Journal of Industrial Intelligence
https://www.acadlore.com/journals/JII

Navigating Complexity: A Multidimensional Neutrosophic Fuzzy Hypersoft Approach to Empowering Decision-Makers

Muhammad Saeed*, Fatima Razaq, Imtiaz Tariq, Irfan Saif Ud Din

Department of Mathematics, University of Management and Technology, 54000 Lahore, Pakistan

* Correspondence: Muhammad Saeed (muhammad.saeed@umt.edu.pk)

Received: 11-23-2023 **Revised:** 12-20-2023 **Accepted:** 12-26-2023

Citation: M. Saeed, F. Razaq, I. Tariq, and I. Saif Ud Din, "Navigating complexity: A multidimensional neutrosophic fuzzy hypersoft approach to empowering decision-makers," *J. Ind Intell.*, vol. 1, no. 4, pp. 219–228, 2023. https://doi.org/10.56578/jii010403.

Abstract: Urban transportation systems, characterized by inherent uncertainty and ambiguity, present a formidable challenge in decision-making due to their complex interplay of factors. This complexity arises from dynamically shifting commuter behaviors, a diverse array of transit options, and variable traffic patterns. Such unpredictability hinders the formulation and implementation of effective strategies. Addressing this challenge necessitates innovative problem-solving methodologies capable of handling the nuanced uncertainties present in these systems. This study introduces the multidimensional neutrosophic fuzzy hypersoft set (MDNFHS) as a groundbreaking method for managing ambiguity in urban transportation planning. MDNFHS, emerging from the integration of neutrosophic fuzzy sets (NFSs) and hypersoft sets (HSs), uniquely encapsulates both the degrees of membership and non-membership. It is demonstrated that the tailored set-theoretic operations and distance measurements specific to MDNFHS enable enhanced manipulation and analysis, making it a potent tool in complex decision-making scenarios. The efficacy of MDNFHS in decision-making is exemplified through a compelling case study, showcasing its ability to offer clarity in situations marred by ambiguity. This novel approach is posited to revolutionize decision-making processes, offering a new level of certainty in environments traditionally dominated by uncertain elements.

Keywords: Ambiguity; Decision-making; Hypersoft set theory; Multidimensional attributes optimization; Uncertainty

1 Introduction

The burgeoning challenges in urban mobility within large metropolises are exacerbated by escalating transportation issues. These challenges stem from rapid urbanization and population growth, manifesting in inadequate public transport systems, escalating traffic congestion, and heightened environmental concerns. The proliferation of vehicles exacerbates commute times, intensifies pollution, and elevates stress levels among residents. Furthermore, suboptimal transportation infrastructure hinders economic growth and adversely impacts the quality of life in urban settings. Addressing these multifaceted issues necessitates innovative and sustainable approaches, ranging from enhanced public transportation systems to the promotion of eco-friendly mobility alternatives such as walking and cycling. The complexity of transportation dilemmas in large cities demands progressive policies and comprehensive urban planning to forge more efficient, accessible, and environmentally responsible transportation systems. Neutrosophic sets (NSs), conceptualized by Smarandache [1], address complex decision-making problems characterized by ambiguities, often accompanied by incompatible and unreliable knowledge. These sets' capacity to express precision, indeterminacy, and contradictions through three association functions in the evaluation of alternatives in multi-criteria decision-making (MCDM) is particularly noteworthy. The concept of NSs finds application across a diverse array of fields, including computational sciences, physical sciences, medical sciences, social sciences, technology, and multi-criteria group decision-making challenges.

Das et al. [2] explored the quadripartitioned neutrosophic closure and the quadripartitioned neutrosophic interior operator of Q-NSs within a quadripartitioned neutrosophic topological space (Q-NTS). Their study delved into various properties of these elements in the context of quadripartitioned NSs (Q-NSs). Karabašević et al. [3] introduced an innovative extension of the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method, applying single-valued NSs in conjunction with Hamming distance for the selection of E-commerce development strategies. This development represents a notable contribution to the field of MCDM under neutrosophic conditions.

The work of Abdel-Basset et al. [4] is particularly noteworthy. They proposed a novel framework integrating computer-assisted examination with the Internet of Things (IoT) for monitoring and diagnosing heart failure patients. Additionally, they developed a neutrosophic MCDM methodology, offering substantial support for patients' care decisions. Said et al. [5] explored various computational methods for traffic flow analysis, employing rough set, fuzzy rough set, and their extensions within the NS framework. This exploration has opened new avenues for understanding and managing traffic systems.

Nafei et al. [6] contributed a modified score function for ranking single-valued neutrosophic numbers. They applied this novel function in a TOPSIS framework, addressing complex hotel location selection issues. In the healthcare sector, Broumi et al. [7] proposed an innovative concept for n-valued interval NSs and their application in healthcare evaluation. Maji [8] introduced the concept of a neutrosophic soft set, along with various operations applicable to these sets. Following this, Karaaslan [9] revisited the conceptual foundations and operational methods of neutrosophic soft sets. His work also encompassed decision-making processes, both individually and in group settings. This area of study has seen a surge in research, particularly focusing on the properties of neutrosophic soft sets and their application in decision-making challenges. For instance, Broumi [10] defined the concept of a generalized neutrosophic soft set. Building on this, Broumi et al. [11] proposed a decision-making strategy based on these generalized neutrosophic soft sets, further expanding the field's scope and applicability. The exploration of neutrosophic soft sets and their applications has seen notable contributions. Sahin and Kücük [12] introduced the concept of generalized neutrosophic soft sets, along with operations on these sets. Their work also encompassed decision-making and similarity measurement methods within a soft neutrosophic setting. Deli [13] defined interval-valued neutrosophic soft sets and elucidated the operations applicable to them. This definition expanded the theoretical framework of NSs. Following this, Deli and Broumi [14] investigated neutrosophic soft matrices, proposing a novel approach to decision-making utilizing these matrices.

A critical distinction has been observed between the practical applications of fuzzy sets (FSs) and NSs. FS primarily handles undetermined and incompatible knowledge, whereas NS is adept at managing incomplete and vague information. Fuzzy set theory has demonstrated efficacy in managing uncertainty, while NS theory excels in handling undetermined and inconsistent data. However, scenarios presenting both uncertain and inconsistent information necessitate a hybrid approach, combining FS and NS. This requirement led to the extension of FS in the context of NS. Hashim et al. [15] conducted research on neutrosophic bipolar FSs and their applications in pharmaceutical formulations. Das et al. [16] developed the concept of NFS by integrating FS and NS. They also explored single valued NFS (SVNFS) and its applications in decision-making, further broadening the scope of neutrosophic applications. Mehmood et al. [17] proposed hybrids of HS and complex FS, along with their generalization frameworks, namely complex intuitionistic FS and complex NS. They provided illustrative examples to demonstrate these concepts. Further researchs [18–21] showcased the application of NSs in various directions.

The amalgamation of FS theory and neutrosophic logic culminates in the creation of NFS, a paradigm shift that captures the essence of uncertainty in decision-making processes. NFS's ability to address both uncertainty and inconsistency revolutionizes decision-making across various fields. These sets, embodying membership grades of truth, indeterminacy, and falsity, offer a versatile framework for navigating the complexities of real-world scenarios. The potential of NFS has captivated researchers, allowing for an in-depth exploration of uncertainty and the derivation of meaningful insights [22]. Embracing the unknown becomes feasible with NFS, facilitating informed and improved decision-making. In the realm of NFS, Khalil et al. [22] proposed the concept of the single-valued neutrosophic fuzzy soft set, blending NFS with the soft set theory. This concept presents an innovative approach in the field. Smarandache's introduction of the HS in 2018 [23] marked a significant advancement. HS is characterized as a mapping from a given set of attributes and the power set of the universal set to their Cartesian product, further subdivided into attributes. Subsequent extensions, such as fuzzy hypersoft sets (FHSs), intuitionistic hypersoft sets (IHSs), and neutrosophic hypersoft sets (NHSs), were developed to accommodate varying degrees of truth, uncertainty, and indeterminacy [24]. In the NHS domain, the concepts of single-valued neutrosophic hypersoft sets (SVNHSs) [25] and various aggregate operators were introduced [26, 27]. The application of various newly proposed MCDM techniques has been explored in numerous decision-making scenarios [28, 29]. Additionally, the representation of complex and parametrized HSs has been proposed [30, 31], further expanding the scope of these mathematical tools in practical applications. In the exploration of MCDM methodologies, neutrosophic and hypersoft set strucutres have been effectively employed, particularly in the group decision making and multi-objective decision making problems [32–36]. Additionally, the introduction of W-structures of HSs [37] marked a significant advancement in the representation of complex data. However, a comprehensive review of the literature reveals a gap in considering the multidimensionality of attributes in these studies, a gap this research aims to address. The urban transportation dilemma, characterized by its multifaceted and unpredictable nature, serves as a prime example of this oversight. Factors such as fluctuating commuter behaviors, diverse transit options, and variable traffic patterns contribute to the complexity of urban transportation networks, thereby amplifying uncertainty and ambiguity in decision-making processes [38].

This paper is organized as follows: Section 1, the Introduction, underscores the limitations of traditional approaches in handling complex decision-making scenarios and emphasizes the pivotal role of MDNFHS in this context. Section 2 presents the preliminaries, laying the groundwork for the subsequent analysis. Section 3 details the proposed distance and similarity measures utilizing MDNFHS. In Section 4, a case study focusing on the transportation issues in Lahore is examined, followed by a discussion of the results. The paper concludes in Section 5, which outlines future research directions in this field.

2 Methodology

In this section, a few fundamental concepts are recalled from previously published literature to justify the proposed study. In this work, $\breve{\mathcal{U}}$ stands for the realm of discourse and $\mathfrak{T}$ for the closed unit interval.

Definition 1 [24]**:** The pair $(\mathfrak{w}, \mathfrak{H})$ is called a hypersoft set over $\breve{\mathcal{U}}$, where $\mathfrak{H}$ is the cartesian product of n disjoint sets $\mathfrak{H}_1, \mathfrak{H}_2, \mathfrak{H}_3, \ldots, \mathfrak{H}_p$ having attribute values of p distinct attributes $\mathfrak{H}^1, \mathfrak{H}^2, \mathfrak{H}^3, \ldots, \mathfrak{H}^p$ respectively and $\mathfrak{w} : \mathfrak{H} \to \mathfrak{p}(\breve{\mathcal{U}})$.

Definition 2 [26]**:** A single-valued neutrosophic hypersoft set (SV-NHSS) over $\breve{\mathcal{U}}$, indicated by $(\mathrm{NHSS})^{\breve{\mathcal{U}}\widehat{\mathbb{W}}}$, in $(\mathrm{HS})^{\breve{\mathcal{U}}\widehat{\mathbb{W}}}$. The pair $\left(\mathfrak{X}_{\mathrm{HS}}, \widehat{\mathcal{W}}\right)$ is NHSS if

$$\mathfrak{X}_{(\widehat{\mathbb{W}})} : \widehat{\mathcal{W}} \to \mathrm{NHSS}(\breve{\mathcal{U}})$$

defined by

$$\mathfrak{X}_{(\widehat{\mathbb{W}})} = \left\{ \begin{array}{c} \dfrac{\left(\mathfrak{P}_{\mathfrak{X}_{(\widehat{\mathbb{W}})}}(\hat{\mathrm{v}}), \mathfrak{Q}_{\mathfrak{X}_{(\widehat{\mathbb{W}})}}(\hat{\mathrm{v}}), \mathfrak{R}_{\mathfrak{X}_{(\widehat{\mathbb{W}})}}(\hat{\mathrm{v}}) : \mathfrak{F}(\hat{\mathrm{v}})\right)}{\hat{\mathrm{v}}}; \\ \widehat{\mathbb{W}} \in \widehat{\mathcal{W}}, \hat{\mathrm{v}} \in \breve{\mathcal{U}} \\ 0 \leq \mathfrak{P}_{\mathfrak{X}_{(\widehat{\mathbb{W}})}} + (\hat{\mathrm{v}}), \mathfrak{Q}_{\mathfrak{X}(\widehat{\mathbb{W}})}(\hat{\mathrm{v}}) + \mathfrak{R}_{\mathfrak{X}_{(\widehat{\mathbb{W}})}}(\hat{\mathrm{v}}) \leq 3 \end{array} \right\}$$

where:

i. For l distinct sets of attribute values, $\widehat{\mathcal{W}} = \mathbb{E}_1 \times \mathbb{E}_2 \times \mathbb{E}_3 \times \ldots \times \mathbb{E}_l$ There are l unique properties in the set $\mathbb{E}$, with corresponding values $\mathbb{E}_{\mathrm{k}}, k = 1, 2, 3, \ldots, l$.

ii. The set of all single-valued neutrosophic fuzzy subsets over $\breve{\mathcal{U}}$ is denoted by the notation NHSS $(\breve{\mathcal{U}})$.

iii. $\mathfrak{P}_{\mathfrak{X}_{(\widehat{\mathcal{W}})}}(\hat{\mathrm{v}}), \mathfrak{Q}_{\mathfrak{X}(\widehat{\omega})}(\hat{\mathrm{v}}), \mathfrak{R}_{\mathfrak{X}(\widehat{\omega})}(\hat{\mathrm{v}})$ and $\mathfrak{F}(\hat{\mathrm{v}})$ are all discrete numbers that lie in the range $[0, 1]$.

iv. $(\mathrm{HS})^{\breve{\mathcal{U}}\widehat{\mathbb{W}}}$ is a hypersoft universe.

Example

Suppose $\breve{\mathcal{U}} = \left\{\breve{\mathfrak{d}}_1, \breve{\mathfrak{d}}_2, \breve{\mathfrak{d}}_3, \breve{\mathfrak{d}}_4\right\}$ be collection available dishes and $\mathfrak{E} = \{\breve{\mathfrak{e}}_1, \breve{\mathfrak{e}}_2, \breve{\mathfrak{e}}_3, \breve{\mathfrak{e}}_4\}$ the set of attributes, where $\breve{\mathfrak{e}}_1$ represents cuisine, $\breve{\mathfrak{e}}_2$ represents spiciness, $\breve{\mathfrak{e}}_3$ represents dietary restriction compatibility of the dishes, and $\breve{\mathfrak{e}}_4$ represents price range of dishes. Their corresponding attribute-valued sets are $\breve{\mathfrak{e}}_1 = \{j_{11} = \text{Italian}, j_{12} = \text{Chinese}, j_{13} = \text{Mexican}, j_{14} = \text{Indian}\}$, $\breve{\mathfrak{e}}_2 = \{j_{21} = \text{Mild}, j_{22} = \text{Medium}, j_{23} = \text{Hot}\}$, $\breve{\mathfrak{e}}_3 = \{j_{31} = \text{Vegetarian}, j_{32} = \text{Vegan}, j_{33} = \text{Gluten free}\}$ and $\breve{\mathfrak{e}}_4 = \{j_{41} = \text{Low}, j_{42} = \text{Medium}, j_{43} = \text{High}\}$.

Now, $\breve{\mathcal{W}} = \breve{\mathfrak{e}}_1 \times \breve{\mathfrak{e}}_2 \times \breve{\mathfrak{e}}_3 \times \breve{\mathfrak{e}}_4$,

$\breve{\mathcal{W}} = \{(\mathbb{W}_{11}, \mathbb{W}_{21}, \mathbb{W}_{31}, \mathbb{W}_{41}), (\mathbb{W}_{11}, \mathbb{W}_{22}, \mathbb{W}_{31}, \mathbb{W}_{42}), (\mathbb{W}_{12}, \mathbb{W}_{21}, \mathbb{W}_{31}, \mathbb{W}_{43}), (\mathbb{W}_{12}, \mathbb{W}_{21}, \mathbb{W}_{32}, \mathbb{W}_{41})\}$.

Or, $\breve{\mathcal{W}} = \left\{\widehat{\mathbb{W}}_1, \widehat{\mathbb{W}}_2, \widehat{\mathbb{W}}_3, \widehat{\mathbb{W}}_4\right\}$. Now,

$$\mathfrak{X}_{(\widehat{\mathbb{W}}_1)} = \left\{\frac{(0.9, 0.2, 0.8, 0.6)}{\breve{\mathfrak{D}}_1}, \frac{(0.6, 0.6, 0.7, 0.4)}{\breve{\mathfrak{D}}_2}, \frac{(0.8, 0.3, 0.5, 0.7)}{\breve{\mathfrak{D}}_3}, \frac{(0.3, 0.6, 0.8, 0.9)}{\breve{\mathfrak{D}}_4}\right\},$$
$$\mathfrak{X}_{(\widehat{\mathbb{W}}_2)} = \left\{\frac{(0.3, 0.5, 0.5, 0.4)}{\breve{\mathfrak{D}}_1}, \frac{(0.3, 0.4, 0.8, 0.4)}{\breve{\mathfrak{D}}_2}, \frac{(0.9, 0.4, 0.6, 0.5)}{\breve{\mathfrak{D}}_3}, \frac{(0.5, 0.1, 0.4, 0.8)}{\breve{\mathfrak{D}}_4}\right\},$$
$$\mathfrak{X}_{(\widehat{\mathbb{W}}_3)} = \left\{\frac{(0.5, 0.7, 0.7, 0.5)}{\breve{\mathfrak{D}}_1}, \frac{(0.6, 0.5, 0.2, 0.4)}{\breve{\mathfrak{D}}_2}, \frac{(0.2, 0.1, 0.5, 0.8)}{\breve{\mathfrak{D}}_3}, \frac{(0.4, 0.6, 0.2, 0.1)}{\breve{\mathfrak{D}}_4}\right\},$$
$$\mathfrak{X}_{(\widehat{\mathbb{W}}_4)} = \left\{\frac{(0.3, 0.8, 0.9, 0.7)}{\breve{\mathfrak{D}}_1}, \frac{(0.4, 0.8, 0.9, 0.1)}{\breve{\mathfrak{D}}_2}, \frac{(0.5, 0.2, 0.4, 0.9)}{\breve{\mathfrak{D}}_3}, \frac{(0.9, 0.2, 0.3, 0.4)}{\breve{\mathfrak{D}}_4}\right\},$$

Its matrix representation is as follows:

$$\mathfrak{X} = \begin{pmatrix} \breve{\mathcal{W}} & \breve{\mathfrak{D}}_1 & \breve{\mathfrak{D}}_2 & \breve{\mathfrak{D}}_3 & \breve{\mathfrak{D}}_4 \\ \widehat{\mathbb{W}}_1 & (0.9, 0.2, 0.8, 0.6) & (0.6, 0.6, 0.7, 0.4) & (0.8, 0.3, 0.5, 0.7) & (0.3, 0.6, 0.8, 0.9) \\ \widehat{\mathbb{W}}_2 & (0.3, 0.5, 0.5, 0.4) & (0.3, 0.4, 0.8, 0.4) & (0.9, 0.4, 0.6, 0.5) & (0.5, 0.1, 0.4, 0.8) \\ \widehat{\mathbb{W}}_3 & (0.5, 0.7, 0.7, 0.5) & (0.6, 0.5, 0.2, 0.4) & (0.2, 0.1, 0.5, 0.8) & (0.4, 0.6, 0.2, 0.1) \\ \widehat{\mathbb{W}}_4 & (0.3, 0.8, 0.9, 0.7) & (0.4, 0.8, 0.9, 0.1) & (0.5, 0.2, 0.4, 0.9) & (0.9, 0.2, 0.3, 0.4) \end{pmatrix}$$

3 Distance and Similarity Measure for MDNFHS

The application of distance and similarity measures in data analysis is pivotal, particularly in the context of MDNFHS. These measures provide a quantitative framework essential for discerning relationships and patterns within datasets, thereby facilitating decision-making across various domains. Their utility in MCDM, pattern recognition, and anomaly detection is particularly significant. Distance and similarity measures are instrumental in evaluating variances and identifying congruities, enabling decision-makers to derive well-informed conclusions. The ability to effectively handle multidimensional data underscores their value. These measures are integral to classification and clustering processes and are crucial for generating personalized recommendations in systems such as recommender algorithms. Furthermore, distance measures play a vital role in data preprocessing, standardizing data to ensure equitable comparisons. This standardization is critical for enhancing the accuracy and reliability of decision outcomes. In essence, distance and similarity measures equip decision-makers with critical insights, thereby augmenting the effectiveness and impartiality of decision-making processes across diverse scenarios.

Consider two MDNFHSs $\mathcal{G} = \left\{\left(\widehat{\mathbb{W}}_i, \mathfrak{P}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}), \mathfrak{Q}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}), \mathfrak{R}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}), \mathfrak{F}_{\mathcal{G}}(\breve{\mathbb{V}})\right)\right\}$ and $\mathcal{H} = \left\{\left(\widehat{\mathbb{W}}_i, \mathfrak{P}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}), \mathfrak{Q}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}), \mathfrak{R}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}), \mathfrak{F}_{\mathcal{H}}(\breve{\mathbb{V}})\right)\right\}$, $i = 1, 2, 3, \ldots, n$ are defined over the universe $\breve{\mathcal{U}} = \{\breve{\mathbb{V}}_1, \breve{\mathbb{V}}_2, \breve{\mathbb{V}}_3, \ldots, \breve{\mathbb{V}}_n\}$. Then following distances can be determined between $\mathcal{G}$ and $\mathcal{H}$.

3.1 Distance Measures

Hamming Distance: Suppose $\mathcal{G}$ and $\mathcal{H}$ are two MDNFHS then Hamming Distance as follows,

$$\mathfrak{D}_1(\mathcal{G}, \mathcal{H}) = \frac{1}{4} \sum_{\breve{\mathbb{V}} \in \breve{\mathcal{U}}} \left[\begin{array}{c} |\mathfrak{P}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{P}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})| + |\mathfrak{Q}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{Q}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})| \\ +|\mathfrak{R}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{R}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})| + |\mathfrak{F}_{\mathcal{G}}(\breve{\mathbb{V}}) - \mathfrak{F}_{\mathcal{H}}(\breve{\mathbb{V}})| \end{array} \right] \tag{1}$$

Normalized Hamming Distance: Suppose $\mathcal{G}$ and $\mathcal{H}$ are two MDNFHS then Normalized Hamming Distance as follows,

$$\mathfrak{D}_2(\mathcal{G}, \mathcal{H}) = \frac{1}{4n} \sum_{\breve{\mathbb{V}} \in \breve{\mathcal{U}}} \left[\begin{array}{c} |\mathfrak{P}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{P}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})| + |\mathfrak{Q}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{Q}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})| \\ +|\mathfrak{R}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{R}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})| + |\mathfrak{F}_{\mathcal{G}}(\breve{\mathbb{V}}) - \mathfrak{F}_{\mathcal{H}}(\breve{\mathbb{V}})| \end{array} \right] \tag{2}$$

Euclidean Distance: Suppose $\mathcal{G}$ and $\mathcal{H}$ are two MDNFHS then Euclidean Distance as follows,

$$\mathfrak{D}_3(\mathcal{G}, \mathcal{H}) = \left[\frac{1}{4} \sum_{\breve{\mathbb{V}} \in \breve{\mathcal{U}}} \left[\begin{array}{c} |\mathfrak{P}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{P}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})|^2 + |\mathfrak{Q}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{Q}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})|^2 \\ +|\mathfrak{R}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{R}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})|^2 + |\mathfrak{F}_{\mathcal{G}}(\breve{\mathbb{V}}) - \mathfrak{F}_{\mathcal{H}}(\breve{\mathbb{V}})|^2 \end{array} \right] \right]^{0.5} \tag{3}$$

Normalized Euclidean Distance: Suppose $\mathcal{G}$ and $\mathcal{H}$ are two MDNFHS then Normalized Euclidean Distance as follows,

$$\mathfrak{D}_4(\mathcal{G}, \mathcal{H}) = \left[\frac{1}{4n} \sum_{\breve{\mathbb{V}} \in \breve{\mathcal{U}}} \left[\begin{array}{c} |\mathfrak{P}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{P}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})|^2 + |\mathfrak{Q}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{Q}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})|^2 \\ +|\mathfrak{R}_{\mathcal{G}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}}) - \mathfrak{R}_{\mathcal{H}_{\widehat{\mathbb{W}}_i}}(\breve{\mathbb{V}})|^2 + |\mathfrak{F}_{\mathcal{G}}(\breve{\mathbb{V}}) - \mathfrak{F}_{\mathcal{H}}(\breve{\mathbb{V}})|^2 \end{array} \right] \right]^{0.5} \tag{4}$$

Proposition 1: The following properties $(\mathfrak{D}_1 - \mathfrak{D}_4)$ hold true for the distances $\mathfrak{D}_{\mathrm{k}}(\mathrm{k} = 1, 2, 3, 4)$ specified above between MDNFHSs $\mathcal{G}$ and $\mathcal{H}$.

$(\mathfrak{D}_1)$ $0 \leq \mathfrak{D}_{\mathrm{k}}(\mathcal{G}, \mathcal{H})$,

$(\mathfrak{D}_2)$ $\mathcal{G} = \mathcal{H}$ iff $\mathfrak{D}_{\mathrm{k}}(\mathcal{G}, \mathcal{H}) = 0$,

$(\mathfrak{D}_3)$ $\mathfrak{D}_{\mathrm{k}}(\mathcal{H}, \mathcal{G}) = \mathfrak{D}_{\mathrm{k}}(\mathcal{G}, \mathcal{H})$,

$(\mathfrak{D}_4)$ If $\mathcal{H} \subseteq \mathcal{K}, \mathcal{G} \subseteq \mathcal{H}$ and $\mathcal{K}$ is MDNFHS in $\breve{U}$, then $\mathfrak{D}_{\mathrm{k}}(\mathcal{G}, \mathcal{H}) \leq \mathfrak{D}_{\mathrm{k}}(\mathcal{G}, \mathcal{K})$ and $\mathfrak{D}_{\mathrm{k}}(\mathcal{H}, \mathcal{K}) \leq \mathfrak{D}_{\mathrm{k}}(\mathcal{G}, \mathcal{K})$.

Proof: We can demonstrate that $\mathfrak{D}_{\mathrm{k}}(\mathcal{G}, \mathcal{H})$ for $(\mathrm{k} = 1, 2, 3, 4)$ meet the characteristics $(\mathfrak{D}_1 - \mathfrak{D}_3)$. As a result, we just check $\mathfrak{D}_4$. For this let α, β, γ be three SV-NFHSs over the universe $\breve{\mathcal{U}} = \left\{\breve{\mathbb{V}}_1, \breve{\mathbb{V}}_2, \breve{\mathbb{V}}_3, \ldots, \breve{\mathbb{V}}_n\right\}$. Let $\alpha \subseteq \beta \subseteq \gamma$. Then,

$$\mathfrak{P}_\alpha\left(\check{\mathbb{V}}_i, w\right) \leq \mathfrak{P}_\beta\left(\check{\mathbb{V}}_i, w'\right) \leq \mathfrak{P}_\gamma\left(\check{\mathbb{V}}_i, w''\right),$$
$$\mathfrak{Q}_\alpha\left(\check{\mathbb{V}}_i, w\right) \geq \mathfrak{Q}_\beta\left(\check{\mathbb{V}}_i, w'\right) \geq \mathfrak{Q}_\gamma\left(\check{\mathbb{V}}_i, w''\right),$$
$$\mathfrak{R}_\alpha\left(\check{\mathbb{V}}_i, w\right) \leq \mathfrak{R}_\beta\left(\check{\mathbb{V}}_i, w'\right) \leq \mathfrak{R}_\gamma\left(\check{\mathbb{V}}_i, w''\right).$$

For all $\check{\mathbb{V}}_i \in \breve{\mathcal{U}}$, for $\mathfrak{s} = 1, 2$, we have:

$$\begin{aligned}
\left|\mathfrak{P}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{P}_\beta\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} &\leq \left|\mathfrak{P}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{P}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} \\
\left|\mathfrak{P}_\beta\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{P}_\gamma\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} &\leq \left|\mathfrak{P}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{P}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} \\
\left|\mathfrak{Q}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{Q}_\beta\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} &\leq \left|\mathfrak{Q}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{Q}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} \\
\left|\mathfrak{Q}_\beta\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{Q}_\gamma\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} &\leq \left|\mathfrak{Q}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{Q}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} \\
\left|\mathfrak{R}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\beta\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} &\leq \left|\mathfrak{R}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} \\
\left|\mathfrak{R}_\beta\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\gamma\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} &\leq \left|\mathfrak{R}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}}
\end{aligned}$$

Hence,

$$\begin{aligned}
&\left|\mathfrak{P}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{P}_\beta\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} + \left|\mathfrak{Q}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{Q}_\beta\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} \\
&+ \left|\mathfrak{R}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\beta\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} \leq \left|\mathfrak{P}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{P}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} \\
&+ \left|\mathfrak{Q}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{Q}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} + \left|\mathfrak{R}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}}
\end{aligned}$$

And,

$$\begin{aligned}
&\left|\mathfrak{P}_\beta\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{P}_\gamma\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} + \left|\mathfrak{Q}_\beta\left(\breve{\approx}_i, w\right) - \mathfrak{Q}_\gamma\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} \\
&+ \left|\mathfrak{R}_\beta\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\gamma\left(\check{\mathbb{V}}_i, w'\right)\right|^{\mathfrak{s}} \leq \left|\mathfrak{P}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{B}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} \\
&+ \left|\mathfrak{Q}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{Q}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}} + \left|\mathfrak{R}_\alpha\left(\check{\mathbb{V}}_i, w\right) - \mathfrak{R}_\gamma\left(\check{\mathbb{V}}_i, w''\right)\right|^{\mathfrak{s}}
\end{aligned}$$

3.2 Similarity Measures

Using the preceding inequality and distance expressions $\mathfrak{D}_{\mathrm{k}}(\alpha, \beta)(\mathrm{k} = 1, 2, 3, 4)$ we get $\mathfrak{D}_{\mathrm{k}}(\alpha, \beta) \leq \mathfrak{D}_{\mathrm{k}}(\beta, \gamma)$ and $\mathfrak{D}_{\mathrm{k}}(\beta, \gamma) \leq \mathfrak{D}_{\mathrm{k}}(\alpha, \gamma)$. Hence proved $(\mathfrak{D}_4)$. Although similarity measures could be readily derived from distance measures [36], we depicted the similarity measure using the available distance measures. $\mathcal{S}_1(\mathcal{G}, \mathcal{H})$ and $\mathcal{S}_2(\mathcal{G}, \mathcal{H})$ similarity measures between MDNFHSs are shown below.

$$\mathcal{S}_1(\mathcal{G}, \mathcal{H}) = 1 - \frac{1}{4n} \sum_{\check{\mathbb{V}} \in \breve{\mathcal{U}}} \left[\begin{array}{c} |\mathfrak{P}_{\mathcal{G}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}}) - \mathfrak{P}_{\mathcal{H}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}})| + |\mathfrak{Q}_{\mathcal{G}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}}) - \mathfrak{Q}_{\mathcal{H}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}})| \\ +|\mathfrak{R}_{\mathcal{G}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}}) - \mathfrak{R}_{\mathcal{H}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}})| + |\mathfrak{F}_{\mathcal{G}}(\check{\mathbb{V}}) - \mathfrak{F}_{\mathcal{H}}(\check{\mathbb{V}})| \end{array} \right] \mathfrak{D}_2(\mathcal{G}, \mathcal{H}) \tag{5}$$

$$\mathcal{S}_2(\mathcal{G}, \mathcal{H}) = 1 - \left[\frac{1}{4n} \sum_{\check{\mathbb{V}} \in \breve{\mathcal{U}}} \left[\begin{array}{c} |\mathfrak{P}_{\mathcal{G}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}}) - \mathfrak{P}_{\mathcal{H}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}})|^2 + |\mathfrak{Q}_{\mathcal{G}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}}) - \mathfrak{Q}_{\mathcal{H}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}})|^2 \\ +|\mathfrak{R}_{\mathcal{G}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}}) - \mathfrak{R}_{\mathcal{H}_{\widehat{\mathbb{W}}_{\mathrm{i}}}}(\check{\mathbb{V}})|^2 + |\mathfrak{F}_{\mathcal{G}}(\check{\mathbb{V}}) - \mathfrak{F}_{\mathcal{H}}(\check{\mathbb{V}})|^2 \end{array} \right] \right]^{0.5} \tag{6}$$

Proposition 2: The following features of the similarity measure $\mathcal{S}_k(\mathcal{G}, \mathcal{H}), \mathrm{k} = 1, 2$ are derived from the associations between distance and similarity measures, and the attributes of distance measurements $(\mathfrak{D}_1 - \mathfrak{D}_4)$.

$(P1)$ $1 \geq \mathcal{S}_k(\mathcal{G}, \mathcal{H}) \geq 0$

$(P2)$ $\mathcal{S}_k(\mathcal{G}, \mathcal{H}) = 1$ if $\mathcal{G} = \mathcal{H}$

$(P3)$ $\mathcal{S}_k(\mathcal{G}, \mathcal{H}) = \mathcal{S}_k(\mathcal{H}, \mathcal{G})$

$(P4)$ If $\mathcal{H} \subseteq \mathcal{K}, \mathcal{G} \subseteq \mathcal{H}, \mathcal{K}$ is a SV-NFHS in $\breve{\mathcal{U}}$, then $\mathcal{S}_k(\mathcal{G}, \mathcal{H}) \geq \mathcal{S}_k(\mathcal{G}, \mathcal{K})$ and $\mathcal{S}_k(\mathcal{H}, \mathcal{K}) \geq \mathcal{S}_k(\mathcal{G}, \mathcal{K})$

4 Application to MCDM

This section delineates the algorithm developed for solving MCDM problems, leveraging the proposed distance and similarity measures in conjunction with the existing aggregate operators of NHSs. The procedural flow of the algorithm is elucidated in Figure 1.

Step 1: Consideration of alternatives, attributes, and goal.

Consider $\hat{\mathcal{V}} = \{\hat{v}_1, \hat{v}_2, \hat{v}_3, \ldots, \hat{v}_{\mathrm{m}}\}$ consists of alternative and $\hat{\varepsilon} = \{\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{\varepsilon}_3, \ldots, \hat{\varepsilon}_{\mathrm{n}}\}$ be attributes set. The sets of their associated attribute values are $\hat{\varepsilon}_1 = \{j_{11}, j_{12}\}, \hat{\varepsilon}_2 = \{j_{21}, j_{22}\}$ and $\hat{\varepsilon}_3 = \{j_{31}\}$.

Let $\hat{\mathcal{W}} = \hat{\varepsilon}_1 \times \hat{\varepsilon}_2 \times \hat{\varepsilon}_3, \hat{\mathcal{W}} = \{(\hat{w}_{11}, \hat{w}_{21}, \hat{w}_{31}), (\hat{w}_{11}, \hat{w}_{22}, \hat{w}_{31}), (\hat{w}_{12}, \hat{w}_{21}, \hat{w}_{31}), (\hat{w}_{12}, \hat{w}_{22}, \hat{w}_{31})\}. \hat{\mathcal{W}} = \{\hat{w}_1, \hat{w}_2, \hat{w}_3, \hat{w}_4\}$ The characteristics of each alternative $\hat{v}_i$, where $i = 1, 2, \ldots, m$, according to the standards $\hat{w}_j$, where $j = 1, 2, 3, 4$.

Step 2: Consideration of ideal alternative along with MDNFHS values. The ideal alternative is specified as:

$$v^* = \{\wedge_i \mathfrak{P}(\hat{w}_j), \wedge_i \mathfrak{Q}(\hat{w}_j), \wedge_i \mathfrak{R}(\hat{w}_j) : \wedge_i \mathfrak{F}(\hat{w}_j); j = 1, 2, 3, \ldots, n\}$$

Step 3: Assign MDNFHS $D_j = (\alpha_{ij})_{m \times n}$ for each attribute.

Step 4: Calculate distance measure d_j using the formula, for each $\hat{v}_i$ with ideal situation v^* for assessing alternatives based on the attributes where v_i is the assessment of the alternative with respect to the multi-attributes $\hat{w}_{\mathrm{j}}$.

Step 5: Calculate similarity measure (SM) using the formula, and rank alternatives based on SM values in ascending order.

Figure 1. The proposed MCDM algorithm based on distance and similarity measures

4.1 Case Study

The case study focuses on Lahore, a city previously plagued by transportation challenges typical of large urban environments. These challenges included limited transit options, an overabundance of vehicles, and high population density. Subsequently, transformative measures were implemented, enhancing traffic management, expanding the availability of buses and trains, and promoting environmentally friendly transportation methods.

Step 1: In this case, we demonstrate how to use similarity measures to MCDM issues in an MDNFHS context. Suppose $\tilde{U} = \{\check{v}_1, \check{v}_2, \check{v}_3, \check{v}_4\}$ is the set of available modes of transport and $E = \{\check{e}_1, \check{e}_2, \check{e}_3, \check{e}_4\}$ attributes set. These are the attribute-valued sets:

$\check{e}_1 = \{j_{11} =$ "Car", $j_{12} =$ "Train", $j_{13} =$ "Bus", $j_{14} =$ "Bicycle"$\}$,

$\check{e}_2 = \{j_{21} =$ "Fast", $j_{22} =$ "Moderate", $j_{23} =$ "Slow" $\}$,

$\check{e}_3 = \{j_{31} =$ "Comfortable", $j_{32} =$ "Average", $j_{33} =$ "Uncomfortable" $\}$,

$\check{e}_4 = \{j_{41} =$ "Expensive", $j_{42} =$ "Affordable", $j_{43} =$ "Cheap"$\}$.

The Cartesian product of the sets with attribute values provides the hypersoft set $\widehat{\mathcal{W}}$:

$$\widehat{\mathcal{W}} = \check{e}_1 \times \check{e}_2 \times \check{e}_3 \times \check{e}_4 = \{(\breve{w}_{11}, \breve{w}_{21}, \breve{w}_{31}, \breve{w}_{41}), (\breve{w}_{11}, \breve{w}_{22}, \breve{w}_{31}, \breve{w}_{42}), (\breve{w}_{12}, \breve{w}_{21}, \breve{w}_{31}, \breve{w}_{43}), (\breve{w}_{12}, \breve{w}_{21}, \breve{w}_{32}, \breve{w}_{41})\}.$$
$$\widehat{\mathcal{W}} = \{\breve{w}_1, \breve{w}_2, \breve{w}_3, \breve{w}_4\}.$$

Step 2: Consideration of Ideal alternative:

$$\hat{v}^* = \{(0.9, 1.0, 0.3, 0.6), (0.6, 0.6, 0.3, 0.4), (0.2, 0.2, 0.3, 0.3), (0.7, 0.9, 0.9, 0.6)\}.$$

Step 3: Assigning of MDNFHS values and presented in Table 1.

Step 4: Calculation of distance measures.

Step 5: Calculation of similarity measure (SM) using the formula, and ranking (Table 2).

The impact of these interventions on urban transportation was analyzed using the proposed similarity and distance measures within the MCDM framework. The algorithm facilitated a comprehensive evaluation of various transportation alternatives, with the final rankings presented in Figure 2. Each alternative was assessed across multiple parameters, including environmental sustainability, accessibility of public transit, and effectiveness in alleviating traffic congestion. The application of the MCDM algorithm yielded a detailed comparison of the performance of each transportation mode, culminating in a ranked list of options. This ranking, accompanied by an in-depth analysis of the strengths and weaknesses of each option in relation to the set criteria, provided valuable insights. The findings demonstrated the adaptability of the proposed measures in addressing the complexities inherent in transportation decision-making. The algorithm accounted for diverse factors and their respective weights, leading to a thorough analysis that accurately captured the intricate dynamics of Lahore's transportation system.

Table 1. MDNFHS based attributive values

	$\breve{\boldsymbol{w}}_1$	$\breve{\boldsymbol{w}}_2$	$\breve{\boldsymbol{w}}_3$	$\breve{\boldsymbol{w}}_4$
$\breve{\boldsymbol{v}}_1$	$(0.6, 0.8, 0.2 : 0.3)$	$(0.7, 0.6, 0.5 : 0.4)$	$(0.8, 0.9, 0.3 : 0.4)$	$(0.7, 0.9, 0.3 : 0.6)$
$\breve{\boldsymbol{v}}_2$	$(0.9, 1.0, 0.1 : 0.2)$	$(0.6, 0.7, 0.3 : 0.2)$	$(0.7, 0.8, 0.2 : 0.3)$	$(0.9, 0.2, 0.5 : 0.5)$
$\breve{\boldsymbol{v}}_3$	$(0.4, 0.5, 0.2 : 0.3)$	$(0.5, 0.6, 0.4 : 0.3)$	$(0.6, 0.7, 0.3 : 0.4)$	$(0.5, 0.29, 0.3 : 0.1)$
$\breve{\boldsymbol{v}}_4$	$(0.6, 0.9, 0.6 : 0.1)$	$(0.8, 0.29, 0.9 : 0.1)$	$(0.2, 0.2, 0.9 : 0.5)$	$(0.1, 0.8, 0.6 : 0.4)$

Table 2. Ranking based on similarity measures

Sr. No.	Similarity Measure	Values	Rank
1	$S_1(\breve{v}_1, \hat{v}^*)$	0.6466	3
2	$S_1(\breve{v}_2, \hat{v}^*)$	0.7798	2
3	$S_1(\breve{v}_3, \hat{v}^*)$	0.9946	1
4	$S_1(\breve{v}_4, \hat{v}^*)$	0.5801	4

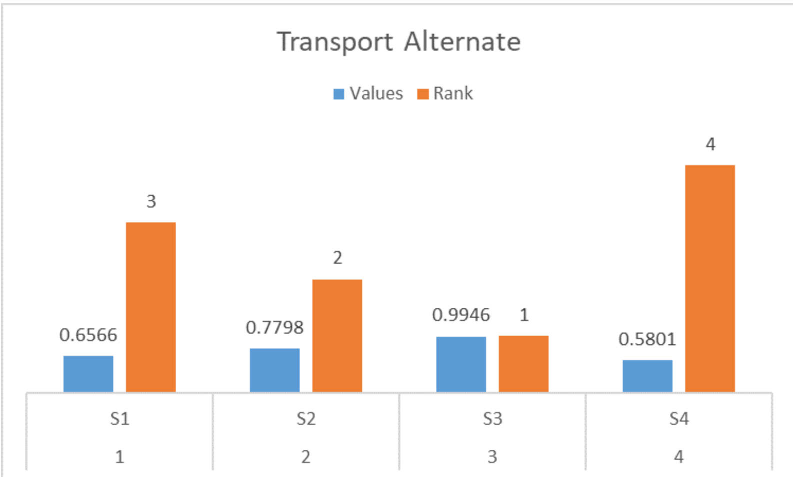

Figure 2. The final ranking for the alternate of transport

The transparency of the final rankings of transportation alternatives, as depicted in Figure 2, is pivotal for ensuring stakeholder and decision-maker comprehension. This clarity facilitates a shared understanding among all parties involved, fostering collaborative efforts in the implementation of the chosen transportation strategy. Such transparency is instrumental in enhancing the decision-making process, as it allows for a clear articulation of the rationale behind the prioritization of certain options over others.

In summation, the integration of the proposed distance and similarity measures with the MCDM algorithm has not only contributed to the mitigation of transportation issues in Lahore but also established a framework that can be adapted for similar urban planning challenges. The outcomes of this study underscore the efficacy of MCDM in navigating complex decision-making scenarios. Furthermore, the findings highlight the critical role of tailored distance and similarity measures in yielding results that are both reliable and relevant to the specific context.

5 Conclusions

This research has introduced the MDNFHS as an innovative extension of NFHSs, specifically tailored for complex decision-making scenarios. MDNFHS integrates NS theory, FS theory, and HS theory, offering a robust model adept at managing imprecision, inconsistency, and hyper-softness in decision-making contexts. This integration enables MDNFHS to accurately capture the gradation or variability in membership, truth, indeterminacy, and falsity, thus facilitating the representation and reasoning of information with varying degrees of membership and truthfulness. The incorporation of Hamming and Euclidean distance measures, along with similarity measures within the MDNFHS framework, significantly enhances its applicability in various engineering and scientific domains. The development of a MCDM approach, underpinned by these measures, allows for the effective ranking of alternatives. An illustrative case study within this research demonstrates the practical effectiveness of MDNFHS in decision-making processes.

Looking forward, there are several promising avenues for future research to augment the MDNFHS framework. These include exploring the integration of NS with other extensions of FS and applying MDNFHS in a wide array of optimization problems. Potential areas of interest encompass language modeling, opinion measurement, choice interaction, information aggregation, and entropy calculation. These future explorations are anticipated to further expand the capabilities and applicability of MDNFHS in addressing the intricate challenges inherent in decision-making processes.

Data Availability

Not applicable.

Acknowledgements

The authors are highly thankful to the editor and referees for the valuable comments and suggestions for improving the quality of the paper.

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] F. Smarandache, "Neutrosophic quadruple numbers, refined neutrosophic quadruple numbers, absorbance law, and the multiplication of neutrosophic quadruple numbers," *Neutrosophic Sets Syst.*, vol. 10, pp. 1–9, 2015.

[2] S. Das, R. Das, and C. Granados, "Topology on quadripartitioned neutrosophic sets," *Neutrosophic Sets Syst.*, vol. 45, pp. 1–5, 2021.

[3] D. Karabašević, D. Stanujkić, E. K. Zavadskas, P. Stanimirović, G. Popović, B. Predić, and A. Ulutaş, "A novel extension of the TOPSIS method adapted for the use of single-valued neutrosophic sets and hamming distance for e-commerce development strategies selection," *Symmetry*, vol. 12, no. 8, p. 1263, 2020. https://doi.org/10.3390/sym12081263

[4] M. Abdel-Basset, A. Gamal, G. Manogaran, L. H. Son, and H. V. Long, "A novel group decision making model based on neutrosophic sets for heart disease diagnosis," *Multimed Tools Appl.*, vol. 79, no. 15-16, pp. 9977–10 002, 2019. https://doi.org/10.1007/s11042-019-07742-7

[5] B. Said, M. Lathamaheswari, P. K. Singh, A. A. Ouallane, A. Bakhouyi, A. Bakali, M. Talea, A. Dhital, and N. Deivanayagampillai, "An intelligent traffic control system using neutrosophic sets, rough sets, graph theory, fuzzy sets and its extended approach: A literature review," *Neutrosophic Sets Syst.*, vol. 50, pp. 10–26, 2022.

[6] A. Nafei, A. Javadpour, H. Nasseri, and W. Yuan, "Optimized score function and its application in group multiattribute decision making based on fuzzy neutrosophic sets," *Int. J. Intell. Syst.*, vol. 36, no. 12, pp. 7522–7543, 2021. https://doi.org/10.1002/int.22597

[7] S. Broumi, I. Deli, and F. Smarandache, "N-valued interval neutrosophic sets and their application in medical diagnosis," *Collected Papers. Volume XIV: Neutrosophics and other Topics*, pp. 45–69, 2022.

[8] P. K. Maji, "Neutrosophic soft set," *Ann. Fuzzy Math. Inf.*, vol. 5, no. 1, pp. 157–168, 2013.

[9] F. Karaaslan, "Neutrosophic soft sets with applications in decision making," *Neutrosophic Sets Syst.*, 2014. https://doi.org/10.5281/ZENODO.23151

[10] S. Broumi, "Generalized neutrosophic soft set," *Int. J. Comput. Sci. Eng. Inf. Technol.*, vol. 3, no. 2, 2013.

[11] S. Broumi, I. Deli, and F. Smarandache, "Neutrosophic parametrized soft set theory and its decision making," *Collected Papers. Volume X: On Neutrosophics, Plithogenics, Hypersoft Set, Hypergraphs, and other Topics*, 2022.

[12] R. Sahin and A. Küçük, "Generalised neutrosophic soft set and its integration to decision making problem," *Appl. Math. Inf. Syst.*, vol. 8, no. 6, pp. 2751–2759, 2014.

[13] I. Deli, "Interval-valued neutrosophic soft sets and its decision making," *Int. J. Mach. Learn. Cyber.*, vol. 8, no. 2, pp. 665–676, 2015. https://doi.org/10.1007/s13042-015-0461-3

[14] I. Deli and S. Broumi, "Neutrosophic soft matrices and NSM-decision making," *J. Intell. Fuzzy Syst.*, vol. 28, no. 5, pp. 2233–2241, 2015. https://doi.org/10.3233/ifs-141505

[15] R. M. Hashim, M. Gulistan, I. Rehman, N. Hassan, and A. M. Nasruddin, "Neutrosophic bipolar fuzzy set and its application in medicines preparations," *Neutrosophic Sets Syst.*, vol. 31, pp. 86–100, 2020.

[16] S. Das, B. K. Roy, M. B. Kar, S. Kar, and D. Pamučar, "Neutrosophic fuzzy set and its application in decision making," *J. Ambient Intell. Humaniz. Comput.*, vol. 11, pp. 5017–5029, 2020.

[17] T. Mahmood, U. U. Rehman, A. Jaleel, J. Ahmmad, and R. Chinram, "Bipolar complex fuzzy soft sets and their applications in decision-making," *Math*, vol. 10, no. 7, p. 1048, 2022. https://doi.org/10.3390/math10071048

[18] R. Şahin and A. Küçük, "Subsethood measure for single valued neutrosophic sets," *J. Intell. Fuzzy Syst.*, vol. 29, no. 2, pp. 525–530, 2015. https://doi.org/10.3233/ifs-141304

[19] E. K. Zavadskas, R. Bausys, A. Kaklauskas, I. Ubarte, A. Kuzminske, and N. Gudiene, "Sustainable market valuation of buildings by the single-valued neutrosophic MAMVA method," *Appl. Soft Comput.*, vol. 57, pp. 74–87, 2017. https://doi.org/10.1016/j.asoc.2017.03.040

[20] J. Zhan, M. Khan, M. Gulistan, and A. Ali, "Applications of neutrosophic cubic sets in multi-criteria decision-making," *Int. J. Uncertainty Quantification*, vol. 7, no. 5, pp. 377–394, 2017. https://doi.org/10.1615/int.j.uncertaintyquantification.2017020446

[21] H. Zhang, J. Wang, and X. Chen, "Interval neutrosophic sets and their application in multicriteria decision making problems," *Sci. World. J.*, vol. 2014, p. 15, 2014.

[22] A. M. Khalil, D. Cao, A. Azzam, F. Smarandache, and W. R. Alharbi, "Combination of the single-valued neutrosophic fuzzy set and the soft set with applications in decision-making," *Symmetry*, vol. 12, no. 8, p. 1361, 2020. https://doi.org/10.3390/sym12081361

[23] F. Smarandache, "Extension of soft set to hypersoft set, and then to plithogenic hypersoft set," *Neutrosophic Sets Syst.*, vol. 22, pp. 168–170, 2018.

[24] M. Saqlain, S. Moin, M. N. Jafar, M. Saeed, and F. Smarandache, "Aggregate operators of neutrosophic hypersoft set," *Neutrosophic Sets Syst.*, vol. 32, pp. 294–306, 2020. https://doi.org/10.5281/ZENODO.3723155

[25] M. Saqlain, N. Jafar, S. Moin, M. Saeed, and S. Broumi, "Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets," *Neutrosophic Sets Syst.*, vol. 32, pp. 317–329, 2020.

[26] A. U. Rahman, M. Saeed, M. A. Mohammed, K. H. Abdulkareem, J. Nedoma, and R. Martinek, "Fppsv-NHSS: Fuzzy parameterized possibility single valued neutrosophic hypersoft set to site selection for solid waste management," *Appl. Soft Comput.*, vol. 140, p. 110273, 2023.

[27] A. Ur Rahman, M. Saeed, M. H. Saeed, D. A. Zebari, M. Albahar, K. H. Abdulkareem, S. Alaa Al-Waisy, and M. A. Mohammed, "A framework for susceptibility analysis of brain tumours based on uncertain analytical cum algorithmic modeling," *Bioengineering*, vol. 10, no. 2, p. 147, 2023. https://doi.org/10.3390/bioengineering10020147

[28] M. N. Jafar, K. Muniba, and M. Saqlain, "Enhancing diabetes diagnosis through an intuitionistic fuzzy soft matrices-based algorithm," *Spectr. Eng. Manage. Sci.*, vol. 1, no. 1, pp. 73–82, 2023. https://doi.org/10.31181/sems1120238u

[29] Ž. Stević, A. Bašić, S. Moslem, and K. Zhong, "An integrated ABC-FUCOM model for product classification," *Spectr. Eng. Manage. Sci.*, vol. 1, no. 1, pp. 83–91, 2023. https://doi.org/10.31181/sems1120239k

[30] M. Ihsan, M. Saeed, and A. U. Rahman, "An intelligent fuzzy parameterized MADM-approach to optimal selection of electronic appliances based on neutrosophic hypersoft expert set," *Neutrosophic Sets and Syst.*, vol. 53, pp. 459–481, 2023. https://doi.org/10.5281/ZENODO.7536071

[31] M. Saeed, M. Ahsan, A. Ur Rahman, M. H. Saeed, and A. Mehmood, "An application of neutrosophic hypersoft mapping to diagnose brain tumor and propose appropriate treatment," *J. Intell. Fuzzy Syst.*, vol. 41, no. 1, pp. 1677–1699, 2021. https://doi.org/10.3233/jifs-210482

[32] A. U. Rahman, M. Saeed, and A. Hafeez, "Theory of bijective hypersoft set with application in decision making," *Punjab Univ. J. Math.*, vol. 53, no. 7, pp. 511–526, 2021. https://doi.org/10.52280/pujm.2021.530705

[33] R. M. Zulqarnain, X. L. Xin, M. Saqlain, M. Saeed, F. Smarandache, and M. I. Ahamad, "Some fundamental operations on interval valued neutrosophic hypersoft set with their properties," *Neutrosophic Sets Syst.*, vol. 40, pp. 134–148, 2021. https://doi.org/10.5281/ZENODO.4549352

[34] M. Saeed, M. Ahsan, M. H. Saeed, A. U. Rahman, M. A. Mohammed, J. Nedoma, and R. Martinek, "An algebraic modeling for tuberculosis disease prognosis and proposed potential treatment methods using fuzzy hypersoft mappings," *Biomed. Signal Process. Control*, vol. 80, p. 104267, 2023.

[35] M. Saeed, M. Ahsan, M. H. Saeed, A. U. Rahman, A. Mehmood, M. A. Mohammed, M. M. Jaber, and R. Damaševičius, "An optimized decision support model for COVID-19 diagnostics based on complex fuzzy hypersoft mapping," *Mathematics*, vol. 10, no. 14, p. 2472, 2022. https://doi.org/10.3390/math10142472

[36] M. Saeed, undefined, and M. I. Harl, "An abstract approach to W-Structures based on hypersoft set with properties," *Neutrosophic Sets Syst.*, vol. 48, 2022. https://doi.org/10.5281/ZENODO.6041593

[37] M. Riaz, A. Habib, M. Saqlain, and M. S. Yang, "Cubic bipolar fuzzy-VIKOR method using new distance and entropy measures and Einstein averaging aggregation operators with application to renewable energy," *Int. J. Fuzzy Syst.*, vol. 25, no. 2, pp. 510–543, 2023.

[38] M. Saqlain, M. Riaz, R. Imran, and F. Jarad, "Distance and similarity measures of intuitionistic fuzzy hypersoft sets with application: Evaluation of air pollution in cities based on air quality index," *AIMS Math.*, vol. 8, no. 3, pp. 6880–6899, 2023.