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A Sustainable Circular EPQ Model with Waste Minimization by Using 3D Technology

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Abstract: Environmental pollution is a headache nowadays. It is observed that waste from industries plays a vital role in environmental pollution. So, in this study, it is tried to reduce the amount of waste materials and try to make it suitable for reuse. In today's market, it is impossible to survive in the market without the help of advertising. Therefore, advertisement for quick information about a product to its customer is used in this study. The demand for 3D printing techniques is increasing day by day for their utilization in the fields of agriculture, education, medicine, automobiles, mobile phones, aircraft, spaceships, etc. In today's era, with the advancement of technology, 3D printing is becoming more and more popular in manufacturing, health care, and many other sectors. Therefore, a production model with waste minimization and utilization of 3D printing is formulated in this study. This study primarily aims to determine optimal cycle length, production rate, and the optimal amount of inventory model technique by synthesizing the carbon emission rate with minimizing the total cost. A numerical example is presented to illustrate the characteristics and progress of the economic production quantity (EPQ) model.

Keywords: Price-advertisement demand; Circularity index; Sustainability; 3D printing; Circular economy; Waste management

1 Introduction

In the modern business, advertising holds a vital role. Attracting customers to a product through advertising is an old technique in business. Advertisement is mainly displayed through TV, radio, road hoarding, vehicles, and internet platforms like Facebook, WhatsApp, email, YouTube, online games, etc. The purpose of advertisement is to increase the visibility of a product by sharing basic information about a product, like color, price, or quality, and to encourage customers to buy the product. As a result, advertisements increase the demand rate of a product, which directly increases the sales volume of the company. Another impact of advertisement is that advertisements guide you to achieve increased revenue and decrease costs by reducing deterioration of inventory (sales before the obsolescence period). For the described reason, advertisement is considered in our study to support profit maximization.

The circularity index measures the efficiency of a system by keeping its resources within its production cycle through remanufacturing, recycling, reuse, or refurbishment. A circularity index is a score that ranks a product's degree of circularity potential throughout its life cycle. The circularity index is useful for manufacturers as it provides quantitative information to support circularity claims. The circularity index is included in our study, as it offers insightful information through sustainability and waste reduction.

Sustainable development aims for balanced growth by considering economic, social, and environmental aspects without compromising future resources. Integration of sustainability in the inventory model is essential as it minimizes the environmental cost and supports resource resiliency. Besides, a sustainable inventory helps to reduce waste through reuse or recycling by using 3D printing technology, which is used in our study also.

In today's rapidly growing world, the idea of the circular economy has gained significant momentum as a powerful alternative to traditional linear economic models with its emphasis on sustainability resources. Circular economy seeks to redefine our approach to production, consumption, and minimization of waste by recycling, reusing, and repairing. It offers a promising solution to the pressing environmental challenges in this paper. The circular economy aims to extend the lifespan of resources and maximize their value throughout their use. This is the first and foremost reason why circular economy is used in this paper.

Waste management is an essential aspect of modern life, as it ensures the proper disposal of waste materials. Poor waste management can pose serious health and environmental risks, including soil pollution, water contamination, and the transmission of diseases. There are various methods of waste management, such as landfill, incineration, and recycling. In this study, waste management is considered as it helps with cost efficiency, enhances sustainable practices, and is responsible for smooth business operations.

The main motivation of this study is to address social, environmental, and economic challenges faced by a traditional manufacturing model by integrating circular economy with 3D technologies. The authors aim to develop a sustainable production model that not only meets demand efficiently but also minimizes the material waste and promotes recycling or reuse of resources. With the help of 3D printing technology, the model seeks to reduce overproduction and lower environmental impact by contributing to a more sustainable system.

1.1 Research question

The following are the research questions that are raised and answered to develop the study:

- How to keep the environment sustainable through a circular economy to protect the environment?
- How to reduce waste materials to protect the environment through 3D printing?
- How has the product demand in the market been implemented according to price-advertisement demand?
- How does the volume of carbon emissions generated by different activities related to the production system make the model sustainable?

1.2 Organization of the Paper

The subsequent sections of this study are structured as follows: Section 2 provides the literature review of the present work. Section 3 contains notations and corresponding assumptions that are utilized in the study. Section 4 presents the mathematical model and calculation of the total cost of this study. A solution procedure is developed in Section 5. Section 6 describes the importance of this study with numerical examples and real case studies. Sensitivity analysis of major parameters and derived managerial insights is presented in section 7. Finally, Section 8 and 9 provides graphical illustration and conclusion part with future research directions.

2 Literature Review

The literature review and motivation of this study are described in the subsequent part:

2.1 Price-Advertisement Dependent Demand

Advertising plays a very important role in today's competitive market. Some of the previous studies have also sought attention for this factor, like Giri and Dash [1] found a shipment policy for imperfect production systems and used the Stackelberg game to determine advertisement effect and optimal pricing. Mandal and Pal [2] formulated an unreliable production system with advertisement and price-dependent demand. Xu et al. [3] showed the advantage of advertisement both by use of manufacturer and buyer. Rad et al. [4] developed a vendor-buyer model by considering imperfect products and found the impact of advertisement on market requirements. Yuan et al. [5] investigated advertising decisions in a manufacturer and retailer model with cost disruptions. Price-advertisement-dependent demand is used in this study to make the model more practical and realistic from a business perspective.

2.2 Circularity Index

Circularity index is needed in almost all areas of economics. Kasztelan [6] presented the aggregate circularity index of the European country with the help of a linear ordering method for an MCDM (multi-criteria decision-making) model. Azevedo et al. [7] suggested a Sustainable Circular Index (SCI) to evaluate the circularity and sustainability of an industry. Corona et al. [8] considered a sustainable economic system where financial development is doubled from the usage of capital through diminution and recycling of natural resources. Rabta [9] formulated an EOQ model with a circular economy indicator. Kirchherr et al. [10] developed the idea of a circular economy using the circularity index as a guiding framework. A sustainable circular economy production model with carbon emission is derived by John and Mishra [11]. Prieto-Sandoval et al. [12] highlighted the primary idea of circular economy and its connection with eco-innovation. The circularity index is used in this study as it helps to measure the sustainability of the system by allowing the reuse or recovery of materials instead of ending up as waste.

2.3 Sustainability

Sustainability in inventory control is nothing but optimization of inventory by minimizing waste, reducing environmental effects, and lowering inventory costs while maintaining the efficiency of the system. Haque et al. [13] discussed a two-phase remanufacturing-manufacturing model where demand was dependent on price and the green level of products, and the number of defective items was also counted. Pervin et al. [14] derived a non-instantaneous deteriorating model with composite demand by following a sustainable production process. Paul et al. [15] formulated a sustainable EPQ model with remanufacturing of returned products. Pervin et al. [16] broaden the application of Paul et al. [15] by integrating a vendor-buyer model. Mishra et al. [17] studied a sustainable inventory model with backorder and controllable carbon emission. Mashud et al. [18] formulated a sustainable inventory model with carbon emission. During the Covid period, Barman et al. [19] formulated an EPQ model by allowing investment in green and preservation technology. Here, sustainability is allowed, as we aim to address both economic and environmental impact in modern manufacturing. Besides, sustainability with the help of 3D printing technology allows minimization of production waste, which reduces environmental impact.

2.4 3D Printing

3D printing has a prominent role in reducing the unnecessary costs in an inventory model. Balletti et al. [20] presented the future perspective of 3D printing technology in an inventory model. Lee et al. [21] summarized the primary perspectives and applications of 3D printing technology in terms of speed, resolution, and energy in an inventory model. Liu et al. [22] evaluated suitable food printing technologies and demonstrated the application of 3D printing across various food sectors while providing in-depth insights into emerging trends and potential challenges in 3D food printing. Shahrubudin [23] developed the kinds of 3D printing technology and ingredients used for 3D printing processing in the industrial industry. Pervin [24] presented an inventory model to monitor carbon emission for achieving sustainability with the help of green technology. In this study, 3D technology is used for minimization of waste of material and to facilitate a circular economy.

2.5 Waste Management

Waste management in the inventory model plays an important role because it affects the cost, efficiency, and sustainability of the model. Wilson [25], in the 19th century, identified six broad groups of operators to increase the formal waste collection methods for reducing waste in developing countries. Jaber et al. [26] found a coordination scheme for waste disposal with remanufacturing. Demirbas [27] studied the ideas and structures of waste management, identified the resources of bio-waste, and classified a category of waste. Nas and Jaffe [28] considered garbage collection as a way of waste reduction and demonstrated an extensive range of locally designed and modified activities. Jauhari et al. [29] studied a closed-loop supply chain model with the effect of carbon emission. Seadon [30] described a sustainable waste management system for New Zealand. Waste management is studied in this study to enhance sustainability and operational efficiency of the system.

2.6 Carbon Emission

Incorporation of carbon emission into a production model ensures that sustainability is addressed properly by allowing waste minimization. In this context, Tiwari et al. [31] presented a vendor-buyer deteriorating model with defective items by allowing carbon emission. Bazan et al. [32] studied a remanufacturing-manufacturing model with energy used and calculation of greenhouse gas releases from manufacturing, remanufacturing, and transport with emissions punishment tax as per the European Union emissions dealing process. Datta [33] formulated a production model with several carbon tax policies and green technology adoption. Li and Hai [34] discussed a multiple retailer model with a single warehouse and noted the effects of carbon emissions. In the model, they have caught the reorder break time for the retailer while minimizing the holding, ordering, and carbon emission costs. Daryanto et al. [35] studied a three-echelon supply chain model with allowable deterioration. Rana et al. [36] studied a growing inventory model with permissible delay in payments and allowable carbon emissions. Das et al. [37] derived a location transportation model with the effect of variable carbon emissions. Carbon emission is allowed in this model, as carbon emission with 3D printing technology contributes to lower energy consumption and reduction in greenhouse gas emissions with a goal of minimization of the environmental impact.

The importance of our study in comparison to the described literature is displayed in Table 1.

Table 1. Contribution of various authors regarding the proposed model

Author(s)	Model Type	Circularity	Carbon Emission	Waste Reduction Due to 3D Printing	Demand
Jaber et al. [26]	EPQ	-	-	-	Constant
Datta [33]	EPQ	-	√	-	Dependent on selling price
Daryanto et al. [35]	SCM	-	√	-	Constant
Jauhari et al. [29]	SCM	-	√	-	Dependent on selling price, green technology and quality of product
Mishra et al. [17]	EPQ	-	√	-	Deterministic
Rabta [9]	EOQ	√	-	-	Cost and price
Mashud et al. [18]	EOQ	-	√	-	Price
Rana et al. [36]	EPQ	-	√	-	Time
John and Mishra [11]	EPQ	-	√	-	Deterministic and constant
This work	EPQ	√	√	√	Advertisement frequency, circularity index, purchasing cost

3 Notations and Assumptions

The following are the notations and assumptions that are utilized in this study:

3.1 Notations

D: Demand rate;
p: Purchasing cost per unit item;
A: Number of advertisement frequency;
α: Circularity index;
c: Intercept of demand;
d: Own demand effect factor;
m: Advertisement frequency;
a: constant and represents additional demand;
V: Proportion of defective items;
β: Unit investment in 3D technology for waste minimization;
λ: Efficacy of 3D printing;
θ: Deterioration rate;
S: Unit Set-up cost ($);
h: Unit holding cost for deteriorating items per unit time;
γ: Carbon tax ($/Kg);
Y: Advertisement cost per frequency;
Q_c: Unit quality checking cost;
P_c: Unit production cost ($);
ζ: Unit production rate of waste material;
θ_0: Unit deterioration cost;
E_s: Production setup process carbon emission (kg/unit);
E_p: Emissions regarding production (kg/unit);
E_h: Emissions regarding holding of items (kg/unit);
E_w: Emission regarding waste (kg/unit);
E_d: Emissions regarding deterioration (kg/unit);
L: Unit cost due to raw materials ($);
M: Unit labor cost ($);
N: Unit instrument cost ($);

Q: Stock level;
W: Waste generation function.

Decision variables

R: Production rate;
T: Inventory cycle length;
t_1: Time at which production stop.

3.2 Assumptions

(i) Demand rate is assumed as advertisement and price dependent and structured as $D(p, A, \alpha) = A^m(c-dp)+a\alpha$, where c is the intercept of the demand function, d is the own demand effect factor, m is the advertisement frequency, and $c, d, m > 0$, a is the additional demand and assumed as constant, and $0 \leq \alpha \leq 1$ [38].
(ii) 3D printing is considered to reduce waste. Further, to minimize the waste, the production rate is also considered as a decision variable.
(iii) Waste generation function $W(\beta) = 1 - e^{-\lambda\beta}$, where β represents the cost regarding minimization of waste by utilizing 3D technology and λ is known as the effectiveness of that 3D technology [39].
(iv) The study is based on a single item.
(v) The model does not allow stockout.
(vi) Replenishment is infinite.
(vii) Carbon emissions due to production, holding, setup, and waste minimization are allowed in this study to make the model environment sustainable.

4 Mathematical Formulation

The production process starts and keeps going until time t_1. Due to demand and deterioration, inventories decreased throughout the time period $[t_1, T]$. Figure 1 shows the graphical demonstration of the model.

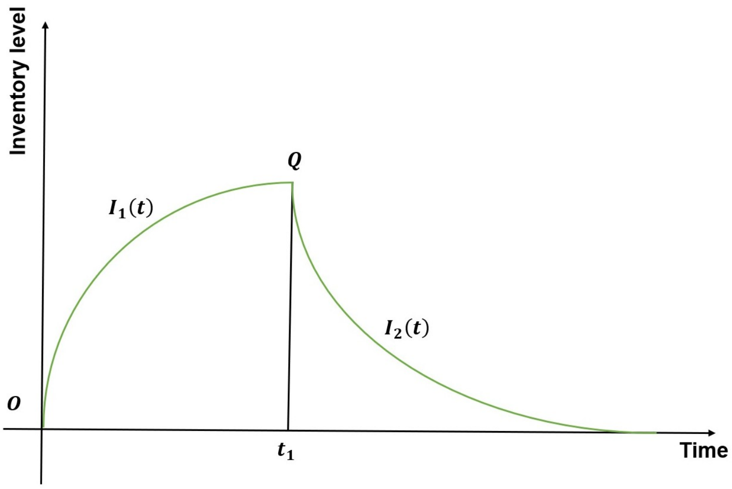

Figure 1. Inventory control model

Now, the inventory level is presented as

$$\frac{dI_1(t)}{dt} + \theta I_1(t) = R(1 - e^{-\lambda\beta}) - D(p, A, \alpha),\ 0 \leq t \leq t_1 \tag{1}$$

with $I_1(0) = 0$;

$$\frac{dI_2(t)}{dt} + \theta I_2(t) = -D(p, A, \alpha),\ t_1 \leq t \leq T \tag{2}$$

with $I_2(T) = 0$.

Solutions of the equations are given as

$$I_1(t) = \frac{1}{\theta}\left(1 - e^{-\theta t}\right)\Big[R(1 - e^{-\lambda\beta}) - A^m(c - dp) - a\alpha\Big] \tag{3}$$

and

$$I_2(t) = \frac{1}{\theta}\big(e^{\theta(T-t)} - 1\big)\Big[A^m(c-dp) + a\alpha\Big] \tag{4}$$

Now, by a suitable choice of t_1, where $I_1(t) = I_2(t)$, the entire function $I_1(t)$ will be guaranteed to be continuous at $t = t_1$ indeed. The value of t_1 is obtained as

$$t_1 = \frac{1}{\theta}log\Bigg[\frac{\big[R(1-e^{-\lambda\beta}) - A^m(c-dp) - a\alpha\big] + e^{\theta T}\big[A^m(c-dp) + a\alpha\big]}{R(1-e^{-\lambda\beta})}\Bigg] \tag{5}$$

Again, stock level at $t = t_1$ is given by

$$Q = I_2(t_1) = \frac{1}{\theta}\big(e^{\theta(T-t_1)} - 1\big)\Big[A^m(c-dp) + a\alpha\Big] \tag{6}$$

Now, the cost functions are calculated as follows:
(i) Set-up cost is $\frac{S}{T}$.
(ii) Production cost is

$$\frac{P_c}{T}\int_0^{t_1} Rdt = \frac{1}{T}\big(L + MR + (\frac{N}{R})\big)Rt_1$$

where, $P_c = L + MR + (\frac{N}{R})$.
(iii) Holding cost is

$$\frac{h}{T}\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right] = \frac{h}{T}\Bigg[\frac{R\left(1-e^{-\lambda\beta}\right) - (A^m(c-dp)+a\alpha)}{\theta}\left(t_1 + \frac{e^{-\theta t_1}}{\theta} - \frac{1}{\theta}\right) + \frac{(A^m(c-dp)+a\alpha)}{\theta}\left(-\frac{1}{\theta} + \frac{e^{\theta(T-t_1)}}{\theta} - T + t_1\right)\Bigg]$$

(iv) Advertisement cost is $\frac{AY}{T}$.
(v) Cost due to waste is the sum of two related costs: the cost of the item produced due to the defective production process and the cost due to deterioration. Therefore, Cost due to waste is calculated as

$$\frac{P_c}{T}\int_0^{t_1} R\zeta e^{-\lambda\beta}dt + \theta\int_{t_1}^{T} I_2(t)\theta_0 dt = \frac{P_c}{T}R\zeta e^{-\lambda\beta}t_1 + \theta_0\left(A^m(c-dp)+a\alpha\right)\left(-\frac{1}{\theta} + \frac{e^{\theta(T-t_1)}}{\theta} - T + t_1\right)$$

(vi) Quality improvement cost is $\frac{VQ_c}{T}$.
(vii) Carbon emits due to several reasons. Therefore, carbon emission costs are
• Emission cost per cycle due to setup is

$$\frac{\gamma}{T}E_s$$

• Emission cost per cycle caused by production is

$$\frac{\gamma}{T}\int_0^{t_1} E_pRdt = \frac{\gamma}{T}E_pRt_1.$$

• Emission cost owing to holding of inventory is

$$\frac{\gamma}{T}E_h\left[\frac{R\left(1-e^{-\lambda\beta}\right) - (A^m(c-dp)+a\alpha)}{\theta}\left(t_1 + \frac{e^{-\theta t_1}}{\theta} - \frac{1}{\theta}\right) + \frac{(A^m(c-dp)+a\alpha)}{\theta}\left(-\frac{1}{\theta} + \frac{e^{\theta(T-t_1)}}{\theta} - T + t_1\right)\right]$$

• Emission cost per cycle because of waste is

$$\frac{\gamma}{T}\Big[E_wP_cR\zeta e^{-\lambda\beta}t_1 + E_d\big(A^m(c-dp)+a\alpha\big)\Big(-\frac{1}{\theta} + \frac{e^{\theta(T-t_1)}}{\theta} - T + t_1\Big)\Big]$$

Therefore, total carbon emission cost is

$$\frac{\gamma}{T}\Bigg[E_s + E_pRt_1 + E_h\Bigg(\frac{R\left(1-e^{-\lambda\beta}\right) - (A^m(c-dp)+a\alpha)}{\theta}\left(t_1 + \frac{e^{-\theta t_1}}{\theta} - \frac{1}{\theta}\right) + \frac{(A^m(c-dp)+a\alpha)}{\theta}\left(-\frac{1}{\theta} + \frac{e^{\theta(T-t_1)}}{\theta} - T + t_1\right)\Bigg) + E_wP_cR\zeta e^{-\lambda\beta}t_1 + E_d\left(A^m(c-dp)+a\alpha\right)\left(-\frac{1}{\theta} + \frac{e^{\theta(T-t_1)}}{\theta} - T + t_1\right)\Bigg]$$

Hence, the average inventory cost can be defined as

$$\begin{aligned}\text{Average inventory cost } &= \text{Setup cost } + \text{ Ordering cost } + \text{ Holding cost } + \text{ Production cost}\\ &+ \text{ Carbon emissions cost } + \text{ Advertisement cost}\\ &+ \text{ Cost due to waste } + \text{ Quality improvement cost.}\end{aligned}$$

Now, inserting the values of the respective parts, the cost function is obtained as

$$\begin{aligned}TC\left(T,t_1,R\right)=&\frac{S}{T}+\frac{AY}{T}+\frac{VQ_c}{T}+\frac{\gamma}{T}\left(E_s+E_pRt_1+E_wP_cR\zeta e^{-\lambda\beta}t_1\right)+\\ &\frac{1}{T}\left(L+MR+\left(\frac{N}{R}\right)\right)Rt_1+\frac{R\left(1-e^{-\lambda\beta}\right)-\left(A^m(c-dp)+a\alpha\right)}{\theta}\left(t_1+\frac{e^{-\theta t_1}}{\theta}-\frac{1}{\theta}\right)\\ &\left(\frac{h}{T}+\frac{E_h\gamma}{T}\right)+\left(A^m(c-dp)+a\alpha\right)\left(\frac{e^{\theta(T-t_1)}}{\theta}-\frac{1}{\theta}-T+t_1\right)\\ &\left(\frac{h}{\theta T}+\frac{h\theta_0}{T}+\frac{E_h\gamma}{\theta T}+\frac{E_d\gamma}{T}\right)\end{aligned} \tag{7}$$

5 Solution Procedure

It is clear that Eq. (7) is a non-linear function of T, t_1 and R. Therefore, the following procedure is applied for finding the optimal solutions:

(i) At first, all the parametric values are entered in Eq. (7).

(ii) Then, critical points are found by deriving Eq. (7) in respect of T, t_1 and R, and equating them to zero.

$$\frac{\partial TC}{\partial t_1}=0,\ \ \frac{\partial TC}{\partial T}=0,\ \ \frac{\partial TC}{\partial R}=0 \tag{8}$$

Now, the values of T, t_1 and R are obtained by solving Eq. (8). The partial derivatives are provided as

$$\begin{aligned}\frac{\partial TC}{\partial t_1}=&\frac{\gamma}{T}\left(E_pR+E_wP_cR\zeta e^{-\lambda\beta}\right)+\left(LR+MR^2+N\right)+\frac{R\left(1-e^{-\lambda\beta}\right)-\left(A^m(c-dp)+a\alpha\right)}{\theta}\\ &\left(1-\frac{e^{-\theta t_1}}{\theta^2}\right)\left(\frac{h}{T}+\frac{E_h\gamma}{T}\right)+\left(A^m(c-dp)+a\alpha\right)\left(-\frac{e^{\theta(T-t_1)}}{\theta^2}+1\right)\\ &\left(\frac{h}{\theta T}+\frac{h\theta_0}{T}+\frac{E_h\gamma}{\theta T}+\frac{E_d\gamma}{T}\right)=0\end{aligned}$$

$$\begin{aligned}\frac{\partial TC}{\partial T}=&-\frac{AY}{T^2}-\frac{VQ_c}{T^2}-\frac{\gamma}{T^2}\left(E_s+E_pRt_1+E_wP_cR\zeta e^{-\lambda\beta}t_1\right)\\ &-\frac{R\left(1-e^{-\lambda\beta}\right)-D(p\cdot A,\alpha)}{\theta}\left(t_1+\frac{e^{-\theta t_1}}{\theta}-\frac{1}{\theta}\right)\left(\frac{h}{T^2}+\frac{E_h\gamma}{T^2}\right)\\ &+\left(A^m(c-dp)+a\alpha\right)\left(\frac{e^{\theta(T-t_1)}}{\theta^2}-1\right)\left(\frac{h}{\theta T}+\frac{h\theta_0}{T}+\frac{E_h\gamma}{\theta T}+\frac{E_d\gamma}{T}\right)\\ &-\left(A^m(c-dp)+a\alpha\right)\left(-\frac{1}{\theta}+\frac{e^{\theta(T-t_1)}}{\theta}-T+t_1\right)\left(\frac{h}{\theta T^2}+\frac{h\theta_0}{T^2}+\frac{E_h\gamma}{\theta T^2}+\frac{E_d\gamma}{T^2}\right)\\ &=0\end{aligned}$$

$$\frac{\partial TC}{\partial R}=\frac{\gamma}{T}E_wP_c\zeta e^{-\lambda}t_1+(L+2MR)t_1+\frac{\left(1-e^{-\lambda\beta}\right)}{\theta}\left(t_1+\frac{e^{-\theta t_1}}{\theta}-\frac{1}{\theta}\right)\left(\frac{h}{T}+\frac{E_h\gamma}{T}\right)=0$$

(iii) Hence, to test whether the objective function is convex or not, the following Hessian matrix is derived

$$H=\begin{pmatrix}\frac{\partial^2TC}{\partial t_1^2} & \frac{\partial^2TC}{\partial t_1\partial T} & \frac{\partial^2TC}{\partial t_1\partial R}\\ \frac{\partial^2TC}{\partial T\partial t_1} & \frac{\partial^2TC}{\partial T^2} & \frac{\partial^2TC}{\partial T\partial R}\\ \frac{\partial^2TC}{\partial R\partial t_1} & \frac{\partial^2TC}{\partial R\partial T} & \frac{\partial^2TC}{\partial R^2}\end{pmatrix}$$

where:

$$M_{11}=\left(\frac{\partial^2TC}{\partial t_1^2}\right)_{(T,t_1,R)}$$

$$M_{22} = \begin{pmatrix} \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial TC}{\partial t_1 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2} \end{pmatrix}_{(T,t_1,R)}$$

$$M_{33} = \begin{pmatrix} \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial T} & \frac{\partial^2 TC}{\partial t_1 \partial R} \\ \frac{\partial^2 TC}{\partial T \partial t_1} & \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T \partial R} \\ \frac{\partial^2 TC}{\partial R \partial t_1} & \frac{\partial^2 TC}{\partial R \partial T} & \frac{\partial^2 TC}{\partial R^2} \end{pmatrix}_{(T,t_1,R)}$$

The values of the derivatives are provided as

$$\begin{aligned} \frac{\partial^2 TC}{\partial t_1^2} =& \frac{R\left(1-e^{-\lambda\beta}\right)-\left(A^m(c-dp)+a\alpha\right)}{\theta^2}\left(\frac{h}{T}+\frac{E_h\gamma}{T}\right)\frac{e^{-\theta t}}{\theta^3}+\left(A^m(c-dp)+a\alpha\right) \\ & \frac{e^{\theta(T-t_1)}}{\theta^3}\left(\frac{h}{\theta T}+\frac{h\theta_0}{T}+\frac{E_h\gamma}{\theta T}+\frac{E_d\gamma}{T}\right) \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 TC}{\partial T \partial t_1} =& \frac{R\left(1-e^{-\lambda\beta}\right)-\left(A^m(c-dp)+a\alpha\right)}{\theta}\left(1-\frac{e^{-\theta t_1}}{\theta^2}\right)\left(-\frac{h}{T^2}-\frac{E_h\gamma}{T^2}\right)+ \\ & \left(A^m(c-dp)+a\alpha\right)\frac{e^{\theta(T-t_1)}}{\theta^3}\left(\frac{h}{\theta T^2}+\frac{h\theta_0}{T^2}+\frac{E_h\gamma}{\theta T^2}+\frac{E_d\gamma}{T^2}\right) \end{aligned}$$

$$\frac{\partial^2 TC}{\partial R \partial t_1} = \frac{\gamma}{T}E_w P_c \zeta e^{-\lambda\beta}+L+2MR+\frac{\left(1-e^{-\lambda\beta}\right)}{\theta}\left(1-\frac{e^{-\theta t_1}}{\theta^2}\right)\left(\frac{h}{T}+\frac{E_h\gamma}{T}\right)$$

$$\begin{aligned} \frac{\partial^2 TC}{\partial T^2} =& \frac{2AY}{T^3}+\frac{2VQ_c}{T^3}+\frac{2\gamma}{T^3}\left(E_s+E_pRt_1+E_wP_cR\zeta e^{-\lambda\beta}t_1\right) \\ & + \frac{R\left(1-e^{-\lambda\beta}\right)-\left(A^m(c-dp)+a\alpha\right)}{\theta}\left(t_1+\frac{e^{-\theta t_1}}{\theta}-\frac{1}{\theta}\right)\left(\frac{2h}{T^3}+\frac{2E_h\gamma}{T^3}\right) \\ & -\left(A^m(c-dp)+a\alpha\right)\left(\frac{h}{\theta T^2}+\frac{h\theta_0}{T^2}+\frac{E_h\gamma}{\theta T^2}+\frac{E_d\gamma}{T^2}\right)\left(\frac{e^{\theta(T-t_1)}}{\theta^2}-1\right) \\ & +\left(A^m(c-dp)+a\alpha\right)\frac{e^{\theta(T-t_1)}}{\theta^3}\left(\frac{h}{\theta T}+\frac{h\theta_0}{T^2}+\frac{E_h\gamma}{\theta T}+\frac{E_d\gamma}{T}\right) \\ & -\left(A^m(c-dp)+a\alpha\right)\left(\frac{e^{\theta(T-t_1)}}{\theta^2}-1\right)\left(\frac{h}{\theta T^2}+\frac{h\theta_0}{T^2}+\frac{E_h\gamma}{\theta T^2}+\frac{E_d\gamma}{T^2}\right) \\ & +2\left(A^m(c-dp)+a\alpha\right)\left(-\frac{1}{\theta}-\frac{e^{\theta(T-t_1)}}{\theta}-T+t_1\right)\left(\frac{h}{\theta T^3}+\frac{h\theta_0}{T^3}+\frac{E_h\gamma}{\theta T^3}+\frac{E_d\gamma}{T^3}\right) \end{aligned}$$

$$\frac{\partial^2 TC}{\partial R \partial T} = -\frac{\gamma}{T^2}E_wP_c\zeta e^{-\lambda\beta}t_1-\frac{\left(1-e^{-\lambda\beta}\right)}{\theta}\left(t_1+\frac{e^{-\theta t_1}}{\theta}-\frac{1}{\theta}\right)\left(\frac{h}{T^2}+\frac{E_h\gamma}{T^2}\right)$$

$$\begin{aligned} \frac{\partial^2 TC}{\partial t_1 \partial T} =& -\frac{\gamma}{T^2}\left(E_pR+E_wP_cRAe^{-\lambda\beta}\right)+\frac{RW\left(\beta-\left(A^m(c-dp)+a\alpha\right)\right)}{\theta}\left(1-\frac{e^{-\theta t_1}}{\theta^2}\right) \\ & \left(-\frac{h}{T^2}-\frac{E_h\gamma}{T^2}\right)-\left(A^m(c-dp)+a\alpha\right)\frac{e^{\theta(T-t_1)}}{\theta^3}\left(\frac{h}{\theta T}+\frac{h\theta_0}{T}+\frac{E_h\gamma}{\theta T}+\frac{E_h\gamma}{T}\right) \\ & -\left(A^m(c-dp)+a\alpha\right)\left(1-\frac{e^{\theta(T-t_1)}}{\theta^2}\right)\left(\frac{h}{\theta T^2}+\frac{h\theta_0}{T^2}+\frac{E_h\gamma}{\theta T^2}+\frac{E_d\gamma}{T^2}\right) \end{aligned}$$

$$\frac{\partial^2 TC}{\partial R^2} = 2Mt_1$$

$$\frac{\partial^2 TC}{\partial t_1 \partial R} = \frac{\gamma}{T}E_wP_c\zeta e^{-\lambda\beta}+L+2MR+\frac{(1-e^{-\lambda\beta})}{\theta}\left(1-\frac{e^{-\theta t_1}}{\theta^2}\right)$$

$$\frac{\partial^2 TC}{\partial T \partial R} = -\frac{\gamma}{T^2}E_wP_c\zeta e^{-\lambda\beta t_1}-\frac{(1-e^{-\lambda\beta})}{\theta}\left(t_1+\frac{e^{-\theta t_1}}{\theta}-\frac{1}{\theta}\right)\left(\frac{h}{T^2}+\frac{E_h\gamma}{T^2}\right)$$

If $M_{11} > 0$, $M_{22} > 0$ and $M_{33} > 0$, it can be concluded that H is positive definite and the objective function TC is minimum at (T, t_1, R).

(iv) Minimum (T, t_1, R) is the extreme value of the objective function and is denoted by (T^*, t_1^*, R^*).

6 Numerical Results

Our team members visited a supermarket situated at Raiganj in India on 30 February 2025 for data collection. Following an extended conversation with the manager, we learned that the products created through 3D technology play an important role in our daily lives. Through discussions with them, we have learned about various processes and policies that are related to our proposed model. The data that we have obtained from the supermarket is provided in Table 2.

Table 2. Numeric value of all parameters

Parameter	Value	Parameter	Value
D	15.26	ζ	0.001
p	5$	θ_0	0.001($/unit/unit time)
α	0.3	A	11
c	80	E_p	0.02 (kg/unit)
m	0.5	a	40
v	0.01	β	1
λ	0.31	θ	0.001
S	400$	h	0.3
γ	20($/Kg)	Y	2
Q_c	4	E_s	0.01 (kg/unit)
E_h	0.02 (kg/unit)	E_w	0.01 (kg/unit)
E_d	0.03 (kg/unit)	L	70
M	15	N	0.7

Optimal results are obtained as: Average cost (TC) = 517036.5$, inventory cycle length (T) = 98.39 , time at which production stops (t_1) = 65.59, stock level (Q) = 809.29, and production rate (R) = 48.54.

7 Sensitivity Analysis

A sensitivity analysis of the model for key parameters and its significance is enclosed in this section. The parametric values are changed by varying the parametric values as -20% to $+20\%$. The impact of the parameters has been observed in Table 3 and Table 4.

The following are the observations derived from Table 3 and Table 4:

- If the parametric value A increases or decreases, then the value of the parameters t_1, T, R, Q and TC increases or decreases, respectively.
- If the parametric value α increases, then the parametric values of R, Q and TC increase. The values will decrease with decreasing of α.
- When the parametric value m increases, then the parameters R, Q and TC increases. The values will decrease with decreasing of m.
- If the parametric value β increases, then the parameter Q, TC increases slightly, and vice versa.
- When the parametric value λ increases, then, Q increase, TC lightly and conversely.
- If the parametric value θ increases, then the parameter Q increases and TC lightly increases, and vice versa.
- If the parametric value S increases, then TC light increases. But, Q is totally unchanged, and conversely.
- If the parameter ζ increases, then TC lightly increase. But there is no change in Q and vice versa.
- If the parameter E_s increases, then Q lightly increases, and there is no change in total cost, TC and vice versa.
- If the parameter E_p increases, then, TC is slightly increased but Q is totally unchanged, and conversely.
- If the parameter γ increases, then Q increases with a slight increment, TC and conversely.

7.1 Managerial Insights

Based on sensitivity analysis, the following managerial insights are derived:

- Using advertising frequency will attract a large number of customers to the product. It will increase the number of potential consumers and gain a good reputation for the product. This will sustain the product's demand in the market.
- According to the study, the rate of carbon emission significantly affects the cost. Hence, reducing carbon emissions is one of the key factors in reducing total product costs. This study suggests that businesses should avoid holding inventory for long periods, which is also effective in reducing product deterioration.
- 3D printing, along with other allied manufacturing processes, has an exclusive importance in controlling material usage and preventing waste.

- The circular economy has a major power in protecting the environment and also influences the demand for products in the market.

Table 3. Sensitivity analysis for several inventory parameters

Parameter	% Change	t_1	T	R	Q	Z
A	+20	68.67	103.13	50.88	888.70	565546.0
	+10	66.67	100.21	49.74	842.99	541532.1
	-10	65.33	98.02	47.28	785.05	491732.6
	-20	63.33	95.37	45.95	739.21	465828.7
α	+20	65.33	98.23	53.34	885.65	619640.8
	+10	68.67	103.09	50.94	889.71	567137.7
	-10	64.67	97.42	46.14	758.17	469096.7
	-20	70.07	105.18	43.74	779.06	423559.0
m	+20	66.05	99.03	55.19	925.89	660595.1
	+10	70.05	105.02	51.67	920.22	582289.0
	-10	67.33	101.11	45.77	783.58	462252.8
	-20	67.67	103.17	43.31	756.40	416284.3
β	+20	64.07	96.07	48.51	789.25	517042.8
	+10	67.33	101.02	48.54	831.05	517039.7
	-10	56.96	85.44	48.54	701.21	517033.2
	-20	50.83	76.25	48.54	624.80	517029.8
λ	+20	67.67	103.31	48.54	847.79	517042.8
	+10	70.08	105.03	48.54	864.54	517039.7
	-10	56.96	85.44	48.54	701.22	517033.2
	-20	50.83	76.25	48.54	624.80	517029.8
θ	+20	59.84	89.76	48.54	739.36	517036.6
	+10	62.52	93.78	48.54	771.93	517036.6
	-10	69.17	103.76	48.54	852.70	517043.5
	-20	73.40	110.10	48.54	904.01	517043.5
S	+20	65.60	98.40	48.54	809.29	517116.5
	+10	65.60	98.40	48.54	809.29	517076.5
	-10	65.60	98.40	48.54	809.29	516996.5
	-20	65.60	98.39	48.54	809.29	516956.5
ζ	+20	65.60	98.40	48.54	809.29	517037.3
	+10	65.60	98.40	48.54	809.29	517036.9
	-10	65.60	98.40	48.54	809.29	517036.1
	-20	65.60	98.40	48.54	809.29	517035.8

Table 4. Sensitivity analysis for several carbon emission-related parameters

Parameter	% Change	t_1	T	R	Q	Z
E_s	+20	65.66	98.49	48.54	810.02	517036.5
	+10	65.63	98.44	48.54	809.66	517035.5
	-10	65.57	98.35	48.54	808.92	517036.5
	-20	65.54	98.31	48.54	810.03	517036.5
E_p	+20	65.60	98.40	48.54	809.29	517039.1
	+10	65.60	98.40	48.54	809.29	517037.8
	-10	65.60	98.40	48.54	809.29	517035.2
	-20	65.60	98.40	48.54	809.29	517033.9
γ	+20	56.83	85.25	48.54	696.57	620366.3
	+10	60.89	91.33	48.54	750.29	568701.3
	-10	71.14	106.71	48.54	878.88	465371.9
	-20	77.77	116.64	48.54	962.32	413707.6

8 Graphical Illustration

To better comprehend the result, several graphs are drawn using the numerical results seen in Table 3 and Table 4.

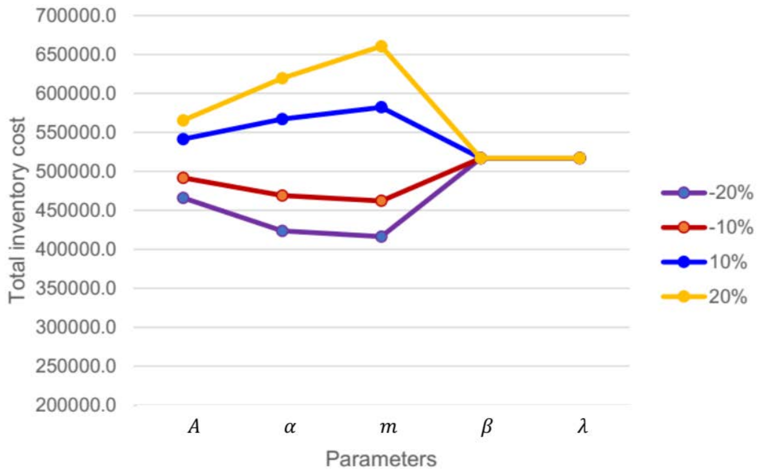

Figure 2. Sensivity analyis of A, α, m and β

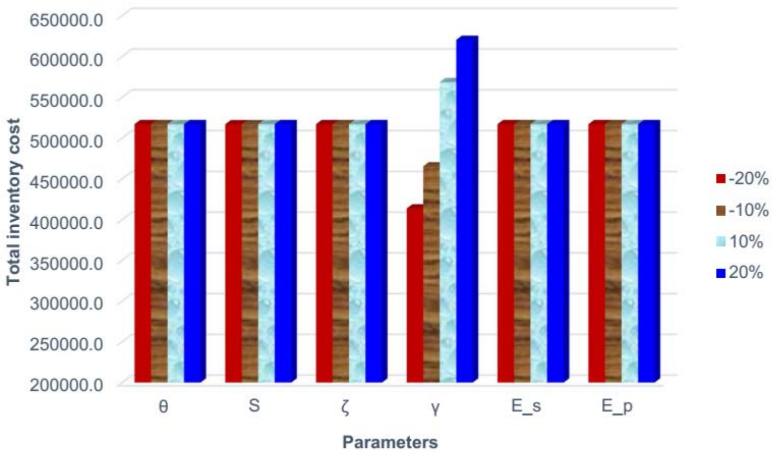

Figure 3. Sensitivity of θ, S, ζ,γ,E_s and E_p

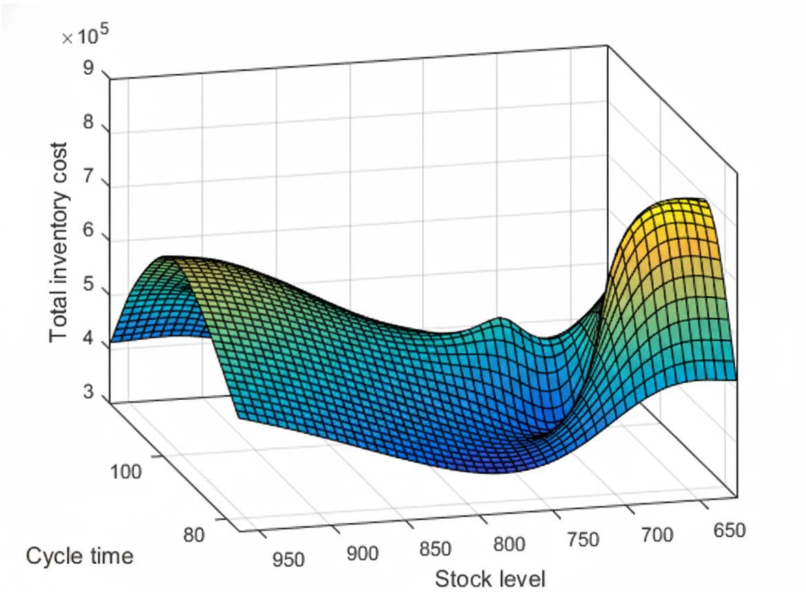

Figure 4. Change of average inventory cost in respect of cycle time and stock level

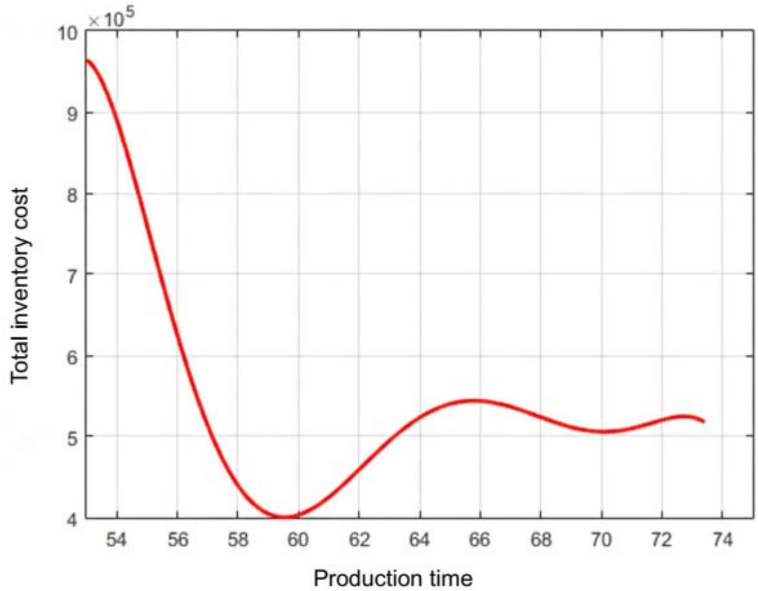

Figure 5. Change of average inventory cost with respect to parameter production time

Figure 2 shows the variation of average inventory cost for changing the parameters from -20% to +20%. This graph clearly indicates that the number of advertisement frequencies A, circularity index α, and advertisement frequency m are highly sensitive parameters.

Figure 3 demonstrates that the deterioration rate θ, setup cost S, waste material production rate ζ, E_s and E_p are insensitive within the changing period of -20% to +20%. Furthermore, advertisement cost per frequency Y is a highly sensitive parameter.

Figure 4 shows clearly how cycle time, stock level, and average inventory cost are related. This surface-like curve indicates that the average cost is minimum when the cycle time approaches 95 units and the stock level approaches 809 units.

Figure 5 indicates the variation of the average cost in respect of changes in production time. This figure shows that when production time gets a value near about 60 units, average inventory cost achieves a minimum value.

9 Conclusions

The primary goal of this study is to find a model with price-advertisement demand, circular economy, carbon emissions, waste management, and production through 3D printing techniques. This model follows a smart production strategy and plays an important role in preventing environmental waste. Here, a mathematical model is developed to evaluate the critical values. Graphical representation is given to support the mathematical model and sensitivity analysis. In reality, data are collected by visiting the market and by highlighting the values in the study. We have tried to reduce the rate of carbon emissions as much as possible due to its negative impact on society. Carbon emissions are shown to be very important so that excess carbon on products can reduce environmental pollution. Here, we have specifically used circular economy for calculating advertising-based demand factors.

This research provides a foundational framework for integrating circular economy principles with sustainable production practices using 3D printing technologies. However, several avenues remain open for further investigation and development. Future work can focus on applying the proposed model to real-world manufacturing systems across various industries, such as automotive, aerospace, and biomedical. This would help validate the model's applicability and uncover practical implementation challenges. Expanding the model to incorporate multi-objective optimization could offer a more holistic decision-making approach. Incorporating dynamic or stochastic demand into the model could better reflect real-life uncertainties. Further research could include a deeper exploration into material flow within circular loops, especially focusing on closed-loop supply chains, remanufacturing potential, and life-cycle assessment of 3D-printed components.

Author Contributions

Conceptualization, D.K.R., S.M., and M.P.; methodology, S.B.M., S.S., and S.P.M.; software, D.K.R., and S.M.; validation, D.K.R., S.P.M., and M.P.; formal analysis, D.K.R., S.S., and S.B.M; investigation, S.B.M., S.P.M., and S.S.; resources, S.B.M. and M.P.; data curation, D.K.R., M.P., and S.P.M.; writing—original draft preparation, S.B.M. and S.M.; writing—revew and editing, M.P., S.P.M., and S.M.; visualization, D.K.R., S.B.M., and S.S.; supervision, S.P.M. and M.P.; project administration, D.K.R., S.M., and S.B.M. All authors have read and agreed to the published version of the manuscript.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

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