Fuzzy Control of Active Vehicle Suspensions for Enhanced Safety in Goods Transport

Georgios K. Tairidis[1] , Konstantinos Marakakis[1,2] , Athanasios Protogerakis[1], Georgios E. Stavroulakis[1]*

[1] Computational Mechanics and Optimization Lab, School of Production Engineering and Management, Technical University of Crete, 73100 Chania, Greece
[2] Vehicle Technical Inspection Centre (IKTEO) of Chania, 73100 Chania, Greece

* Correspondence: Georgios E. Stavroulakis (gestavroulakis@tuc.gr)

Abstract: Suspension systems play a critical role in ensuring the safety, comfort, and stability of vehicles during the transportation of both passengers and goods. Among various suspension technologies, active or electronic suspensions have emerged as the most advanced due to their ability to dynamically adjust damping characteristics, thereby optimizing vehicle performance. This is typically achieved by modulating the pressure or flow of air or oil within the damper, or by altering its physical properties. To facilitate such dynamic adjustments, an effective control system is essential. Soft computing techniques, such as fuzzy logic controllers, are increasingly employed for their robustness and adaptability in providing the required control forces. In this study, the active suspension system was controlled via a fuzzy logic controller, with a piezoelectric actuator employed to generate the control force. A comparative analysis was conducted with traditional control methods, including the proportional-integral-derivative (PID) controller, to evaluate the performance of the fuzzy logic approach. Simulation results demonstrated that both control strategies were capable of achieving stable and smooth suspension behavior. However, fuzzy control was found to respond more quickly to dynamic changes, while the PID controller exhibited superior performance during the initial stages of vibration, offering enhanced safety during the commencement of transport. These findings underscore the potential of fuzzy logic control in optimizing the active suspension systems for improved vehicle dynamics and the safe transport of sensitive goods.

Keywords: Active suspensions; Fuzzy systems; Control; Quarter-vehicle; Safe transport

1 Introduction

A suspension is a vital part for a vehicle's proper and smooth functioning. The kinematics and the geometry of a suspension system are among the most complex concepts in the automotive field. The suspension is the mechanism that keeps the wheels in contact with the road when the car is turning, accelerating or braking. It consists of shock absorbers, springs (as many as the wheels) and metal rods, which control the movement of the wheels through mechanical joints. The suspension determines the overall handling and comfort of the car. There are many types of suspensions, but they all have three main functions, namely, keeping the wheels as vertical as possible, regardless of the movement of the car, absorbing road abnormalities and ensuring the grip on the road, i.e., the stability and road behavior of the vehicle.

The most advanced suspension systems are the so-called active or electronic suspensions, where the damping of shock absorbers can be changed, modifying the dynamic characteristics and thus, the quality of the car's rolling. This is achieved by changing the pressure or flow of air or oil in the shock absorber, or the properties of the fluid (cf. magnetorheological dampers). Some such suspensions even have the ability to change the height of the vehicle. All these features are controlled by a computer through a control system, which collects information from sensors about the quality of the road, the speed or acceleration of the vehicle, the steering wheel angle and the rotation speed, the accelerator pedal pressure, and the engine speed or pressure on the brake pedal, and, subsequently, configures the suspension.

An effective control of the suspension is important for the safety of the vehicle and the comfort of the passengers. For this reason, several methods have been proposed in the current literature to control the vibrations caused by the roadway in the suspension. Among others, fuzzy control [1–5], proportional-integral (PI) control [6], neural networks [7] and hybrid techniques [8–10] have been considered. For example, Phu and Choi [11] proposed an adaptive fuzzy-PID controller for the vibration control of a vehicle's seat suspension. Moaaz and Ghazaly [12] compared fuzzy control with PID control for active suspensions in the frequency domain. If necessary, the fuzzy controller can be fine-tuned using global optimization methods, as analytically described by Tairidis et al. [13]. Optimal design of active suspension controllers with respect to safety requirements for goods transport seems to be an open field for research.

In most studies, the problem-solving approach is usually seen as a model that includes an axis, a suspension and a wheel. This system receives stimuli which depend on the typical unevenness of the ground and the speed of the system, and the response to these disturbances is observed. Fuzzy control is a very powerful soft computing tool, which is used not only in modern active suspension systems but also in several other engineering applications, such as the vibration control of smart structures, as described by Tairidis et al. [14], the wind energy sector [15], and robotics [16–18]. Emam [19] used an automatic control system with a fuzzy logic controller in order to minimize vertical acceleration for better passenger comfort and determine the position of the damper to avoid collision with the limits of the suspension, thereby reducing wear. A similar concept was proposed in the present study as it has realistic approaches and is easy to apply in a real-life environment.

2 Materials and Methods

2.1 Active Suspension Problem Modelling

The design and modelling of an active suspension system of a vehicle constitutes an interesting automated control problem. In this study, a quarter-vehicle suspension model, i.e., one of the four wheels, was considered. As seen in Figure 1, this concept simplifies the problem to a two-degree-of-freedom spring-mass-damper dynamic problem.
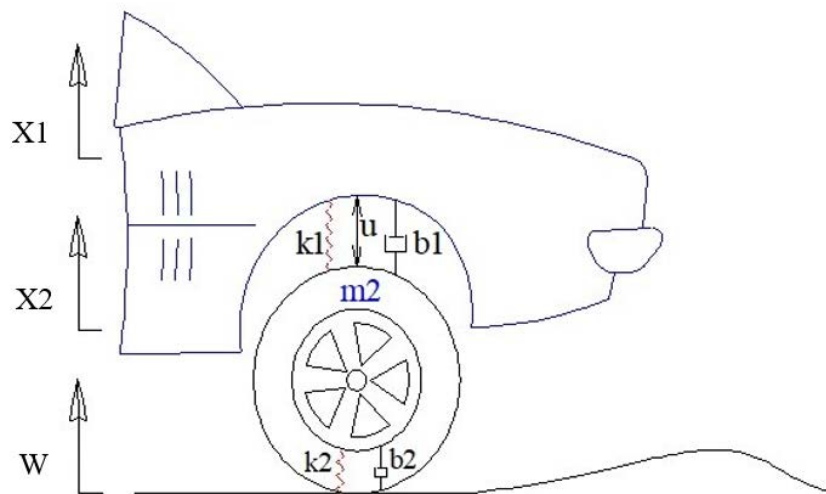

Figure 1. The quarter-vehicle suspension system

The equations of motion of the problem are given by Newton's second law of motion in terms of generalized coordinates as:

$$m_1 \ddot{x}_1 = -k_1 (x_1 - x_2) - b_1 (\dot{x}_1 - \dot{x}_2) + u \tag{1}$$

$$m_2 \ddot{x}_2 = k_1 (x_1 - x_2) + k_2 (W - x_2) + b_1 (\dot{x}_1 - \dot{x}_2) + b_2 \left(\dot{W} - \dot{x}_2 \right) - u \tag{2}$$

where, m_1 is the mass of the quarter-vehicle; m_2 is the mass of the whole system of the wheel, including steering, suspension, and so on; k_1 is the spring constant of the suspension system; k_2 is the spring constant of the wheel; b_1 is the damping constant of the suspension system; b_2 is the damping constant of the wheel; x_1 is the deflection of the vehicle; x_2 is the deflection of the suspension system; the first and second derivatives of x_1 and x_2 are the respective velocities and accelerations; u is the control actuator force; and W is the disturbance from the road. In this

study, the masses, spring constants and damping constants were selected as follows: $m_1 = 500 \text{ kg}, m_2 = 200 \text{ kg}, k_1 = 70500 \text{ N/m}, k_2 = 150000 \text{ N/m}, b_1 = 370 \text{ N} \cdot \text{s/m}$ and $b_2 = 8620 \text{ N} \cdot \text{s/m}$. The disturbance of the road is $W = -0.1 \text{ m}$.

2.2 Fuzzy Control

A Mamdani-type fuzzy controller was developed within the MATLAB environment using the Fuzzy Logic Toolbox. The inputs of the controller were the deflection, the velocity and the acceleration of the body, while the output of the control was the actuator force which is necessary for the control of the active suspension. The membership functions of the fuzzy variables are depicted in Figure 2. The range of the first and the second input variables, i.e., the deflection and the velocity, was [-1, 1], and five membership functions (categories), namely, negative medium (NM), negative small (NS), zero (ZE), positive small (PS) and Positive Medium (PM) were considered for both variables. For the third input, i.e., the acceleration of the body, the range was [-0.1, 0.1] and three membership functions were considered, namely, negative (N), ZE and positive (P). The output variable (actuator force) had a range of values in the interval [-5000, 5000] and nine membership functions were used, namely, negative very big (NV), negative big (NB), NM, NS, ZE, PS, PM, positive big (PB) and positive very big (PV).
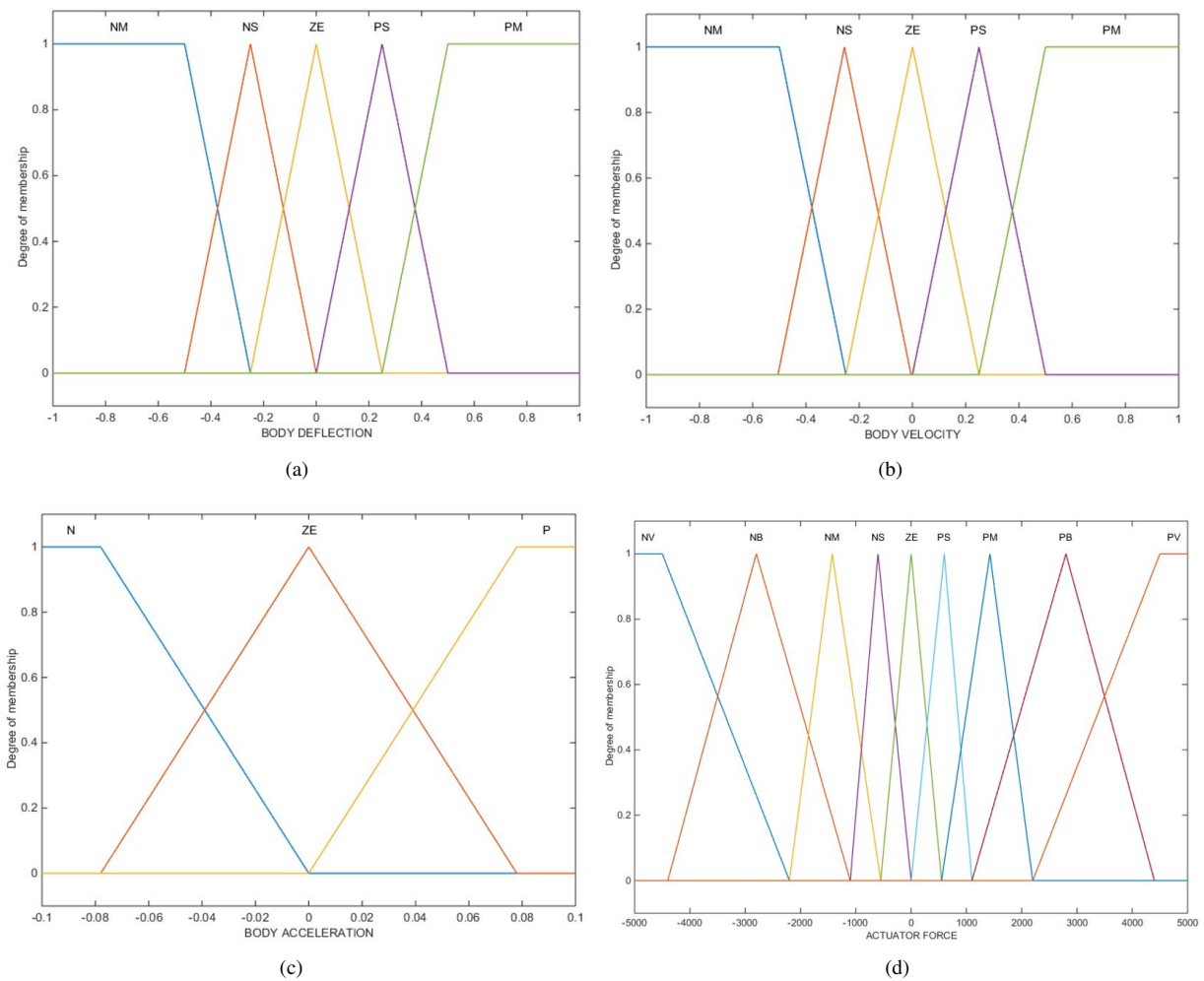
(a)

(b)

(c)

(d)

Figure 2. Membership functions of the fuzzy variables: (a) Membership function of deflection; (b) Membership function of velocity; (c) Membership function of acceleration; (d) Membership function of actuator force

The fuzzy rule base was written considering the movement and the control of a pendulum. The number of rules which are necessary for the decision-making unit of the fuzzy system was derived according to the number of the fuzzy input variables and its membership functions. In the controller proposed in this study, 50 if-then rules were used for the description of the fuzzy inference system. These verbal rules are given analytically in Table 1.

The graphic representation (surface) of the rules for the three inputs and the one output is depicted in pairs of inputs over the output, as shown in Figure 3.

Table 1. Verbal if-then rules for the fuzzy inference system

Rule	$X_1 - X_2$	$V_1 - V_2$	$A_1 - A_2$	F	Rule	$X_1 - X_2$	$V_1 - V_2$	$A_1 - A_2$	F
R_1	PM	PM	ZE	ZE	R_{26}	PM	PM	not ZE	NS
R_2	PS	PM	ZE	NS	R_{27}	PS	PM	not ZE	NM
R_3	ZE	PM	ZE	NM	R_{28}	ZE	PM	not ZE	NB
R_4	NS	PM	ZE	NM	R_{29}	NS	PM	not ZE	NB
R_5	NM	PM	ZE	NB	R_{30}	NM	PM	not ZE	NV
R_6	PM	PS	ZE	ZE	R_{31}	PM	PS	not ZE	NS
R_7	PS	PS	ZE	NS	R_{32}	PS	PS	not ZE	NM
R_8	ZE	PS	ZE	NS	R_{33}	ZE	PS	not ZE	NM
R_9	NS	PS	ZE	NM	R_{34}	NS	PS	not ZE	NB
R_{10}	NM	PS	ZE	NM	R_{35}	NM	PS	not ZE	NB
R_{11}	PM	ZE	ZE	PS	R_{36}	PM	ZE	not ZE	PM
R_{12}	PS	ZE	ZE	ZE	R_{37}	PS	ZE	not ZE	PS
R_{13}	ZE	ZE	ZE	ZE	R_{38}	ZE	ZE	not ZE	ZE
R_{14}	NS	ZE	ZE	ZE	R_{39}	NS	ZE	not ZE	NS
R_{15}	NM	ZE	ZE	NS	R_{40}	NM	ZE	not ZE	NM
R_{16}	PM	NS	ZE	PM	R_{41}	PM	NS	not ZE	PB
R_{17}	PS	NS	ZE	PM	R_{42}	PS	NS	not ZE	PB
R_{18}	ZE	NS	ZE	PS	R_{43}	ZE	NS	not ZE	PM
R_{19}	NS	NS	ZE	PS	R_{44}	NS	NS	not ZE	PM
R_{20}	NM	NS	ZE	ZE	R_{45}	NM	NS	not ZE	PS
R_{21}	PM	NM	ZE	PB	R_{46}	PM	NM	not ZE	PV
R_{22}	PS	NM	ZE	PM	R_{47}	PS	NM	not ZE	PB
R_{23}	ZE	NM	ZE	PM	R_{48}	ZE	NM	not ZE	PB
R_{24}	NS	NM	ZE	PS	R_{49}	NS	NM	not ZE	PM
R_{25}	NM	NM	ZE	ZE	R_{50}	NM	NM	not ZE	PS

Note: As for Rule 1, if the deflection $X_1 - X_2$ is PM , the velocity $V_1 - V_2$ is PM , and the acceleration $A_1 - A_2$ is ZE , then the actuator force is ZE .
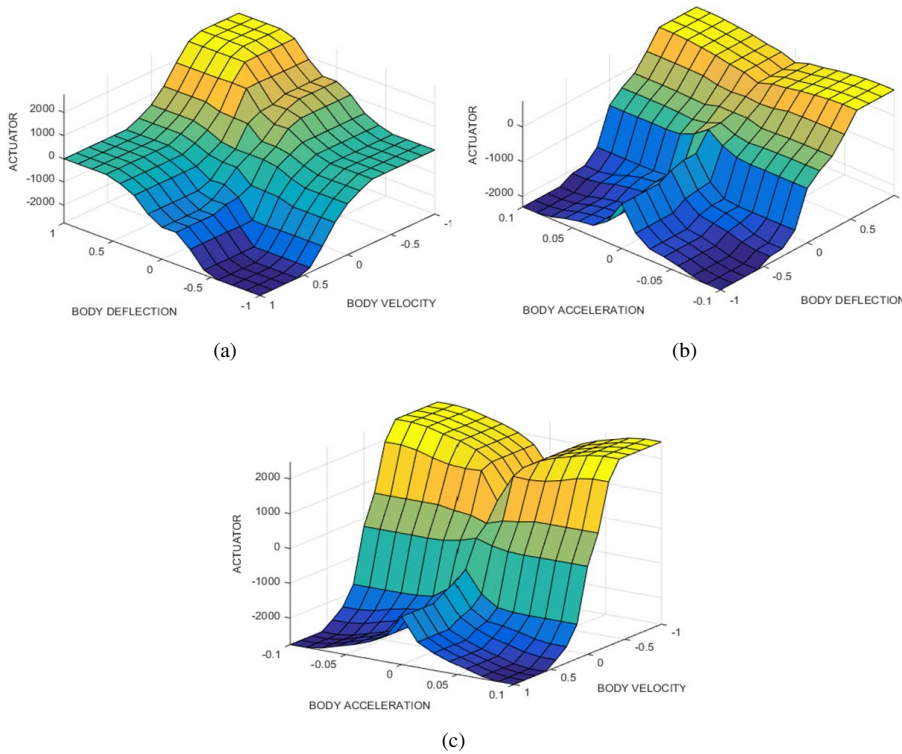

(a)

(b)

(c)

Figure 3. Fuzzy surface for (a) Deflection-velocity; (b) Deflection-acceleration; and (c) Velocity-acceleration

2.3 PID Control

A PID controller was developed in order to check the efficiency of the control in comparison to a more classical control approach. In order to apply the PID controller to the system, it is necessary to calculate the transfer function of the plant (suspension model) and the disturbance (road abnormalities). Considering that all the initial conditions are zero, that is, the moment that the wheel of the vehicle receives a disturbance from the road, Eqs. (1) and (2) can be reformulated through Laplace transformation, yielding the transfer functions $G_1(s)$ and $G_2(s)$ with output $x_1 - x_2$ and inputs u and W, as follows:

$$\left(M_1 s^2 + b_1 s + K_1\right) X_1(s) - \left(b_1 s + K_1\right) X_2(s) = U(s) \tag{3}$$

$$-\left(b_1 s + K_1\right) X_1(s) + \left(M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)\right) X_2(s) = (b_2 s + K_2) W(s) - U(s) \tag{4}$$

$$\begin{bmatrix} \left(M_1 s^2 + b_1 s + K_1\right) & -\left(b_1 s + K_1\right) \\ -\left(b_1 s + K_1\right) & \left(M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)\right) \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} U(s) \\ (b_2 s + K_2) W(s) - U(s) \end{bmatrix} \tag{5}$$

If

$$A = \begin{bmatrix} \left(M_1 s^2 + b_1 s + K_1\right) & -\left(b_1 s + K_1\right) \\ -\left(b_1 s + K_1\right) & \left(M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)\right) \end{bmatrix} \tag{6}$$

$$\Delta = \det(A) = \left(M_1 s^2 + b_1 s + K_1\right) \cdot \left(M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)\right) - (b_1 s + K_1) \cdot (b_1 s + K_1) \tag{7}$$

Taking the inverse of matrix A, the following can be obtained by using its determinant Δ and multiplying the inputs $U(s)$ and $W(s)$ with $1/\Delta$:

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \left(M_2 s^2 + (b_1 + b_2) s + (K_1 + K_2)\right) & (b_1 s + K_1) \\ (b_1 s + K_1) & \left(M_1 s^2 + b_1 s + K_1\right) \end{bmatrix} \begin{bmatrix} U(s) \\ (b_2 s + K_2) W(s) - U(s) \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \left(M_2 s^2 + b_2 s + K_2\right) & \left(b_1 b_2 s^2 + (b_1 K_2 + b_2 K_1) s + K_1 K_2\right) \\ -M_1 s^2 & \left(M_1 b_2 s^3 + (M_1 K_2 + b_1 b_2) s^2 + (b_1 K_2 + b_2 K_1) s + K_1 K_2\right) \end{bmatrix} \begin{bmatrix} U(s) \\ W(s) \end{bmatrix} \tag{9}$$

When only the study of input $U(s)$ is needed, then $W(s) = 0$ can be set. As a result, the transfer function $G_1(s)$ is given as:

$$G_1(s) = \frac{X_1(s) - X_2(s)}{U(s)} = \frac{(M_1 + M_2) s^2 + b_2 s + K_2}{\Delta} \tag{10}$$

When the study of the disturbance $W(s)$ is of interest, $U(s) = 0$ can be set, obtaining the transfer function $G_2(s)$ as:

$$G_2(s) = \frac{X_1(s) - X_2(s)}{W(s)} = \frac{-M_1 b_2 s^3 - M_1 K_2 s^2}{\Delta} \tag{11}$$

The transfer function of the PID controller $G_c(s)$ is given as:

$$G_c(s) = K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s} \tag{12}$$

where, K_P, K_I and K_D are the proportional, the integral and the derivative terms, respectively, of the PID controller.

The feedback control system with the respective transfer functions for each element (plant, controller and external disturbance) is shown in Figure 4.

The transfer function $F(s)$ of the disturbance can be calculated from the block diagram of Figure 4 as follows:

$$G_2(s) = F(s)G_1(s) \Leftrightarrow F(s) = \frac{G_2(s)}{G_1(s)} = \frac{\frac{-M_1 b_2 s^3 - M_1 K_2 s^2}{\Delta}}{\frac{(M_1 + M_2) s^2 + b_2 s + K_2}{\Delta}} = \frac{-M_1 b_2 s^3 - M_1 K_2 s^2}{(M_1 + M_2) s^2 + b_2 s + K_2} \tag{13}$$

The three gains of the PID controller, i.e., the numerical values for the three terms, were selected with trial and error as $K_P = 1200$, $K_I = 500$, and $K_D = 450$. Several methods, such as the Ziegler-Nichols method, or optimization procedures (e.g., a genetic algorithm) can also be used for the selection and/or fine-tuning of the values for the three terms of the PID controller if necessary.
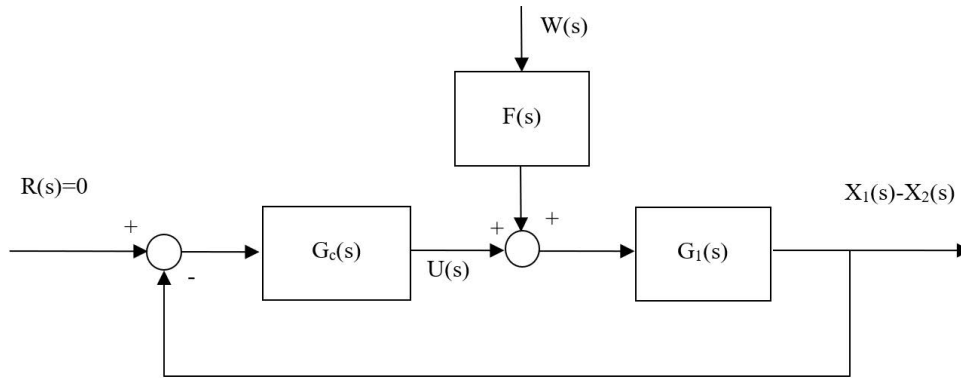
Figure 4. The control system with the PID controller and the external disturbance

3 Results

For the simulation of the active suspension, as described in Subsection 2.1, the Simulink package of MATLAB was used. More details about the Simulink modelling are provided in Appendix.

Table 2 shows the maximum and minimum diversion of the deflection, velocity and acceleration of the body from the equilibrium point for the passive system, i.e., the one without any control.

For the control of these vibrations, two different configurations were considered: one with a controller based on fuzzy logic, and one with a three-term (PID) controller, as shown in Subsections 3.1 and 3.2, respectively.

Table 2. Maximum and minimum diversion of the deflection, velocity and acceleration for the passive, fuzzy and PID systems

	Passive			Fuzzy			PID		
	$X_1 - X_2$ (m)	$V_1 - V_2$ (m/s)	$A_1 - A_2$ $\left(\text{m/s}^2\right)$	$X_1 - X_2$ (m)	$V_1 - V_2$ (m/s)	$A_1 - A_2$ $\left(\text{m/s}^2\right)$	$X_1 - X_2$ (m)	$V_1 - V_2$ (m/s)	$A_1 - A_2$ $\left(\text{m/s}^2\right)$
Max	4.96×10^{-2}	1.003	37.18	3.42×10^{-2}	0.907	33.64	0.98×10^{-2}	0.056	1.47
Min	-5.03×10^{-2}	-0.675	-19.32	-3.94×10^{-2}	-0.973	-19.66	-0.79×10^{-2}	-0.498	-1.42

3.1 Use of Fuzzy Controller for the Active Suspension System

In the first layout, the fuzzy controller which is analytically described in Subsection 2.2 was used. The simulation time was set to 5 seconds. The results for the deterioration of vibrations in terms of deflection, velocity and acceleration of both passive (without control) and active (with fuzzy control) systems are graphically presented in Figures 5, 6, and 7, respectively. The actuator control force obtained by the fuzzy controller is depicted in Figure 8.
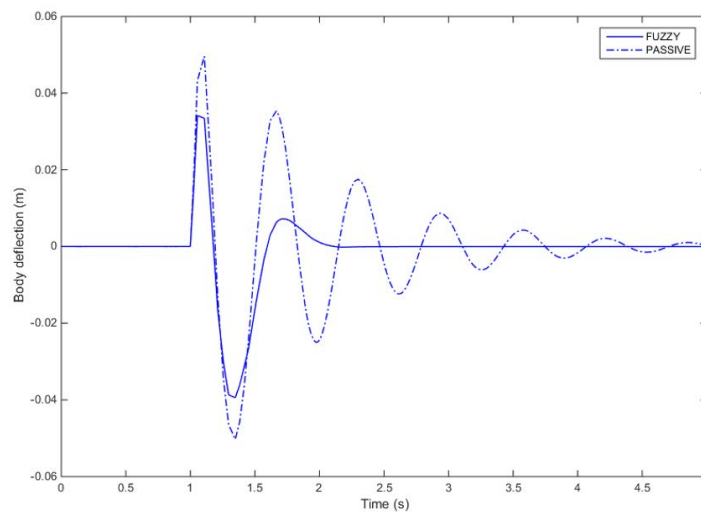

Figure 5. The deflection $(x_1 - x_2)$ with (-) and without (- -) the fuzzy controller
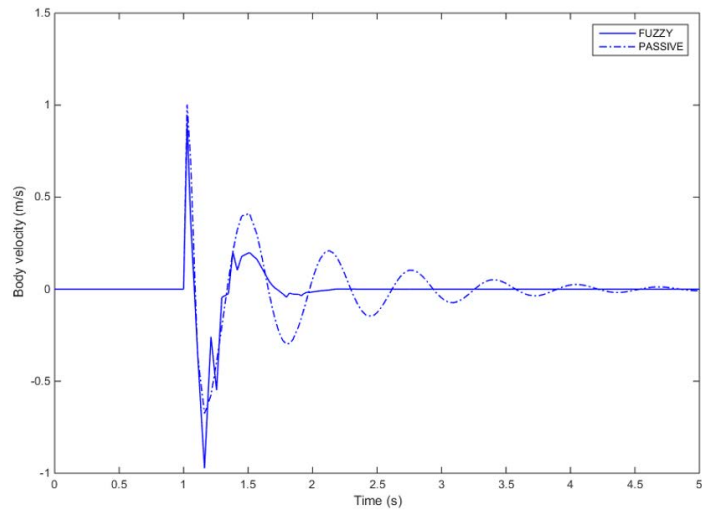
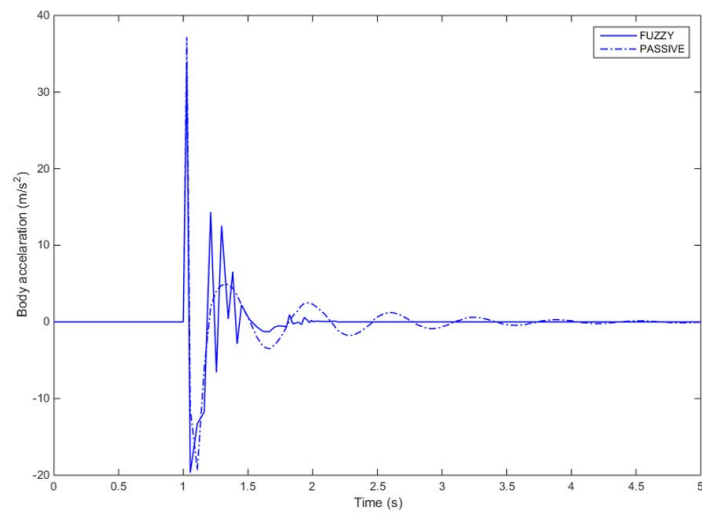**Figure 6.** The velocity $(v_1 - v_2)$ with (-) and without (- -) the fuzzy controller

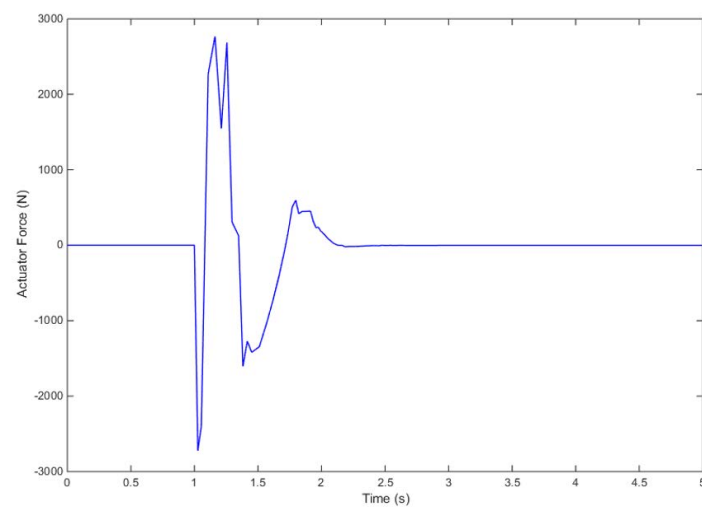**Figure 7.** The acceleration $(a_1 - a_2)$ with (-) and without (- -) the fuzzy controller

Figure 8. The actuator force of the fuzzy controller

The maximum and minimum diversion of the parameters of interest for the system with the fuzzy controller are given in Table 2. Moreover, as seen in Table 3, the vibrations in terms of deflection were suppressed by 26.35% after the application of control. Velocity was increased by 12.02%, due to the high values between 1 and 1.5 seconds, as seen in Figure 6, while acceleration was reduced by 5.66%.

Table 3. Percentages of reduction after the application of fuzzy and PID control

Passive vs. Fuzzy			Passive vs. PID		
$X_1 - X_2(\%)$	$V_1 - V_2(\%)$	$A_1 - A_2(\%)$	$X_1 - X_2(\%)$	$V_1 - V_2(\%)$	$A_1 - A_2(\%)$
26.35	-12.02	5.66	82.43	93.51	94.31

Note: The minus sign in "-12.02" denotes that instead of reduction, there is an increase of velocity $V_1 - V_2$ by 12.02%.

3.2 Use of PID Controller for the Active Suspension System

In the second setup, the PID controller, as described in Subsection 2.3, was applied to the active suspension model. The simulation time was again set to 5 seconds and the control results in terms of deflection, velocity and acceleration of both passive and PID controlled systems are presented in Figures 9, 10, and 11, respectively.
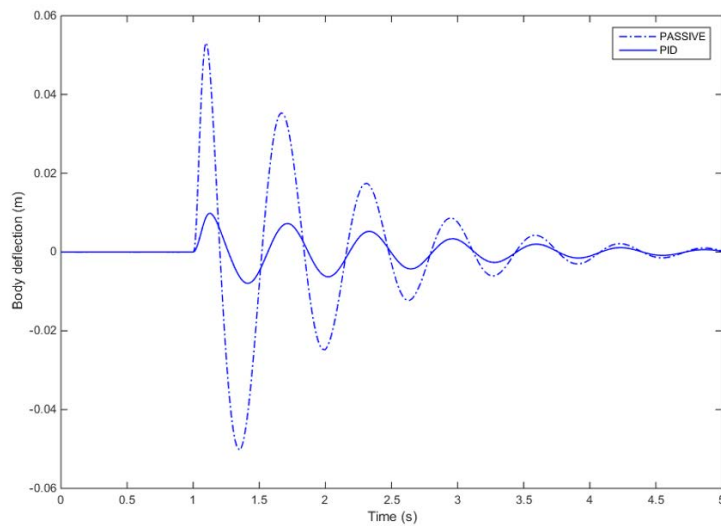

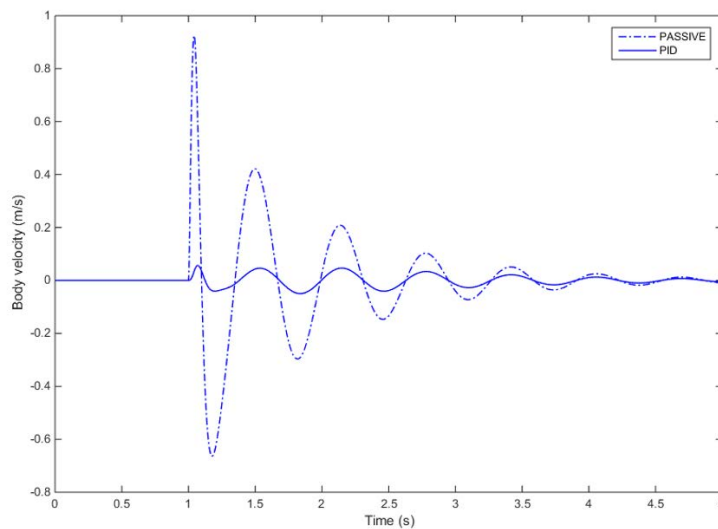**Figure 9.** The deflection $(x_1 - x_2)$ with (-) and without (- -) the PID controller

Figure 10. The velocity $(v_1 - v_2)$ with (-) and without (- -) the PID controller
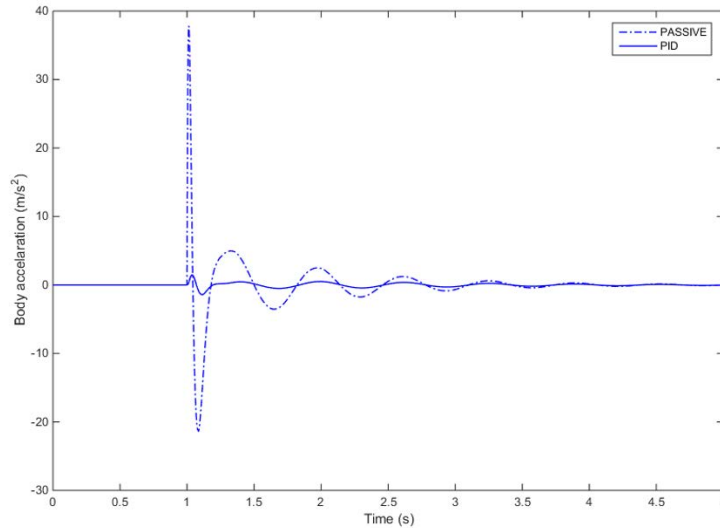
Figure 11. The acceleration $(a_1 - a_2)$ with (-) and without (- -) the PID controller

The maximum and minimum deflection, velocity and acceleration for the active suspension with the PID controller are shown in Table 2. The reduction percentage of vibrations in this case for the three studied variables is given in Table 3. The deflection was suppressed by 82.43% after the application of the PID controller to the system, and, at the same time, velocity and acceleration were also reduced by 93.51% and 94.31%, respectively.

4 Discussion

The passive system, i.e., the suspension model prior to the application of any control, presents significant vibrations. More specifically, the maximum width of these vibrations in terms of deflection, velocity and acceleration is 9.99×10^{-2} m, 1.68 m/s, and 56.5 m/s^2, respectively.

From the numerical experiments of the first control configuration, i.e., the one with the fuzzy system, it showed that the vibrations in terms of deflection settled down right after 1.5 seconds with a maximum width of 7.36×10^{-2} m. The system was also damped in terms of velocity right after 2 seconds of time with a width of 1.88 m/s, while the acceleration settled down before 2 seconds after the external disturbance, presenting a vibration width which equals 53.3 m/s^2.

Correspondingly, in the case where the PID controller was used, it showed that the deflection did not settle down completely within the 5 seconds of simulation, but, instead, it followed the one of the passive system after 2.5 seconds. The maximum width in this case was 1.78×10^{-2} m, which is significantly lower than the corresponding value of the previous case. However, the damping was much slower. A similar behavior was seen also for the velocity of the system, with a maximum vibration width which equals 0.1 m/s. Finally, the acceleration in the case of the system with the PID controller was very smooth, even if it seemed not to settle down within the 5 seconds of the simulation. The width in this case was 2.89 m/s^2 which is an impressive performance.

5 Conclusions

In summary, it can be observed that the active suspension controlled by a fuzzy system damped the vibrations, which were caused by road abnormalities, significantly faster compared to the one equipped with a PID controller. More specifically, the fuzzy controller managed to reduce the vibrations in less than two seconds, while almost 5 seconds or more were required by the PID system for the same assignment. However, PID control seemed to work smoother, especially in the first one to two seconds, as the width of vibrations within this time frame was significantly smaller for all three variables of interest, namely, the deflection, the velocity and the acceleration. For this reason, a hybrid scheme based on the combination of fuzzy systems and PID control may provide better results and thus, it can be an object of further study. Moreover, this hybrid control scheme can be optimized using neural networks, genetic algorithms, or other nature-inspired optimization methods, if necessary. The requirements and the choice of the controller may depend on the need of transported goods or equipment.

Author Contributions

Data Availability

Acknowledgements

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] F. J. D'Amato and D. E. Viassolo, "Fuzzy control for active suspensions," *Mechatronics*, vol. 10, no. 8, pp. 897–920, 2000. https://doi.org/10.1016/s0957-4158(99)00079-3

[2] T. Sun, Z. Huang, D. Chen, and L. Tang, "Signal frequency based self-tuning fuzzy controller for semi-active suspension system," *J. Zhejiang Univ. Sci. A*, vol. 4, no. 4, pp. 426–432, 2003. https://doi.org/10.1631/jzus.2003.0426

[3] S. Foda, "Fuzzy control of a quarter-car suspension system," in *Proceedings of the 12th International Conference on Microelectronics, Piscataway, NJ, USA*, 2000, pp. 231–234. https://doi.org/10.1109/icm.2000.916451

[4] R. Kalaivani, P. Lakshmi, and K. Sudhagar, "Vibration control of vehicle active suspension system using novel fuzzy logic controller," *Int. J. Enterp. Network Manage.*, vol. 6, no. 2, p. 139, 2014. https://doi.org/10.1504/ijenm.2014.066824

[5] S. Palanisamy and S. Karuppan, "Fuzzy control of active suspension system," *J. Vibroeng.*, vol. 18, no. 5, pp. 3197–3204, 2016. https://doi.org/10.21595/jve.2016.16699

[6] Y. M. Sam, J. H. S. Osman, and M. R. A. Ghani, "A class of proportional-integral sliding mode control with application to active suspension system," *Syst. Control Lett.*, vol. 51, no. 3-4, pp. 217–223, 2004. https://doi.org/10.1016/j.sysconle.2003.08.007

[7] F. Zhao, S. S. Ge, F. Tu, Y. Qin, and M. Dong, "Adaptive neural network control for active suspension system with actuator saturation," *IET Control Theor. Appl.*, vol. 10, no. 14, pp. 1696–1705, 2016. https://doi.org/10.1049/iet-cta.2015.1317

[8] G. I. Y. Mustafa, H. P. Wang, and Y. Tian, "Vibration control of an active vehicle suspension systems using optimized model-free fuzzy logic controller based on time delay estimation," *Adv. Eng. Softw.*, vol. 127, pp. 141–149, 2019. https://doi.org/10.1016/j.advengsoft.2018.04.009

[9] O. Demir, I. Keskin, and S. Cetin, "Modeling and control of a nonlinear half-vehicle suspension system: A hybrid fuzzy logic approach," *Nonlinear Dyn.*, vol. 67, no. 3, pp. 2139–2151, 2011. https://doi.org/10.1007/s11071-011-0135-y

[10] V. Bhattacharjee, D. Chatterjee, and O. Karabasoglu, "Hybrid control strategy for a semi active suspension system using fuzzy logic and bio-inspired chaotic fruit fly algorithm," *arXiv*, p. arXiv:1703.08878, 2017. https://doi.org/10.48550/ARXIV.1703.08878

[11] D. X. Phu and S. Choi, "A new adaptive fuzzy PID controller based on Riccati-like equation with application to vibration control of vehicle seat suspension," *Appl. Sci.*, vol. 9, no. 21, p. 4540, 2019. https://doi.org/10.3390/app9214540

[12] A. O. Moaaz and N. M. Ghazaly, "Fuzzy and PID controlled active suspension system and passive suspension system comparison," *Int. J. Adv. Sci. Technol.*, vol. 28, no. 16, pp. 1721–1729, 2019.

[13] G. Tairidis, G. Foutsitzi, P. Koutsianitis, and G. Stavroulakis, "Fine tuning of a fuzzy controller for vibration suppression of smart plates using genetic algorithms," *Adv. Eng. Softw.*, vol. 101, pp. 123–135, 2016. https://doi.org/10.1016/j.advengsoft.2016.01.019

[14] G. K. Tairidis, G. E. Stavroulakis, D. G. Marinova, and E. C. Zacharenakis, "Classical and soft robust active control of smart beams," in *Computational Structural Dynamics and Earthquake Engineering*. Taylor & Francis, 2008, pp. 165–177.

[15] A. Tamaarat, "Active and reactive power control for DFIG using PI, fuzzy logic and self-tuning PI fuzzy controllers," *Adv. Modell. Anal.*, vol. 74, no. 2-4, pp. 95–102, 2019. https://doi.org/10.18280/ama_c.742-408

[16] W. Guo, R. Li, Y. Zhu, T. Yang, R. Qin, and Z. Hu, "A robotic deburring methodology for tool path planning

and process parameter control of a five-degree-of-freedom robot manipulato," *Appl. Sci.*, vol. 9, no. 10, p. 2033, 2019. https://doi.org/10.3390/app9102033

[17] K. B. Phan, H. T. Ha, and S. V. Hoang, "Eliminating the effect of uncertainties of cutting forces by fuzzy controller for robots in milling process," *Appl. Sci.*, vol. 10, no. 5, p. 1685, 2020. https://doi.org/10.3390/app10051685

[18] G. E. Stavroulakis and G. K. Tairidis, *Introduction to Mechatronics.* Kallipos, Open Academic Publications, 2023.

[19] A. S. Emam, "Fuzzy self tuning of PID controller for active suspension system," *Adv. Powertrains Automot.*, vol. 1, no. 1, pp. 34–41, 2015.

Appendix

The Simulink model diagrams which were developed for this study are presented below. The results are presented by means of the Scope blocks; however, if necessary, they can be extracted to MATLAB's workspace using the Simout command. The model for the active suspension with fuzzy control is depicted in Figure A1. Two subsystems, one for the passive and one for the fuzzy system, were developed, as shown in Figures A2 and A3, respectively. The Simulink model for the PID-controlled suspension is depicted in Figure A4. The subsystem of this model is the one of the passive systems, as seen in Figure A3.
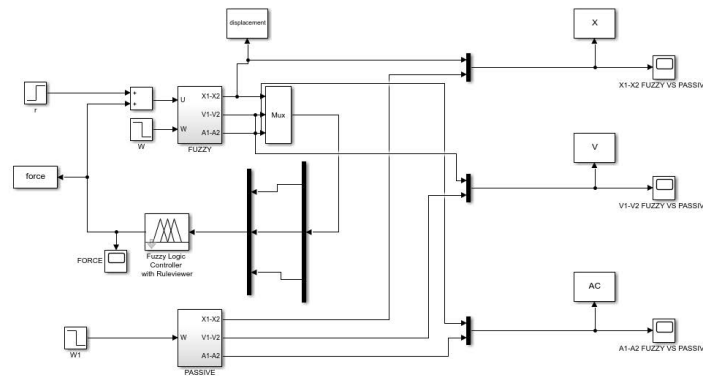

Figure A1. The Simulink model for the comparison between the passive and the fuzzy system
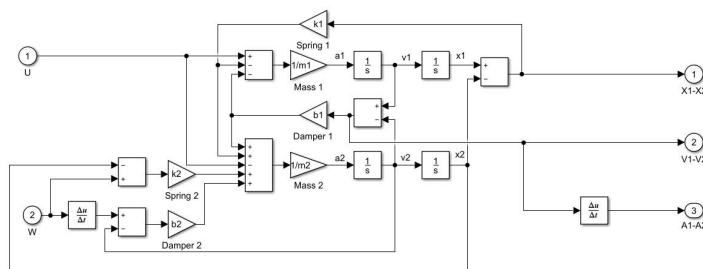

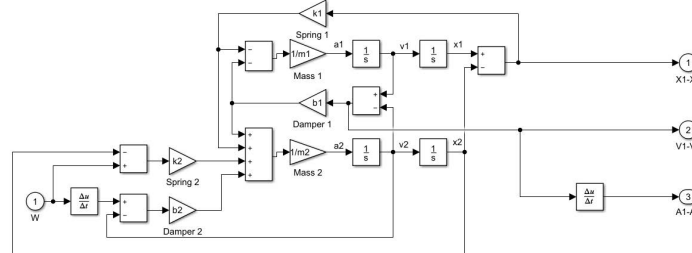**Figure A2.** The Simulink model for the fuzzy subsystem

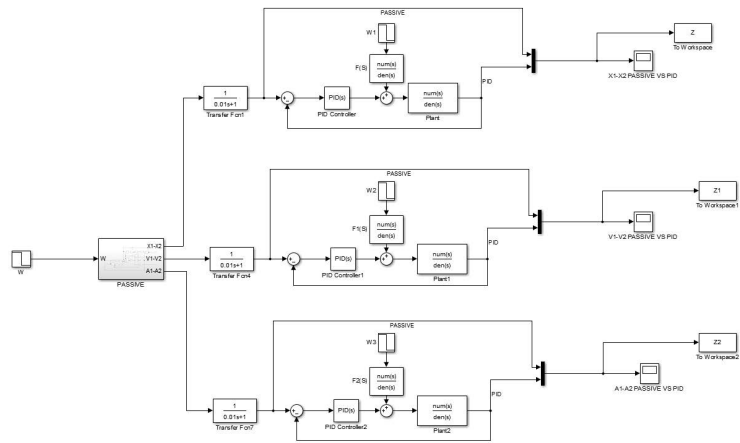**Figure A3.** The Simulink model for the passive subsystem

Figure A4. The Simulink model for the comparison between the passive and the PID system