS and IVIFS. However, a significant issue with the results obtained is their lack of flexibility. This article's primary contribution is the creation of a flexible method for compiling data into IVIFVs. Because a parameter is involved, the flexible operational laws for numbers expressed as unit intervals are FTRM and FTCRM. The following are some of the developed operators' motivations.

The result reached through decision-making may not be conveyed with suitable numerical demonstration due to the complexity and uncertainty of the data, especially because of human assessment. In order to accommodate the fuzziness and complexity of the data, Zadeh [1] introduced the concept of a fuzzy set (FS), in which the membership grade (MG) whose value falls between [0, 1] was introduced in place of a classical set. Following Atanassov [2], introduce the non-membership grade (NG) to expand the concept of Zadeh's FS. It's suggested that intuitionistic FS (IFS) might be possible with the NG [2]. Compared to the FS, the IFS system addressed the complexity and uncertainty of the data with greater accuracy. Because of the complexity and ambiguity of the issues, researchers used the IFS to adapt. From the grave list of options in the form of alternatives, the MAGDM helps select the best choice. MAGDM is considered more reliable than MADM due to its comprehensive view of expert perspectives on the outcome. On the other hand, the MADM relies on the opinion of a single expert to determine the output. Owing to the importance of MAGDM, a number of authors have created methods for dealing with it. MAGDM is thought to be a more successful and efficient process for making decisions. The MAGDM has been used in the fields of business [3–5], clinical science [6] and so forth.

Garg and Arora [7] develop the priorities for IFS. Deschrijver and Kerre [8] aggregated the IFS. Mahanta and Panda [9] aggregated the distance measure in IFS. Khan et al. [10] used Dombi AO in IFS. Zhang [11] introduced the frank AOs based on the MAGDM for IFS. Wu et al. [12] introduced the interval valued Dombi Hamy Mean AO in IFS. The study [13] introduced the AOs based on the Frank operator for IFS. Garg [14] introduced the Hamacher AOs with entropy weights for IFS. Wang [15] introduced the AOs based on MAGDM. Liu et al. [16] introduced the concept of power MSM AOs in IVIFS. Hussain et al. [17] developed the AOs based on the MSM for IFS. Garg and Arora [18] gave the concept of MSM AOs for soft set. Contingent upon the AOs, numerous methods can be used to create the aggregate information. Deschrijver and Kerre [8] developed the AOs for IFS. Garg and Kumar [19] introduced similarity measures for IFS. Sarfraz et al. [20] gave the concept of Aczel-Alsina AOs for IFS. Hussain and Pamucar [21] gave the concept of Schweizer-Sklar. Hussain et al. developed the idea of Aczel-Alsina AOs using the application of medical diagnosis. Sarfraz [22] developed an AO for water recycling using the Schweizer-Sklar algorithm. Ullah et al. [23] gave the concept of prioritized Aczel-Alsina AOs for IFS. Mahmood and ur Rehman [24] developed the theory of bipolar complex fuzzy. Ali et al. [25] gave the theory of Aczel-Alsina AOs on IFS. Tešić and Marinković [26] presented the application on fuzzy weight operators. Sarfraz [27] given the concept of Dombi Hamy Mean aggregation operator. Sarfraz [28] developed the theory of Maclaurin Symmetric Mean Aggregation Operators.

Nasini et al. [29] proposed applying market portfolio optimization at risk. Liu and Yu [30] analyzed market risk based on machine learning. Sarfraz [31] introduced the theory of Aczel-Alsina Aggregation Operators on Spherical Fuzzy Rough Set. Zhang [32] controlled the market based on an optimization algorithm. Sarfraz [33] developed the theory of Maclaurin Symmetric Mean Aggregation Operators. Leonel et al. [34] discussed the risk control of hydro-market intelligence and stochastic optimization. Kostadinova et al. [35] developed the theory of market chains for stock price and risk portfolio optimization. Sarfraz and Azeem [36] developed the theory of similarity measure. Chen et al. [37] proposed the concept of market risk optimization with the application of real industry in China. Sarfraz and Pamucar [38] given the concept of similarity measure. Faia et al. [39] developed the theory of electricity market using forecasting errors in risk formulation. Song et al. [40] discussed the risk of block chain financial market based on swarm optimization. Pradhan et al. [41] optimized the market risk of major crypto currencies.

The above-mentioned AOs combine several numbers into a single value. Recent authors defined the MSM and developed various AOs whose effectiveness depends on the relationships between the attributes they aggregate. Some AOs to the information between the attributes were created by Mesiar [42]. Power MSM AOs that can aggregate data in interval form are presented by Maclaurin [43] for IVIFS. These AOs are hindered by their reliance on traditional functional regulations. Several researchers have used alternative functional regulations to achieve results that are more favorable and flexible than the AOs that were previously discussed.

Among authors, IVIFS has become a well-known framework for handling data uncertainty. A number of authors have introduced a number of AOs for the IVIFS framework. However, either the AOs are unable to handle the variety of information, or the outcomes they produce are not adaptable.

By using a parameter as the base of the logarithm, Maclaurin Symmetric Mean [44] introduced the operational laws to obtain flexibility in the information fusion. The suggested operators are therefore generalized using the FTRM and FTCRM as a basis.

The MSM operator can handle the data by making use of all of its features. IVIFVS's traditional operations are used in the development of the current AOs, which are based on the MSM operator. Therefore, utilizing the FTRM, FTCRM, and MSM operators to generalize the AOs for the IVIFS can be very beneficial in resolving the flexibility issues.

There are five more sections in this paper. We present our article's thorough literature review in Section 2. The article's core ideas are covered in Section 3. The proposed AOs are based on some new operational laws for IVIFVs that are developed in Section 3. The establishment of IVIFFMSM operators and an examination of their properties are covered. The IVIFFWMSM operator is constructed, and its fundamental characteristics are stated. The application of the suggested AOs with the help of an example is covered in Section 4. A comparison between the suggested operators and a few current operators is also included in Section 4. The comprehensive discussion is conclusively addressed in Section 5.

2 Preliminaries

The concepts of IVIFS, score function, FTRM, FTCRM, and MSM are defined in this section for a better understanding of this article.

Definition 1: [2] Let $\ddot{\Gamma}$ is an all-inclusive set. Then an IVIFS $\dot{A}$ can be characterized as:

$$\dot{\mathrm{A}} = \left\{ \langle \mathrm{P}, \left[{\gamma_{\dot{\mathrm{B}}}}^{\mathrm{L}}, \gamma_{\dot{\mathrm{B}}}^{\mathrm{U}} \right], \left[{\theta_{\dot{\mathrm{B}}}}^{\mathrm{L}}, {\theta_{\dot{\mathrm{B}}}}^{\mathrm{U}} \right] \rangle \mid \mathrm{P} \in \ddot{\Gamma} \right\}$$

where, ${\gamma_{\dot{B}}}^{\mathrm{L}}, {\gamma_{\dot{B}}}^{\mathrm{U}} : \ddot{\Gamma} \to [0,1], {\theta_{\dot{\mathrm{B}}}}^{\mathrm{L}}, {\theta_{\dot{\mathrm{B}}}}^{\mathrm{U}} : \ddot{\Gamma} \to [0,1]$ are MG and NG, individually, with ${\gamma_{\dot{\mathrm{B}}}}^{\mathrm{U}} + {\theta_{\dot{\mathrm{B}}}}^{\mathrm{U}} \in [0,1]$.

Definition 2: [2] Assume $\mathrm{E}=\left(\left[\gamma_{\dot{\mathrm{B}}}{}^{\mathrm{L}},\gamma_{\dot{\mathrm{B}}}{}^{\mathrm{U}}\right],\left[\theta_{\dot{\mathrm{B}}}{}^{\mathrm{L}},\theta_{\dot{\mathrm{B}}}{}^{\mathrm{U}}\right]\right)$ be an IVIFV. The score value of E is characterized as follows:

$$\mathrm{Sc(E)}=\frac{\gamma_{\dot{\mathrm{B}}}^{\mathrm{L}}-\theta_{\dot{B}}^{\mathrm{L}}+\gamma_{\dot{\mathrm{B}}}^{\mathrm{U}}-\theta_{\dot{B}}^{\mathrm{U}}}{2} \tag{1}$$

where, $\mathrm{Sc}(E)\in[-1,1]$. The smaller the $Sc(E)$, the more modest the IVIFVs E are.

It was found that some IVIFVs couldn't be situated with the help of the score value described above. Consequently, the accuracy value is defined for the ranking of the IVIFVs as follows.

Definition 3: Let $\mathrm{E}=\left(\left[\gamma_{\dot{\mathrm{B}}}{}^{\mathrm{L}},\gamma_{\dot{\mathrm{B}}}^{\mathrm{U}}\right],\left[\theta_{\dot{\mathrm{B}}}^{\mathrm{L}},\theta_{\dot{\mathrm{B}}}^{\mathrm{U}}\right]\right)$ be an IVIFV. The grade of accuracy (GA) of E is characterized as the accompanying $\mathrm{GA(E)}\in[0,1]$.

$$\mathrm{GA(E)}=\frac{\gamma_{\dot{\mathrm{B}}}{}^{\mathrm{L}}+\theta_{\dot{\mathrm{B}}}{}^{\mathrm{L}}+\gamma_{\dot{\mathrm{B}}}{}^{\mathrm{U}}+\theta_{\dot{\mathrm{B}}}{}^{\mathrm{U}}}{2} \tag{2}$$

Definition 4: [42] There are mappings from the Frank TRM and TCRM $[0,1]^2$ to $[0,1]$ such that

$$T_F(r,s)=\log_\varrho\left(1+\frac{(\varrho^r-1)(\varrho^s-1)}{\varrho-1}\right),r,s\in[0,1],\varrho>1$$

$$S_F(r,s)=1-\log_\varrho\left(1+\frac{(\varrho^{1-r}-1)(\varrho^{1-s}-1)}{\varrho-1}\right),r,s\in[0,1],\varrho>1$$

Definition 5: Let $\mathrm{E}_1=\left(\left[\gamma_{\mathrm{B}_1}^{\mathrm{L}},\gamma_{\mathrm{B}_1}^{\mathrm{U}}\right],\left[\theta_{\mathrm{B}_1}^{\mathrm{L}},\theta_{\mathrm{B}_1}^{\mathrm{U}}\right]\right)$ $\mathrm{E}_2=\left(\left[\gamma_{\mathrm{B}_2}^{\mathrm{L}},\gamma_{\mathrm{B}_2}^{\mathrm{U}}\right],\left[\theta_{\mathrm{B}_2}^{\mathrm{L}},\theta_{\mathrm{B}_2}^{\mathrm{U}}\right]\right)$ and $\mathrm{E}=\left(\left[\gamma_{\mathrm{B}}^{\mathrm{L}},\gamma_{\mathrm{B}}^{\mathrm{U}}\right],\left[\theta_{\mathrm{B}^{\mathrm{L}}}^{\mathrm{L}},\theta_{\mathrm{B}}^{\mathrm{U}}\right]\right)$ be three IVIFVs and $n>0$, the tasks of IVIFVs in light of Frank TCRM and TRM are characterized,

$$\begin{aligned}E_1\oplus_{\mathrm{F}}E_2=\Bigg(&\Bigg[1-\log_\varrho\left(1+\frac{(\varrho^{1-\gamma_{\mathrm{B}_1}^{\mathrm{L}}}-1)(\varrho^{1-\gamma_{\mathrm{B}_2}^{\mathrm{L}}}-1)}{\varrho-1}\right),\\&1-\log_\varrho\left(1+\frac{(\varrho^{1-\gamma_{\mathrm{B}_1}^{\mathrm{U}}}-1)(\varrho^{1-\gamma_{\mathrm{B}_2}^{\mathrm{U}}}-1)}{\varrho-1}\right)\Bigg]\\&\Bigg[\log_\varrho\left(1+\frac{(\varrho^{\theta_{\mathrm{B}_1}^{\mathrm{L}}}-1)(\varrho^{\theta_{\mathrm{B}_2}^{\mathrm{L}}}-1)}{\varrho-1}\right),\\&\log_\varrho\left(1+\frac{(\varrho^{\theta_{\mathrm{B}_1}^{\mathrm{U}}}-1)(\varrho^{\theta_{\mathrm{B}_2}^{\mathrm{U}}}-1)}{\varrho-1}\right)\Bigg]\Bigg)\end{aligned}$$

$$\begin{aligned}E_1\otimes_{\mathrm{F}}\mathrm{E}_2=\Bigg(&\Bigg[\log_\varrho\left(1+\frac{(e^{\theta_{\mathrm{B}_1}^{\mathrm{L}}}-1)(e^{\theta_{\mathrm{B}_2}^{\mathrm{L}}}-1)}{\varrho-1}\right),\\&\log_\varrho\left(1+\frac{(e^{\theta_{\mathrm{B}_1}^{\mathrm{U}}}-1)(e^{\theta_{\mathrm{B}_1}^{\mathrm{U}}}-1)}{\varrho-1}\right)\Bigg],\\&\Bigg[1-\log_\varrho\left(1+\frac{(\varrho^{1-\gamma_{\mathrm{B}_1}^{\mathrm{L}}}-1)(\varrho^{1-\gamma_{\mathrm{B}_1}^{\mathrm{L}}}-1)}{\varrho-1}\right),\\&1-\log_\varrho\left(1+\frac{(\varrho^{1-\gamma_{\mathrm{B}_1}^{\mathrm{U}}}-1)(\varrho^{1-\gamma_{\mathrm{B}_1}^{\mathrm{U}}}-1)}{\varrho-1}\right)\Bigg]\Bigg)\end{aligned} \tag{3}$$

$$n\cdot E=\left(\begin{matrix}\left[1-\log_\varrho\left(1+\frac{\left(\varrho^{1-\gamma^{\mathrm{L}}}-1\right)^{\mathrm{n}}}{(\varrho-1)^{\mathrm{n}-1}}\right),1-\log_\varrho\left(1+\frac{\left(\varrho^{1-\gamma^{\mathrm{U}}}-1\right)^{\mathrm{n}}}{(\varrho-1)^{\mathrm{n}-1}}\right)\right]\\\left[\log_\varrho\left(1+\frac{\left(\varrho^{\theta^{\mathrm{L}}}-1\right)^{\mathfrak{n}}}{(\varrho-1)^{\mathrm{n}-1}}\right),\log_\varrho\left(1+\frac{\left(\varrho^{\theta^{\mathrm{U}}}-1\right)^{\mathrm{n}}}{(\varrho-1)^{\mathrm{n}-1}}\right)\right]\end{matrix}\right)$$

$$E^{\mathrm{n}}=\left(\begin{matrix}\left[\log_\varrho\left(1+\frac{\left(\theta^{e^{\mathrm{L}}}-1\right)^{\mathrm{n}}}{(\varrho-1)^{\mathrm{n}-1}}\right),\log_\varrho\left(1+\frac{\left(\varrho^{\theta^{\mathrm{U}}}-1\right)^{\mathrm{n}}}{(\varrho-1)^{\mathrm{n}-1}}\right),\right]\\\left[1-\log_\varrho\left(1+\frac{\left(\varrho^{1-\gamma^{\mathrm{L}}}-1\right)^{\mathfrak{n}}}{(\varrho-1)^{\mathfrak{n}-1}}\right),1-\log_\varrho\left(1+\frac{\left(\varrho^{1-\gamma^{\mathrm{U}}}-1\right)^{\mathfrak{n}}}{(\varrho-1)^{\mathrm{n}-1}}\right)\right]\end{matrix}\right)$$

Definition 6: [43] Let h_m be positive numbers. After that, the MSM operator has the following definition.

$$\mathrm{MSM}\left(\mathrm{h}_1,\ \mathrm{h}_2,\ \mathrm{h}_3, \cdots \mathrm{h}_{\mathrm{n}}\right)=\left(\frac{\sum_{1 \leq \mathrm{m}_1<\mathrm{m}_2<\mathrm{m}_3<\cdots<\mathrm{m}_\omega \leq \mathrm{n}} \prod_{l=1}^{\omega} \mathrm{h}_{\mathrm{m}_1}}{\mathrm{C}_\omega^{\mathrm{n}}}\right)^{1 / \omega} \tag{4}$$

where, $\mathrm{C}_{\mathrm{w}}^{\mathrm{n}}$ is the binomial coefficient, $1 \leq \mathrm{m}_1<\mathrm{m}_2<\mathrm{m}_3<\cdots<\mathrm{m}_{\mathrm{w}} \leq \mathrm{n}$ shows the w-tuple of the positive numbers.

3 The Interval-Valued Intuitionistic Fuzzy Frank MSM Operator

This section comprises of the formulization of the IVIFFMSM operator based on the Frank TRM and Frank TCRM for the aggregation of IVIFVs. This segment can additionally be concentrated on a few positive properties of the IVIFFMSM operator.

Definition 7: Let $\mathrm{E}_i=\left(\left[\gamma_i^L, \gamma_i^U\right],\left[\theta_i^L, \theta_i^U\right]\right)$ be the assortment of the IVIFVs, Then IVIFFFMSM $\ddot{\Gamma} n \rightarrow \ddot{\Gamma}$, the operator is characterized for $(i=1,2,3 \cdots, n)$ and $\omega=1,2,3, \cdots, n$ as follows:

$$\mathrm{IVIFFMSM}\left(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \cdots \mathrm{E}_{\mathrm{n}}\right)=\left(\frac{\underset{1 \leq i_1<i_2<i_3<\cdots<i_\omega \leq n}{\oplus_F} \ \underset{\mathrm{j}=1}{\overset{\omega}{\otimes_F}} \mathrm{E}_{\mathrm{i}_{\mathrm{j}}}}{\mathrm{C}_\omega^{\mathrm{n}}}\right)^{1 / \omega} \tag{5}$$

where, $\mathrm{C}_\omega^{\mathrm{n}}$ is the coefficient binomial $1 \leq \mathrm{i}_1<\mathrm{i}_2<\mathrm{i}_3<\cdots<\mathrm{i}_\omega \leq \mathrm{n}$ represents the $\mathrm{w}-$ tuple combination of the IVIFVs.

Because of their flexibility, the Frank TRM and Frank TCRM are the better operational laws. Compared to the current operators, the IVIFMSM operator has greater flexibility in aggregating the information into IVIFVs. Some properties are discussed in the following.

Theorem 1: The $\mathrm{E}_i=\left(\left[\gamma_i^L, \gamma_i^U\right],\left[\theta_i^L, \theta_i^U\right]\right)$, is the assortment of IVIFVs. Then $\mathrm{IVIFFMSM}\left(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \cdots \mathrm{E}_{\mathrm{n}}\right)$ is an IVIFV as follows:

$$\mathrm{IVIFFMSM}\left(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \cdots \mathrm{E}_{\mathrm{n}}\right)=\left(\begin{array}{c}\left[\begin{array}{l}\log_\varrho\left(1+\left(\varrho^A-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right), \\ \log_\varrho\left(1+\left(\varrho^B-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right),\end{array}\right] \\ \left[\begin{array}{l}1-\log_\varrho\left(1+\left(\varrho^C-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right), \\ 1-\log_\varrho\left(1+\left(\varrho^D-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right)\end{array}\right]\end{array}\right)$$

where,

$$A=1-\log_\varrho\left(1+\left(\varrho^{1-\log_\varrho\left(1+\prod_{1 \leq i_1<i_2<i_3<\cdots<i_w \leq n}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^n\left(\varrho^{\gamma_i^L}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_\omega^n}}(\varrho-1)^{1-\frac{1}{C_\omega^n}}\right)$$

$$B=1-\log_\varrho\left(1+\left(\varrho^{1-\log_\varrho\left(1+\prod_{1 \leq i_1<i_2<i_3<\cdots<i_w \leq n}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^n\left(\varrho^{\gamma_i^U}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_\omega^n}}(\varrho-1)^{1-\frac{1}{C_\omega^n}}\right)$$

$$C=\log_{\varrho}\left(1+\left(\varrho^{\log_{\varrho}\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_{\varrho}\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\theta_i^L}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_{\omega}^{n}}}(\varrho-1)^{1-\frac{1}{C_{\omega}^{n}}}\right)$$

And

$$D=\log_{\varrho}\left(1+\left(\varrho^{\log_{\varrho}\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_{\varrho}\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\theta_i^U}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_{\omega}^{n}}}(\varrho-1)^{1-\frac{1}{C_{\omega}^{n}}}\right)$$

Proof: we can find $\overset{\omega}{\underset{j=1}{\otimes_F}}\mathrm{E}_{\mathrm{i_j}}$ first, as follows:

$$\overset{\omega}{\underset{j=1}{\otimes_F}}\mathrm{E}_{\mathrm{i_j}}=\left(\begin{array}{c}\left[\log_{\varrho}\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{\theta_i^L}-1\right)\right)(\varrho-1)^{1-n}\right),\log_{\varrho}\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{\theta_i^U}-1\right)\right)(\varrho-1)^{1-n}\right)\right]\\ \left[1-\log_{\varrho}\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{1-\gamma_i^L}-1\right)\right)(\varrho-1)^{1-n}\right),1-\log_{\varrho}\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{1-\gamma_i^U}-1\right)\right)(\varrho-1)^{1-n}\right)\right]\end{array}\right)$$

This is an IVIFV. Now, we calculate $\underset{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}{\bigoplus_F}\overset{\omega}{\underset{j=1}{\otimes_F}}\mathrm{E}_{\mathrm{i_j}}$ as follows:

$$\underset{1\leq i_1<i_2<i_3<\cdots<i_\omega\leq n}{\oplus_F}\overset{\omega}{\underset{j=1}{\otimes_F}}\mathrm{E}_{\mathrm{i_j}}$$

$$=\left(\begin{array}{c}\left[\begin{array}{c}1-\log_{\varrho}\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_{\varrho}\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\gamma_i^{\mathrm{L}}}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right),\\ 1-\log_{\varrho}\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_{\varrho}\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\gamma_i^{\mathrm{U}}}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)\end{array}\right],\\ \left[\begin{array}{c}\log_{\varrho}\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_{\varrho}\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\gamma_i^{\mathrm{L}}}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right),\\ \log_{\varrho}\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_{\varrho}\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\gamma_i^{\mathrm{L}}}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)\end{array}\right]\end{array}\right)$$

$\underset{1\leq i_1<i_2<i_3<\cdots<i_\omega\leq n}{\oplus_F}\overset{\omega}{\underset{j=1}{\otimes_F}}\mathrm{E}_{\mathrm{i_j}}$ is also clearly an IVIFV. Now, we calculate $\left(\frac{\underset{1\leq i_1<i_2<i_3<\cdots<i_\omega\leq n}{\oplus_F}\overset{\omega}{\underset{j=1}{\otimes_F}}\mathrm{E}_{\mathrm{i_j}}}{C_w^n}\right)^{\frac{1}{w}}$ as follows:

$$\left(\frac{\underset{1\leq i_1<i_2<i_3<\cdots<i_\omega\leq n}{\bigoplus_F}\ \overset{\omega}{\underset{\mathrm{j=1}}{\bigotimes_F}}\mathrm{E_{i_j}}}{C_w^n}\right)^{\frac{1}{w}}=\left(\begin{array}{c}\left[\begin{array}{c}\log_\varrho\left(1+\left(\varrho^A-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right),\\ \log_\varrho\left(1+\left(\varrho^B-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right)\end{array}\right],\\ \left[\begin{array}{c}1-\log_\varrho\left(1+\left(\varrho^C-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right),\\ 1-\log_\varrho\left(1+\left(\varrho^D-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right)\end{array}\right]\end{array}\right)$$

$$A=1-\log_\varrho\left(1+\left(\varrho^{1-\log_\varrho\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^n\left(\varrho^{\gamma_i^L}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_\omega^n}}(\varrho-1)^{1-\frac{1}{C_\omega^n}}\right)$$

$$B=1-\log_\varrho\left(1+\left(\varrho^{1-\log_\varrho\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^n\left(\varrho^{\gamma_i^U}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_\omega^n}}(\varrho-1)^{1-\frac{1}{C_\omega^n}}\right)$$

$$C=\log_\varrho\left(1+\left(\varrho^{\log_\varrho\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^n\left(\varrho^{\theta_i^L}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_\omega^n}}(\varrho-1)^{1-\frac{1}{C_\omega^n}}\right)$$

And

$$D=\log_\varrho\left(1+\left(\varrho^{\log_\varrho\left(1+\prod_{1\leq i_1<i_2<i_3<\cdots<i_w\leq n}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^n\left(\varrho^{\theta_i^U}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)}-1\right)^{\frac{1}{C_\omega^n}}(\varrho-1)^{1-\frac{1}{C_\omega^n}}\right)$$

The IVIFV. Thus, the proof is finished.

Theorem (Idempotency) 2: Let $\mathrm{E}_i=\left(\left[\gamma_i^L,\gamma_i^U\right],\left[\theta_i^L,\theta_i^U\right]\right)$ $(i=1,2,3,\cdots,n)$ be the assembly of IVIFVs, and if E_i=$\left(\left[\gamma_i^L,\gamma_i^U\right],\left[\theta_i^L,\theta_i^U\right]\right)=\mathrm{E}=\left(\left[\gamma^L,\gamma^U\right],\left[\theta^L,\theta^U\right]\right)\forall(i=1,2,3,\cdots,n)$, then IVIFFMSM $(\mathrm{E_1},\mathrm{E_2},\mathrm{E_3},\cdots\mathrm{E_n})$ $=\mathrm{E}$ is called the idempotency.

Proof: As $\mathrm{E}_i = \left(\left[\gamma_{\mathrm{i}}^{\mathrm{L}}, \gamma_{\mathrm{i}}^{\mathrm{U}}\right], \left[\theta_{\mathrm{i}}^{\mathrm{L}}, \theta_{\mathrm{i}}^{\mathrm{U}}\right]\right) = \left(\left[\gamma^{\mathrm{L}}, \gamma^{\mathrm{U}}\right], \left[\theta^{\mathrm{L}}, \theta^{\mathrm{U}}\right]\right) = \mathrm{E} \forall (i = 1, 2, 3, \cdots, n)$, by Theorem 1 , we have

$$= \left(\begin{array}{c} \left[\log_\varrho\left(1+\left(\varrho^{\theta^L}-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right), \log_\varrho\left(1+\left(\varrho^{\theta^U}-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right)\right] \\ \left[1-\log_\varrho\left(1+\left(\varrho^{1-\gamma^L}-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right), 1-\log_\varrho\left(1+\left(\varrho^{1-\gamma^U}-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right)\right] \end{array} \right)$$

$$= \left(\left[\gamma^L, \gamma^U\right], \left[\theta^L, \theta^U\right]\right) = \mathrm{E}$$

Thus, the proof is finished.

Theorem (Monotonicity) 3: Let $\mathrm{E}_i = \left(\left[\gamma_i^L, \gamma_i^U\right], \left[\theta_i^L, \theta_i^U\right]\right)$ and $\mathrm{E}_i^a = \left(\left[\left(\gamma_i^L\right)^a, \left(\gamma_i^U\right)^a\right], \left[\left(\theta_i^L\right)^a, \left(\theta_i^U\right)^a\right]\right)$ $(i = 1, 2, 3, \cdots, n)$, find the IFVs and if $\mathrm{E}_i \geq \mathrm{E}_i^a$ for all $(i = 1, 2, 3, \cdots, n)$. Then $\mathrm{IVIFFMSM}\left(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \cdots \mathrm{E}_\mathrm{n}\right) \geq \mathrm{IVIFFMSM}\left(\mathrm{E}_1^\mathrm{a}, \mathrm{E}_2^\mathrm{a}, \mathrm{E}_3^\mathrm{a}, \cdots \mathrm{E}_\mathrm{n}^\mathrm{a}\right)$

Proof: Given that $\mathrm{E}_i \geq \mathrm{E}_i^a$ so, for IFVs $\varkappa_i \geq \varkappa_i^a$ and $\theta_i \leq \theta_i^a$, we can write:

$$\left[\log_\varrho\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{\theta_i^L}-1\right)\right)(\varrho-1)^{1-n}\right)\right] \geq \left[\log_\varrho\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{\left(\theta_i^L\right)^a}-1\right)\right)(\varrho-1)^{1-n}\right)\right]$$

And

$$\left[1-\log_\varrho\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{1-\gamma_i^L}-1\right)\right)(\varrho-1)^{1-n}\right)\right] \leq \left[1-\log_\varrho\left(1+\left(\prod_{i=1}^{n}\left(\varrho^{1-\left(\gamma_i^L\right)^a}-1\right)\right)(\varrho-1)^{1-n}\right)\right]$$

Further, we have

$$\left[1-\log_\varrho\left(1+\prod_{1\leq \mathrm{i}_1<\mathrm{i}_2<\mathrm{i}_3<\cdots<\mathrm{i}_\mathrm{w}\leq \mathrm{n}}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\gamma_i^L}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)\right]$$

$$\geq 1-\log_\varrho\left(1+\prod_{1\leq \mathrm{i}_1<\mathrm{i}_2<\mathrm{i}_3<\cdots<\mathrm{i}_\mathrm{w}\leq \mathrm{n}}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^{n}\left(e^{\left(v_i^L\right)^a}-1\right)}{(e-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)\Bigg]$$

And

$$\left[\log_\varrho\left(1+\prod_{1\leq \mathrm{i}_1<\mathrm{i}_2<\mathrm{i}_3<\cdots<\mathrm{i}_\mathrm{w}\leq \mathrm{n}}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{1-\theta_i^L-1}\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)\right]$$

$$\leq \left[\log_\varrho\left(1+\prod_{1\leq \mathrm{i}_1<\mathrm{i}_2<\mathrm{i}_3<\cdots<\mathrm{i}_\mathrm{w}\leq \mathrm{n}}\left(\varrho^{1-\left(\log_\varrho\left(1+\frac{\prod_{i=1}^{n}\left(\varrho^{\left(1-\theta_i^L\right)^a}-1\right)}{(\varrho-1)^{\omega-1}}\right)\right)}-1\right)(\varrho-1)^{1-\omega}\right)\right]$$

Then we have

$$1-\log_\varrho\left(1+\left(\varrho^{C^{LA}}-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right) \geq 1-\log_\varrho\left(1+\left(\varrho^{C^{LB}}-1\right)^{\frac{1}{\omega}}(\varrho-1)^{1-\frac{1}{\omega}}\right)$$

Similarly, we can do for upper value of MG.

Hence, the proof is completed.

Theorem (Boundedness) 4: Let $\mathrm{E}_i = \left(\left[\gamma_i^L, \gamma_i^U\right], \left[\theta_i^L, \theta_i^U\right]\right)$ be the assortments of the IVIFVs, and then $\mathrm{E}^- = \left(\min\left[\gamma_i^L, \gamma_i^U\right], \max\left[\theta_i^L, \theta_i^U\right]\right), \mathrm{E}^+ = (\max \gamma_i, \min \varrho_i)$. Then $\mathrm{E}_i^- \leq \mathrm{IVIFFMSM}\left(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \cdots \mathrm{E}_\mathrm{n}\right) \leq \mathrm{E}_i^+$.

Proof: Since $\varkappa^- = \min\left[\gamma_i^L, \gamma_i^U\right]$ and $\varrho^+ = \max\left[\theta_i^L, \theta_i^U\right]$. Therefore, we can write $\gamma^- \leq \gamma_i \leq \gamma^+, \theta^- \leq \theta_i \leq \theta^+$ for all $i = 1, 2, 3, \cdots, n$. Therefore, by Theorems 3 and 4 , we can write:

$$\mathrm{E}_i^- \leq \mathrm{IVIFFMSM}\left(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3, \cdots \mathrm{E}_\mathrm{n}\right) \leq \mathrm{E}_i^+$$

Thus, the proof is finished.

Example 1: Consider $\mathrm{E}_1 = (0.2, 0.3), \mathrm{E}_2 = (0.4, 0.5)$, and $\mathrm{E}_3 = (0.4, 0.5)$ are three IVIFVs. Here, $\gamma^- = 0.2$ and $\varrho^+ = 0.5$. Hence, $\mathrm{E}^- = (0.2, 0.5)$ and same $\mathrm{E}^+ = (0.4, 0.3)$. Now, we can aggregate the values.

$$\mathrm{IVIFFMSM}\left(\mathrm{E}_1, \mathrm{E}_2, \mathrm{E}_3\right) = (0.4, 0.4)$$

Consequently, the Example 3.7 is the monotonic of IVIFFMSM operator. It is critical to note that monotonicity cannot be demonstrated by the benefits the scores provide to IVIFVs.

3.1 MSM Operator of Interval-Value Fuzzy Frank Weighted

Section 3 deals with the aggregation of the attributes without their weights. The aggregation of the attributes with the help of the attributes is significant. Hence, we use the IVIFFWMSM operator to deal with the aggregation of the attributes by using their weights. We assume R is given as the weight vectors where $R = \{\varphi_i\}, i = 1, 2, \cdots n$ and $\sum_{i=1}^n \varphi_i = 1$. Note that we will use symbol to describe the weights of the attributes in further discussion unless otherwise stated.

The AO for the aggregation of the attributes by using their weights is defined as follows.

Definition 8: Let E_i be the assortment of IVIFVs. Then IVIFFWMSM: $\ddot{\Gamma}^n$ to $\ddot{\Gamma}$ the operator is defined as and R are weight vector.

$$\mathrm{IVIFFMSM}\left(E_1, E_2, E_3, \cdots E_n\right) = \left(\frac{\underset{1 \leq i_1 < i_2 < i_3 < \cdots < i_\omega \leq n}{\oplus_F} \overset{\omega}{\underset{\mathrm{j}=1}{\otimes_F}} \varphi_{\mathrm{i}_j} \mathrm{E}_{\mathrm{i}_\mathrm{j}}}{C_w^n}\right)^{\frac{1}{w}} \tag{6}$$

where, C_ω^n is the binomial coefficient, $1 \leq i_1 < i_2 < i_3 < \cdots < i_\mathrm{w} \leq \mathrm{n}$ represent $\mathrm{w}-$ tuple combination of the IVIFVs.

We now list a few fundamental characteristics of the IVIFFWMSM operator. Recall that the properties of the MSM operator and the weighted MSM operator are the same. It can be demonstrated that the IVIFFWMSM operator's properties are comparable to those of the IVIFMSM operator. Therefore, the following theorems' proofs are omitted.

Theorem 5: Let $\mathrm{E}_i = \left(\left[\gamma_i^L, \gamma_i^U\right], \left[\theta_i^L, \theta_i^U\right]\right)$ be the assortment of IVIFVs. Then the value aggregated by the IVIFFWMSM operator,

$$\mathrm{IVIFFWMSM}\left(E_1, E_2, E_3, \cdots E_n\right)$$

$$= \left(\begin{array}{c} \left[\log_\varrho\left(1 + \frac{\left(\varrho^{1-\log_\varrho\left(1 + \frac{\left(\varrho^{1-\varepsilon} - 1\right)^{C_\omega^n}}{(\varrho-1)^{C_\omega^n - 1}}\right)} - 1\right)^{\frac{1}{\omega}}}{(\varrho-1)^{\omega-1}}\right), \log_\varrho\left(1 + \frac{\left(\varrho^{1-\log_\varrho\left(1 + \frac{\left(\varrho^{1-\varepsilon} - 1\right)^{C_\omega^n}}{(\varrho-1)^{C_\omega^n - 1}}\right)} - 1\right)^{\frac{1}{\omega}}}{(\varrho-1)^{\omega-1}}\right)\right] \\ \left[1 - \log_\varrho\left(1 + \frac{\left(\varrho^{1-\log_{\varrho_\varrho}\left(1 + \frac{\left(\varrho^{\psi} - 1\right)^{C_\omega^n}}{(\varrho-1)^{C_\omega^n - 1}}\right)} - 1\right)^{\frac{1}{\omega}}}{(\varrho-1)^{\frac{1}{\omega}-1}}\right), 1 - \log_\varrho\left(1 + \frac{\left(\varrho^{1-\log_{\varrho_\varrho}\left(1 + \frac{\left(\varrho^{\psi} - 1\right)^{C_\omega^n}}{(\varrho-1)^{C_\omega^n - 1}}\right)} - 1\right)^{\frac{1}{\omega}}}{(\varrho-1)^{\frac{1}{\omega}-1}}\right)\right] \end{array}\right)$$

where,

$$\left[\mathcal{E}^L, \mathcal{E}^U\right] =$$

$$\left[\begin{array}{c} 1-\log_{\varrho}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(\Pi_{i=1}^{n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(e^{1-\gamma_{i-1}^{L}}\right)^{\frac{1-\varphi_{i_j}}{\varphi_{i_j}}}(\varrho-1)^{1-\varphi_{i_j}}\right)}-1\right)\right)(\varrho-1)^{-1}\right)}-1\right)(\varrho-1)^{1-\omega}\right) \\ 1-\log_{\varrho}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(\Pi_{i=1}^{n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(e^{1-\gamma_{i-1}^{U}}\right)^{\frac{1-\varphi_{i_j}}{\varphi_{i_j}}}(\varrho-1)^{1-\varphi_{i_j}}\right)}-1\right)\right)(\varrho-1)^{-1}\right)}-1\right)(\varrho-1)^{1-\omega}\right) \end{array}\right]$$

And

$$\left[\psi^{L},\psi^{U}\right]=$$

$$\left[\begin{array}{c} \log_{\varrho}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(\Pi_{i=1}^{n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(e^{\theta_{i}^{L}}-1\right)^{\frac{1-\varphi_{i_j}}{\varphi_{i_j}}}(\varrho-1)^{1-\varphi_{i_j}}\right)}-1\right)\right)(\varrho-1)^{-1}\right)}-1\right)(\varrho-1)^{1-\omega}\right) \\ \log_{\varrho}\left(1+\prod_{1\le i_1<i_2<i_3<\cdots<i_w\le n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(\Pi_{i=1}^{n}\left(\varrho^{1-\log_{\varrho}\left(1+\left(e^{\theta_{i}^{L}}-1\right)^{\frac{1-\varphi_{i_j}}{\varphi_{i_j}}}(\varrho-1)^{1-\varphi_{i_j}}\right)}-1\right)\right)(\varrho-1)^{-1}\right)}-1\right)(\varrho-1)^{1-\omega}\right) \end{array}\right]$$

Theorem (Idempotency) 6: Let $\mathrm{E}_i=\left(\left[\gamma_i^L,\gamma_i^U\right],\left[\theta_i^L,\theta_i^U\right]\right)(i=1,2,3,\cdots,n)$ be the assortment of IVIFVs, and if $\mathrm{E}_i=\left(\left[\gamma_i^L,\gamma_i^U\right],\left[\theta_i^L,\theta_i^U\right]\right)=\left(\left[\gamma^L,\gamma^U\right],\left[\theta^L,\theta^U\right]\right)=\mathrm{E}\forall(i=1,2,3,\cdots,n)$. Then IVIFFWMSM $(\mathrm{E}_1,\mathrm{E}_2,\mathrm{E}_3,\cdots\mathrm{E}_\mathrm{n})=\mathrm{E}$ is called the idempotency

Theorem (Monotonicity) 7: Let $\mathrm{E}_i=\left(\left[\gamma_i^L,\gamma_i^U\right],\left[\theta_i^L,\theta_i^U\right]\right)$ and $\mathrm{E}_i^a=\left(\left[\left(\gamma_i^L\right)^a,\left(\gamma_i^U\right)^a\right],\left[\left(\theta_i^L\right)^a,\left(\theta_i^U\right)^a\right]\right)(i=1,2,3,\cdots,n)$ be the assortment of IFVs, and if $\mathrm{E}_i\ge\mathrm{E}_i^a$ for all $(i=1,2,3,\cdots,n)$, then

$$\mathrm{IVIFFWMSM}\left(\mathrm{E}_1,\mathrm{E}_2,\mathrm{E}_3,\cdots\mathrm{E}_\mathrm{n}\right)\ge\mathrm{IVIFFWMSM}\left(\mathrm{E}_1^\mathrm{a},\mathrm{E}_2^\mathrm{a},\mathrm{E}_3^\mathrm{a},\cdots\mathrm{E}_\mathrm{n}^\mathrm{a}\right)$$

Theorem (Boundedness) 8: The $\mathrm{E}_i=\left(\left[\gamma_i^L,\gamma_i^U\right],\left[\theta_i^L,\theta_i^U\right]\right)(i=1,2,3,\cdots,n)$ is the assortment of the IVIFVs, $\mathrm{E}^-=\min\left(\mathrm{E}_i\right),\mathrm{E}^+=\max\left(\mathrm{E}_i\right)$, then:

$$\mathrm{E}_i^-\le\mathrm{IVIFFWMSM}\left(\mathrm{E}_1,\mathrm{E}_2,\mathrm{E}_3,\cdots\mathrm{E}_\mathrm{n}\right)\le\mathrm{E}_i^+$$

4 The Application Approach of the MAGDM

In this section, the application of the recommended approach is discussed. First, the stepwise methodology of the application of the proposed novel model is discussed with the help of the flowchart. Then, the model is applied to the MADM problem in real life.

Now, we provide the MADM process using the developed model. Consider the list of alternatives $Z = \{\zeta_1, \zeta_2, \ldots, \zeta_n\}$. For this list, we have to indicate the most appropriate alternative based on the list of attributes, say $\mathrm{K} = \{\beta_1, \beta_2, \ldots, \beta_m\}$ having the weights from $R = \{\varphi_1, \varphi_2, \ldots, \varphi_m\}$ with $\sum_{i=1}^{m} \varphi_i \forall i = 1, 2, 3, \cdots, m$. Let $\Gamma = \{\kappa_1, \kappa_2, \ldots, \kappa_j\}$ be the weights of j specialists and $T^s = [r^s{}_{ij}]_{m\times n}$ be the decision matrix where $r^s{}_{ij} = \left(\gamma^s_{ij}, \theta^s_{ij}\right)$ be the IVIF information.

The MAGDM process using the developed model can be illustrated through the following steps.

Step 1: Compile the data for the actual scenario using the IVIFVs. In this case, the information is supplied by the experts as IVIFVs. Initially, we standardize the data gathered from the actual situation when the cost type attribute is present. The normalization's specifics are listed below.

The attributes are characterized by benefit and cost type. We need normalization if the cost type attributes exist to convert the type of the attribute. The normalization of the data can be done as follows:

$$(T^s)^c = \begin{cases} \left(\gamma^s_{ij}, \theta^s_{ij}\right) & \text{for benefit attribute} \\ \left(\theta^s_{ij}, \gamma^s_{ij}\right) & \text{for cost attribute} \end{cases}$$

Step 2: The IVIFVs, or normalized information, were compiled with the assistance of the IVIFFWMSM operator. We aggregate each expert attribute separately if there are multiple experts. Subsequently, the developed model is used to determine the total aggregated value of every alternative. In the event that a single expert provides the information, we determine the total aggregated value of the attributes for every option.

Step 3: Eq. (1) is used to determine each alternative's score after the total aggregated values for all the alternatives have been obtained. This helps with the ranking of the alternatives.

Step 4: In the end, we arrange the options according to their score values, either in ascending or descending order.

Step 5: Using the obtained score values, choose the best optimized option from the given list.

The following flowchart (Figure 1) describes the stepwise process of the MAGDM.

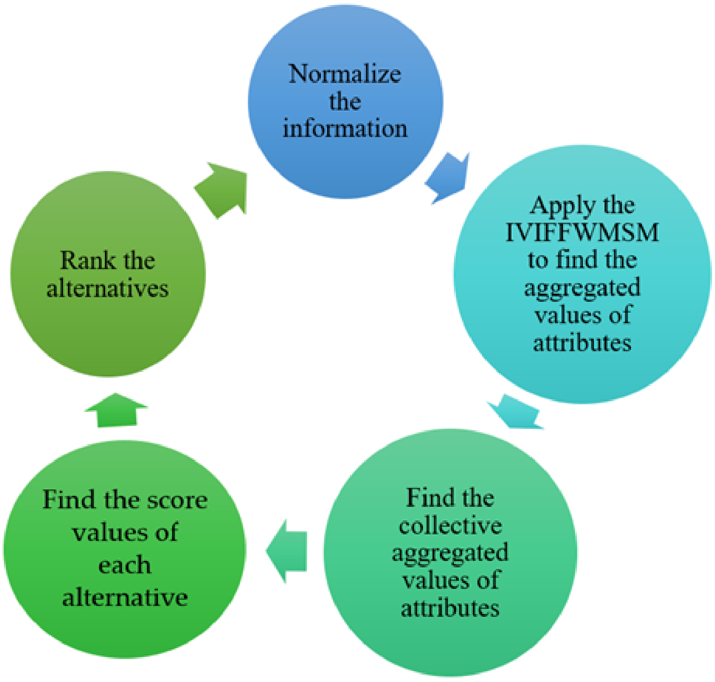

Figure 1. Flowchart of the methodology of the MAGDM process based on the IVIFFWMSM operator

4.1 Application in Market Risk Optimization

Portfolio optimization, in which investors aim to create portfolios that maximize returns while minimizing risks, is a prominent application of optimization in managing market risks. Investors can utilize mathematical optimization techniques, like modern portfolio theory or mean-variance optimization, to strategically allocate assets in a way that maximizes return while minimizing risk. In this process, historical market data is analyzed, correlations between various assets are evaluated, and variables like volatility and liquidity are taken into account. By means of optimization, investors can discern diversified portfolios that present the highest projected returns for a particular degree of risk, consequently augmenting their capacity to maneuver through turbulent market conditions and curtail possible losses.

4.2 Impact of Market Risk

Optimizing market risks has a significant and wide-ranging impact on our lives. Effective risk management protects investments and retirement funds by maintaining market stability. Institutions and individuals can reduce the effects of market swings, recessions, and unanticipated events by using optimization techniques, which will increase confidence and trust in the financial system. Additionally, optimization encourages innovation in financial services and products, which results in the creation of customized solutions that meet a range of risk tolerances and investing goals. In the end, managing market risks improves financial stability, gives investors the ability to make wise choices, and advances general economic prosperity by creating a more stable and secure financial environment for people, companies, and society at large.

4.3 Source of Market Risk

Variations in prices are the source of market risk. The term "price volatility" refers to standard deviations in stock, currency, or commodity prices. Annualized terms are used to measure volatility, which can be expressed as an absolute value (e.g., $10) or as a percentage of the starting value (e.g., 10%). The Securities and Exchange Commission (SEC), which oversees US public companies, requires them to disclose any possible relationships between their productivity and outcomes and the financial markets' performance. The purpose of this requirement is to describe the financial risk that a company faces. A company that provides foreign exchange futures or derivative investments, for example, might be more financially exposed than one that doesn't. With this information, traders and investors can make decisions based on their own risk management policies.

4.4 Investing Diversification Can Help

Investing in diversification can help you avoid specific risk, also referred to as unsystematic risk, which is unrelated to the overall performance of the market but specifically linked to the performance of a particular asset. One way that unsystematic risk manifests itself is when a company files for bankruptcy, in which case investors lose all of their investment in the company. Some of the most common types of market risks are interest rate risk, equity risk, currency risk, and commodity risk. Interest rate risk is the volatility that could accompany changes in interest rates caused by underlying factors, like announcements from central banks regarding changes to monetary policy. Particularly vulnerable to this risk are fixed-income assets such as bonds. The risk posed by shifting stock prices is known as equity risk. The volatility of commodity prices, such as those of corn and crude oil, is a component of commodity risk. Currency risk, or exchange-rate risk, is the result of changes in the value of one currency relative to another. Foreign investments by individuals and businesses are susceptible to currency risk. The volatility of commodity prices, such as those of corn and crude oil, is a component of commodity risk. Commodity risk covers the price fluctuations of goods like corn and crude oil. Changes in one currency's exchange rate to another give rise to currency risk, also referred to as exchange-rate risk. Currency risk can have an impact on foreign-holding companies or investors. The following examines an example and assesses certain risk factors according to certain characteristics.

4.5 Example

Accept that $1 \leq K_i \leq 4$ are considered as four market risk factors, i.e., price variation risk K_1, policy adjustment risk K_2, equity risk K_3, and commodity risk K_4. The risk factors are analyzed based on the attributes mentioned above. Consider the weights of the attributes are $(0.2, 0.3, 0.4, 0.1)$. Initially, some experts are hired to investigate the risk factors and to assign the IVIFVs to each alternative with admiration to each attribute. The information is provided in Table 1, Table 2, and Table 3 as follows:

Table 1. Interval fuzzy decision matrix $D1$

K	DD_1		DD_2		DD_3		DD_4	
K_1	$[0.30, 0.32]$	$[0.33, 0.40]$	$[0.10, 0.28]$	$[0.35, 0.42]$	$[0.14, 0.32]$	$[0.24, 0.45]$	$[0.23, 0.24]$	$[0.31, 0.35]$
K_2	$[0.1, 0.20]$	$[0.28, 0.30]$	$[0.35, 0.40]$	$[0.44, 0.50]$	$[0.23, 0.25]$	$[0.20, 0.43]$	$[0.22, 0.34]$	$[0.35, 0.45]$
K_3	$[0.20, 0.21]$	$[0.32, 0.38]$	$[0.20, 0.40]$	$[0.48, 0.50]$	$[0.15, 0.24]$	$[0.23, 0.35]$	$[0.33, 0.34]$	$[0.42, 0.44]$
K_4	$[0.25, 0.33]$	$[0.40, 0.44]$	$[0.22, 0.38]$	$[0.28, 0.40]$	$[0.11, 0.34]$	$[0.22, 0.35]$	$[0.22, 0.25]$	$[0.28, 0.33]$

Step 1: In the above data, every one of the qualities is of a similar type. Consequently, we needn't bother with the standardization since nothing of the properties is the expense characteristic.

Table 2. Interval fuzzy decision matrix $D2$

$\boldsymbol{K}$	$\boldsymbol{DD_1}$		$\boldsymbol{DD_2}$		$\boldsymbol{DD_3}$		$\boldsymbol{DD_4}$	
$\boldsymbol{K_1}$	$[0.20, 0.21]$	$[0.30, 0.36]$	$[0.22, 0.40]$	$[0.41, 0.50]$	$[0.22, 0.32]$	$[0.41, 0.46]$	$[0.24, 0.34]$	$[0.17, 0.45]$
$\boldsymbol{K_2}$	$[0.10, 0.31]$	$[0.20, 0.34]$	$[0.26, 0.30]$	$[0.32, 0.40]$	$[0.21, 0.23]$	$[0.32, 0.44]$	$[0.21, 0.33]$	$[0.23, 0.44]$
$\boldsymbol{K_3}$	$[0.10, 0.21]$	$[0.24, 0.30]$	$[0.18, 0.35]$	$[0.21, 0.40]$	$[0.13, 0.35]$	$[0.21, 0.44]$	$[0.21, 0.35]$	$[0.21, 0.44]$
$\boldsymbol{K_4}$	$[0.15, 0.21]$	$[0.22, 0.40]$	$[0.11, 0.30]$	$[0.24, 0.40]$	$[0.21, 0.25]$	$[0.24, 0.33]$	$[0.23, 0.34]$	$[0.45, 0.55]$

Table 3. Interval fuzzy decision matrix $D3$

$\boldsymbol{K}$	$\boldsymbol{DD_1}$		$\boldsymbol{DD_2}$		$\boldsymbol{DD_3}$		$\boldsymbol{DD_4}$	
$\boldsymbol{K_1}$	$[0.28, 0.31]$	$[0.41, 0.50]$	$[0.22, 0.30]$	$[0.35, 0.50]$	$[0.22, 0.31]$	$[0.38, 0.44]$	$[0.23, 0.27]$	$[0.32, 0.45]$
$\boldsymbol{K_2}$	$[0.21, 0.29]$	$[0.30, 0.51]$	$[0.20, 0.40]$	$[0.42, 0.40]$	$[0.22, 0.24]$	$[0.32, 0.33]$	$[0.35, 0.35]$	$[0.41, 0.44]$
$\boldsymbol{K_3}$	$[0.24, 0.42]$	$[0.28, 0.50]$	$[0.21, 0.34]$	$[0.34, 0.42]$	$[0.21, 0.35]$	$[0.37, 0.44]$	$[0.23, 0.24]$	$[0.35, 0.35]$
$\boldsymbol{K_4}$	$[0.21, 0.48]$	$[0.30, 0.32]$	$[0.19, 0.25]$	$[0.23, 0.35]$	$[0.24, 0.32]$	$[0.35, 0.35]$	$[0.32, 0.33]$	$[0.34, 0.44]$

Step 2: We apply the IVIFFWMSM operator to total the all upsides of qualities in the given Table 1, Table 2, and Table 3.

Step 3: Eq. (1) is used to determine each alternative's score after the total aggregated values for all the alternatives have been obtained. This helps with the ranking of the alternatives.

Table 4. Aggregated information matrix

$\boldsymbol{DD_1}$		$\boldsymbol{DD_2}$	
$[0.733, 0.744]$	$[0.011, 0.010]$	$[0.727, 0.670]$	$[0.032, 0.045]$
$\boldsymbol{DD_3}$		$\boldsymbol{DD_4}$	
$[0.881, 0.808]$	$[0.208, 0.167]$	$[0.746, 0.703]$	$[0.052, 0.040]$

Step 4: From Table 4 we can see the classification order with the help of the score function alternatives.

Step 5: Using the obtained score values, choose the best optimized option from the given list. We arrange the options according to their score values, either in ascending or descending order in Table 5.

Table 5. Score values

$\mathbf{sce}\,(\boldsymbol{K_1})$	$\mathbf{sce}\,(\boldsymbol{K_2})$	$\mathbf{sce}\,(\boldsymbol{K_3})$	$\mathbf{sce}\,(\boldsymbol{K_4})$
0.728	0.661	0.657	0.679

$\text{sce}\,(K_1) > \text{sce}\,(K_4) > \text{sce}\,(K_2) > \text{sce}\,(K_3)$, this means that K_1 alternative is the most appropriate.

Table 6. IFFWMSM operator variation the ranking orders with different parameter

N	Score Values Ranking	Orders of Preference
2	$\text{sce}\,(K_1) > \text{sce}\,(K_4) > \text{sce}\,(K_2) > \text{sce}\,(K_3)$	$K_1 \succ K_4 \succ K_2 \succ K_3$
3	$\text{sce}\,(K_1) > \text{sce}\,(K_4) > \text{sce}\,(K_2) > \text{sce}\,(K_3)$	$K_1 \succ K_4 \succ K_2 \succ K_3$
4	$\text{sce}\,(K_1) > \text{sce}\,(K_4) > \text{sce}\,(K_2) > \text{sce}\,(K_3)$	$K_1 > K_4 \succ K_2 \succ K_3$
5	$\text{sce}\,(K_1) > \text{sce}\,(K_4) > \text{sce}\,(K_2) > \text{sce}\,(K_3)$	$K_1 \succ K_4 \succ K_2 \succ K_3$
6	$\text{sce}\,(K_1) > \text{sce}\,(K_4) > \text{sce}\,(K_2) > \text{sce}\,(K_3)$	$K_1 \succ K_4 \succ K_2 \succ K_3$
7	$\text{sce}\,(K_1) > \text{sce}\,(K_3) > \text{sce}\,(K_4) > \text{sce}\,(K_2)$	$K_1 > K_3 > K_4 > K_2$
8	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$
9	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$
10	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$
15	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$
20	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$
30	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$
40	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$
50	$\text{sce}\,(K_1) > \text{sce}\,(K_2) > \text{sce}\,(K_4) > \text{sce}\,(K_3)$	$K_1 \succ K_2 \succ K_4 \succ K_3$

4.6 Comparative Analysis

The fundamental characteristic of Frank TCRM and FTRM is their flexible results, which are attributed to the parameter ϱ. Nonetheless, a variation in the parameter's value could result in a variation in the results' value. As a result, Table 6 presents the altered results along with the parameter values.

We have investigated those results gained by IVIFFWMSM executives by vitiating the potential gains of δ. We have taken the values of ϱ since 2 to 50. The characterized consequences are provided in Table 6. At all values of parameter ϱ, the results are same.

4.7 Pre-existing Operators Can be Comparison

The AOs for the IVIFS were developed by Qin and Liu [44] based on algebraic TRM and algebraic TCRM. Xu and Yager [45] developed geometric aggregation operators based on intuitionistic fuzzy sets. Zhang et al. [46] fostered some AOs for uncertainties in light of Frank power AOs to tackle the MAGDM issue. Senapati et al. [47] Presented AOs based on Aczel-Alsina TCRM and TRM (IFAAWA), addressing uncertainties. Nevertheless, the AOs presented in our study are the most reliable because they utilize the most generalized form of the mean operator, namely the MSM operator, which ensures that the attributes are aggregative. Additionally, the developed model is derived based on Candid's operational regulations, which are more flexible and generalized. As a result, the developed model provides a more advantageous method for aggregating data. The proposed approach has been compared with the aforementioned previous methods to justify this methodology. The results, which can be found in Table 6, are graphically represented in Figure 1. Key findings of this analysis are outlined below. The IVIFFWMSM operator emerges as the best option, represented by K_1. Similarly, the IVIFPWG operator and the IVIFPWA operator both identify K_1 as the optimal choice. The IVIFWHM operator also suggests K_1 as the top alternative. However, the most reliable approach is considered to be the MSM-based method. The IFFWMSM operator collects data using straightforward operations where several parameters are defined. Consequently, the IFFWMSM operator is deemed the most suitable for addressing MAGDM issues due to its remarkable adaptability, enhanced by parameter involvement. Table 7 presents a ranking of risk factors using different operators.

Using various AOs, the score values are calculated and presented in Table 7. These operators yield the same ranking.

Table 7. Pre-existing operators can be compared

Operator	Ranking of Score Values	Preference Orders
IVIFFWMSM	$sc(K_1) > sc(K_4) > sc(K_2) > sc(K_3)$	$K_1 > K_4 > K_2 > K_3$
IVIFWHM [44]	$sc(K_1) > sc(K_2) > sc(K_3) > sc(K_4)$	$K_1 > K_2 > K_3 > K_4$
IVIFWDHM [45]	$sc(K_4) > sc(K_3) > sc(K_2) > sc(K_1)$	$K_4 > K_3 > K_2 > K_1$
IVIFPWG [46]	$sc(K_1) > sc(K_3) > sc(K_2) > sc(K_4)$	$K_1 > K_3 > K_2 > K_4$
IVIFPWA [47]	$sc(K_1) > sc(K_2) > sc(K_3) > sc(K_4)$	$K_1 > K_2 > K_3 > K_4$

5 Conclusions

The MSM chairman has given the IFFWMSM and IFFMSM executives more latitude in section composition. The IVIFS design is based on the FTCRM and FTRM. The managers' primary applications are visible and demonstrated. Then, by taking into consideration the numerical model, directors are applied to a real problem. This model's betting factors are industry improvements that are analyzed and provided as IVFVs with the chairman's assistance. Four wagering factors are examined and evaluated in light of the research. Therefore, the outcomes obtained can be utilized by distinct and previous assortment heads, such as the IFMSM, IFFWA, and IFWG overseers. A few enormous centers have been communicated below. The FTCRM and FTRM exercises, as well as the complex limit δ, are what allow the proposed IFFWMSM overseer to be flexible. The bet factors are improving the studied business with the aid of research. Most likely, we can take a chance on the elements that the research community and association truly want to look into, and we can take the necessary steps to compel those risk factors that will ultimately ensure the success of the business. In summary, we can avoid expected risk by surveying and recommending the bet factors. Our immediate objectives are to focus on spherical fuzzy set, T- spherical fuzzy set, artificial intelligence, machine learning, and game theory.

Author Contributions

Mehwish Sarfaraz: Conceptualization, writing, review, and Darko Božanić editing: Validation, Supervision.

Data Availability

Not Available.

Conflicts of Interest

The authors declare that none of the work reported in this paper could have been influenced by any known competing financial interests or personal relationships.

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