Enhancing Multi-Criteria Decision-Making with Fuzzy Logic: An Advanced Defining Interrelationships Between Ranked II Method Incorporating Triangular Fuzzy Numbers

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Received: 01-05-2024 Revised: 02-09-2024 Accepted: 02-20-2024


Abstract: In multi-criteria decision-making (MCDM), accurately quantifying qualitative data and simulating real-world scenarios remains a significant challenge, particularly in the presence of inherent imprecision and incompleteness of information. Fuzzy logic, recognized for its capacity to model uncertainty and ambiguity, emerges as a pivotal theory in decision-making processes. This study introduces an enhancement to the Defining Interrelationships Between Ranked Criteria II (DIBR II) method, employing triangular fuzzy numbers with variable confidence intervals for the determination of criteria weight coefficients—essential for assessing their significance and impact on final decisions. The enhanced method, hereafter referred to as the Fuzzy-DIBR II (F-DIBR II), is elaborated upon through a comprehensive description of its algorithmic steps, underscored by a numerical example that highlights its potential. Validation of F-DIBR II is undertaken via a comparative analysis against the traditional DIBR II approach, placing particular emphasis on its application within the Fuzzy Complex Proportional Assessment (COPRAS) framework, geared towards evaluating sustainable mobility measures. This focal point not only reaffirms the necessity of integrating fuzzy logic into the DIBR II methodology but also validates its practical applicability in addressing real-world issues. Contributions of this research extend beyond the theoretical enhancements of fuzzy logic within the MCDM landscape, offering tangible implications for the application of F-DIBR II in sustainable mobility analyses. The consistency in professional terminology throughout the study ensures clarity and coherence, aligning with the stringent standards of top-tier academic journals.

Keywords: Fuzzy numbers; Defining Interrelationships Between Ranked Criteria II (DIBR II); Fuzzy-Defining Interrelationships Between Ranked Criteria II (F-DIBR II); Multi-criteria decision-making (MCDM); COPRAS

1 Introduction

Decision-making is a daily challenge for every individual or organization. The future of both depends on the quality of the decision made. In order to make a decision, it is necessary to have certain parameters (criteria) that determine the choice between two or more options (alternatives). Defining the mentioned selection criteria and the significance of each of them represents one of the most important steps in this process. To define the above, experts from the field are generally hired who will give their opinions regarding the decision-making problem. The evaluation of expert opinions, initially and in the case of simpler decision-making problems, is mainly reduced to quantitative methods that statistically process the obtained data without considering the degree of knowledge of the field by the experts or their objective or subjective feelings related to the answers they give during the survey regarding the subject of research. By developing the field of MCDM and theories that handle inaccuracies and uncertainties well, the above problem is overcome.

Given that the essence of all mathematical models is the imitation of existing reality, it can be concluded that defining the criteria itself is a bigger problem than defining the significance of each of them. In the field of MCDM, this process is based on the application of some of the objective or subjective methods for defining the weight coefficients of the criteria, i.e., their importance and influence on the final decision. Until now, many methods
have been developed that are applied to the aforementioned. One of the first methods used to determine the weight coefficients of criteria was the Analytic Hierarchy Process (AHP), developed by Saaty [1]. This method is based on pairwise comparisons of criteria, using Saaty’s scale. With the need for a better imitation of human thinking when making decisions, considering all the circumstances that influence the decision, as well as the development of new theories that define this area, various improvements to this method have been made. One of the basic theories that deals with the vague, imprecise, and undefined is the fuzzy theory [2]. One of the first improved AHP methods with fuzzy theory was presented in the study of Laarhoven and Pedrycz [3]. In the following years, numerous improvements were made using different types of fuzzy numbers [4–9]. In addition to the AHP method in the previous years, and especially in the last ten years, numerous methods were developed for defining the weight coefficients of the criteria. Some of them are presented below. In 2018, the Full Consistency Method (FUCOM) was developed [10]. Like many other MCDM methods, it has been enhanced by fuzzy theory for application in various decision problems [11–14]. One of the methods that has been recently developed is the level-based weight assessment (LBWA) method [15]. The improvement of the method with fuzzy numbers has been shown in numerous studies [16–19]. One of the methods for defining criteria weights developed in the specified period is the Logarithm Methodology of Additive Weights (LMAW) method [20]. Also, the mentioned method was improved using fuzzy theory and applied in numerous areas [21–23]. The DIBR method was introduced in 2021 [24] and has since found wide application in various fields [25–28], while the fuzzy DIBR method was presented in only one study [29]. The DIBR method was improved by developing the new DIBR II method [30]. Although it is a very young method, it quickly found application in various research studies [31, 32] and has not yet been improved by theories that handle imprecision and indeterminacy well.

This paper presents a new fuzzy DIBR II method as well as the application of this method to the MCDM problem from existing research. In addition, the proposed method was validated by comparative analysis with the results of the classical DIBR II method.

2 Methodology

The improvement of the DIBR II method in a fuzzy environment and its validation, as well as its application in solving various decision-making problems, and specifically the definition of weight coefficients of the criteria, were carried out using the methodology shown in Figure 1.

![Figure 1. Methodology for improving the DIBR II method](image)

Considering that in the introduction of the paper, an analysis of previous research in the field was performed, the selected fuzzifications and the Fuzzy DIBR II method itself are presented below.
2.1 Triangular Fuzzy Numbers with a Variable Confidence Interval

The expansion of fuzzy theory caused the development of different types of fuzzy numbers: triangular, trapezoidal, pentagonal, hexagonal, heptagonal [33], nanogonal, decagonal [34], hexadecagonal [35], sequential [36], diamond [37], picture [38], etc. The focus of the paper is on triangular fuzzy numbers, the basic settings of which are shown below.

Triangular fuzzy numbers have the form \( \tilde{F} = (f_1, f_2, f_3) \), which is shown in Figure 2 [9, 39].

Values \( f_1 \) and \( f_3 \) represent the left and right distribution of the confidence interval of fuzzy number \( \tilde{F} \), while \( f_2 \) represents the place where \( \mu(x) \) has a maximum value (1). The membership function of the fuzzy number \( \tilde{F} \) is defined by the following expressions [39]:

\[
\mu_{\tilde{F}}(x) = \begin{cases} 
0, & x < f_1 \\
\frac{x - f_1}{f_2 - f_1}, & f_1 \leq x \leq f_2 \\
1, & x = f_2 \\
\frac{f_3 - x}{f_3 - f_2}, & f_2 \leq x \leq f_3 \\
0, & x > f_3
\end{cases}
\]

(1)

The value of the degree of confidence ranges in the interval \( \sigma \in [0, 1] \), where the value 1 describes the absolute conviction (100%) of the expert in the given statement. The aforementioned fuzzification will represent the starting point for the development of the fuzzy DIBR II method.

2.2 Fuzzy DIBR II Method

Respecting the steps of the existing DIBR II method and the previously described fuzzification using triangular fuzzy numbers with a variable degree of confidence, the steps of the fuzzy DIBR II method are presented below [9, 30].

Step 1: Identification of the criteria \( K = \{K_1, K_2, ..., K_n\} \), where \( n \) represents the total number of identified criteria.

Step 2: Determining the importance of each of the identified criteria \( K_1 > K_2 > ... > K_n \).

Step 3: Defining the relationship between criteria \( (\theta_{n-1,n}) \):
\[ \tilde{\omega}_1 : \tilde{\omega}_2 = \hat{\theta}_{1,2} : 1 \mapsto \frac{\tilde{\omega}_1}{\tilde{\omega}_2} = \hat{\theta}_{1,2} \quad (3) \]

\[ \tilde{\omega}_2 : \tilde{\omega}_3 = \hat{\theta}_{2,3} : 1 \mapsto \frac{\tilde{\omega}_2}{\tilde{\omega}_3} = \hat{\theta}_{2,3} \quad (4) \]

\[ \ldots \]

\[ \tilde{\omega}_{n-1} : \tilde{\omega}_n = \hat{\theta}_{n-1,n} : 1 \mapsto \frac{\tilde{\omega}_{n-1}}{\tilde{\omega}_n} = \hat{\theta}_{n-1,n} \quad (5) \]

\[ \tilde{\omega}_1 : \tilde{\omega}_n = \hat{\theta}_{1,n} : 1 \mapsto \frac{\tilde{\omega}_1}{\tilde{\omega}_n} = \hat{\theta}_{1,n} \quad (6) \]

where, the relationship between the criteria \( (\hat{\theta}_{n-1,n}) \) is defined by Eq. (2), using the degree of confidence \( (\sigma) \) when defining the relationship (must satisfy the following condition \( \hat{\theta}_{n-1,n} \geq 1 \)), and \( \tilde{\omega} \) represents the fuzzified value of the weight coefficient of the criterion.

Step 4: Defining the relationship between the most significant and other criteria (Eqs. (7) to (9)).

\[ \tilde{\omega}_2 = \frac{\tilde{\omega}_1}{\theta_{1,2}} \quad (7) \]

\[ \tilde{\omega}_3 = \frac{\tilde{\omega}_1}{\theta_{1,2} \odot \theta_{2,3}} \quad (8) \]

\[ \ldots \]

\[ \tilde{\omega}_n = \frac{\tilde{\omega}_1}{\theta_{1,2} \odot \theta_{2,3} \odot \ldots \odot \theta_{n-1,n}} \quad (9) \]

Step 5: Determination of the value of the weight coefficient of the most significant criterion (Eq. (10)).

\[ \tilde{\omega}_1 = \frac{1}{1 + \frac{1}{\theta_{1,2}} + \frac{1}{\theta_{1,2} \odot \theta_{2,3}} + \ldots + \frac{1}{\theta_{1,2} \odot \theta_{2,3} \odot \ldots \odot \theta_{n-1,n}} \quad (10) \]

Step 6: Determination of the value of the weight coefficient of the other criteria (Eqs. (7) to (9)).

Step 7: Defuzzification of the value of the weight coefficient of the criteria (Eqs. (11) to (13)) [9].

\[ def 1F = \frac{(f_3 - f_1) + (f_2 - f_1))}{3} + f_1 \quad (11) \]

\[ def 2F = \frac{\rho f_3 + f_2 + (1 - \rho) f_1}{2} \quad (12) \]

\[ F = \frac{def 1F + def 2F}{2} \quad (13) \]

where, \( \rho \) represents an index of optimism \( \rho \in [0, 1] \), that is, decision-makers’ belief in risk when making decisions, and is used to represent pessimistic, moderate, and optimistic attitudes [40].

Step 8: Determining the quality of the relationship between the criteria, that is, the relationship between the deviation values \( V_n \) (14) and the control value \( \omega^k_n \) (15), which must satisfy the condition that \( 0 \leq V_n \leq 0.1 \):

\[ V_n = \left| 1 - \frac{\omega_k}{\omega^k_n} \right| \quad (14) \]

\[ \omega^k_n = \frac{\omega_1}{\theta_{1,n}} \quad (15) \]
3 Numerical Example

Let five criteria be identified $K = \{K_1, K_2, ..., K_5\}$ in Step 1 and their importance be $K_1 > K_2 > ... > K_5$ in Step 2. Let expert $E_1$ define the relationships between the criteria as follows (Step 3):

$$
\theta_{1,2} = \frac{\omega_1}{\omega_2} = 1.30; \theta_{2,3} = \frac{\omega_2}{\omega_3} = 1.20; \theta_{3,4} = \frac{\omega_3}{\omega_4} = 1.10; \theta_{4,5} = \frac{\omega_4}{\omega_5} = 1.10; \theta_{1,5} = \frac{\omega_1}{\omega_5} = 1.70
$$

Let the expert $E_1$ be sure of the given claims at 90% (the degree of confidence is 0.9). Applying Eq. (2), the following fuzzified values of the relationship between the criteria are obtained:

$$
\tilde{\theta}_{1,2} = \frac{\tilde{\omega}_1}{\tilde{\omega}_2} = (1.2, 1.3, 1.4); \tilde{\theta}_{2,3} = \frac{\tilde{\omega}_2}{\tilde{\omega}_3} = (1.1, 1.2, 1.3); \tilde{\theta}_{3,4} = \frac{\tilde{\omega}_3}{\tilde{\omega}_4} = (1.1, 1.1, 1.2); \tilde{\theta}_{4,5} = \frac{\tilde{\omega}_4}{\tilde{\omega}_5} = (1, 1.1, 1.2);
\tilde{\theta}_{1,5} = \frac{\tilde{\omega}_1}{\tilde{\omega}_5} = (1.5, 1.7, 1.9)
$$

In Step 4, the following relationships between the most important and other criteria are defined, using Eqs. (7) to (9):

$$
\tilde{\omega}_2 = \frac{\tilde{\omega}_1}{\tilde{\theta}_{1,2}} = \frac{\tilde{\omega}_1}{(1.2, 1.3, 1.4)}; \tilde{\omega}_3 = \frac{\tilde{\omega}_1}{\tilde{\theta}_{1,2} \circ \tilde{\theta}_{2,3}} = \frac{\tilde{\omega}_1}{(1.2, 1.3, 1.4) \circ (1.1, 1.2, 1.3)}; \\
\tilde{\omega}_4 = \frac{\tilde{\omega}_1}{\tilde{\theta}_{1,2} \circ \tilde{\theta}_{2,3} \circ \tilde{\theta}_{3,4}} = \frac{\tilde{\omega}_1}{(1.2, 1.3, 1.4) \circ (1.1, 1.2, 1.3) \circ (1, 1.1, 1.2)}; \\
\tilde{\omega}_5 = \frac{\tilde{\omega}_1}{\tilde{\theta}_{1,2} \circ \tilde{\theta}_{2,3} \circ \tilde{\theta}_{4,4} \circ \tilde{\theta}_{4,5}} = \frac{\tilde{\omega}_1}{(1.2, 1.3, 1.4) \circ (1.1, 1.2, 1.3) \circ (1, 1.1, 1.2) \circ (1, 1.1, 1.2)}
$$

Applying Eq. (10) the fuzzified value of the weight coefficient of the most significant criterion is obtained (Step 5).

$$
\tilde{\omega}_1 = \frac{1}{1 + \frac{1}{\tilde{\theta}_{1,2}} + \frac{1}{\tilde{\theta}_{1,2} \circ \tilde{\theta}_{2,3}} + \cdots + \frac{1}{\tilde{\theta}_{1,2} \circ \tilde{\theta}_{2,3} \circ \cdots \circ \tilde{\theta}_{4,5}}} \\
= \frac{1}{1 + \frac{1}{(1.2, 1.3, 1.4)} + \frac{1}{(1.2, 1.3, 1.4) \circ (1.1, 1.2, 1.3)} + \cdots + \frac{1}{(1.1, 1.2, 1.3) \circ (1, 1.1, 1.2) \circ (1, 1.1, 1.2)}} \\
= (0.236, 0.284, 0.330)
$$

In Step 6, the weights of the other criteria are defined, using Eqs. (7) to (9).

$$
\tilde{\omega}_2 = \frac{(0.236, 0.284, 0.33)}{(1.2, 1.3, 1.4)} = (0.202, 0.218, 0.231); \\
\tilde{\omega}_3 = \frac{(0.236, 0.284, 0.33)}{(1.2, 1.3, 1.4) \circ (1.1, 1.2, 1.3)} = (0.187, 0.182, 0.175); \\
\tilde{\omega}_4 = \frac{(0.236, 0.284, 0.33)}{(1.2, 1.3, 1.4) \circ (1.1, 1.2, 1.3) \circ (1.1, 1.1, 1.2)} = (0.187, 0.165, 0.145); \\
\tilde{\omega}_5 = \frac{(0.236, 0.284, 0.33)}{(1.2, 1.3, 1.4) \circ (1.1, 1.2, 1.3) \circ (1, 1.1, 1.2) \circ (1, 1.1, 1.2)} = (0.187, 0.150, 0.119)
$$

After obtaining the fuzzified values of the weight coefficients of all criteria, it is necessary to perform their defuzzification (Step 7), using Eqs. (11) to (13). Applying the above equations and if $\rho = 0.5$, the following values are obtained (Table 1):

| Table 1. The values of the weight coefficients of the criteria using fuzzy DIBR II method |
|-----------------|---------|
| **Criterion**   | **Weight (\(\omega\))** |
| \(C_1\)        | 0.284   |
| \(C_2\)        | 0.217   |
| \(C_3\)        | 0.181   |
| \(C_4\)        | 0.166   |
| \(C_5\)        | 0.152   |
In order to check the obtained results and defined relationships (Step 8), it is necessary to calculate the deviation and the control value (Eqs. (14) to (15)).

\[
\omega_k^5 = \frac{\omega_1}{\theta_{1.5}} = \frac{0.284}{1.7} = 0.167; \quad V_5 = \left| 1 - \frac{\omega_5}{\omega_k^5} \right| = \left| 1 - \frac{0.152}{0.167} \right| = 0.088
\]

Given that the condition from Step 8 is met, it can be stated that the relationships between the criteria are well defined.

4 Validation of the Proposed Methodology

In order to validate the proposed methodology, a comparative analysis of the results of the proposed methodology was performed with the results obtained by the classic DIBR II method [30].

The application of the classic DIBR II method will be shown in the input parameters given in the numerical example of this paper. Steps 1 and 2 are the same as I in the numerical example.

Step 3: Defuzzified values of defined relationships between criteria will be used as input.

\[
\theta_{1.2} = \frac{\omega_1}{\omega_2} = 1.30; \theta_{2.3} = \frac{\omega_2}{\omega_3} = 1.20; \theta_{3.4} = \frac{\omega_3}{\omega_4} = 1.10; \theta_{4.5} = \frac{\omega_4}{\omega_5} = 1.10; \theta_{1.5} = \frac{\omega_1}{\omega_5} = 1.70
\]

By applying Steps 3 to 6 of the DIBR II method [30], the following values of the weight coefficients of the criteria are obtained Table 2:

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Criterion Weight ((\omega)) - DIBR II</th>
<th>Criterion Weight ((\omega)) - Fuzzy DIBR II</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>0.284</td>
<td>0.284</td>
</tr>
<tr>
<td>C_2</td>
<td>0.218</td>
<td>0.217</td>
</tr>
<tr>
<td>C_3</td>
<td>0.182</td>
<td>0.181</td>
</tr>
<tr>
<td>C_4</td>
<td>0.165</td>
<td>0.166</td>
</tr>
<tr>
<td>C_5</td>
<td>0.151</td>
<td>0.152</td>
</tr>
</tbody>
</table>

By checking the relationships, it was found that they are well defined, that is

\[
\omega_k^5 = 0.167; \quad V_5 = 0.099
\]

As can be seen from Table 2, the values obtained by DIBR II are minimally different compared to the values obtained by the fuzzy DIBR II method, i.e., due to the level of expert confidence in the given claims of 90%, in the numerical example, the values of the criteria weights are more nuanced. By reducing the expert’s degree of conviction, the influence of the most important criterion on the final decision increases, so in the case that the expert is 10% convinced of the given claims, the weight of the criteria will be (Table 3):

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Criterion Weight ((\omega))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_1</td>
<td>0.347</td>
</tr>
<tr>
<td>C_2</td>
<td>0.219</td>
</tr>
<tr>
<td>C_3</td>
<td>0.164</td>
</tr>
<tr>
<td>C_4</td>
<td>0.142</td>
</tr>
<tr>
<td>C_5</td>
<td>0.128</td>
</tr>
</tbody>
</table>

In the continuation of the text, the use of the fuzzy DIBR II method will be presented when solving decision problems in the MCDM model with fuzzy COPRAS method.

5 Application of the Fuzzy DIBR II Method to the MCDM Problem

The MCDM problem represents the choice of the best (optimal) alternative from a set of admissible ones, viewed from the perspective of two or more competing criteria. The problems mentioned include defining the criteria that determine the choice, determining their significance, and identifying optimal alternatives using one of the MCDM methods [41–59]. The application of the fuzzy DIBR II method is shown on the problem of sustainable mobility measure evaluation from the existing research presented in the study of Parezanovic et al. [60]. In their research, the
authors defined five criteria (Table 4) that determine the choice of the optimal alternative from a set of 26 alternatives. All criteria are of the benefit type.

**Table 4.** Criteria that determine the choice of the optimal alternative [60]

<table>
<thead>
<tr>
<th>Criterion Label</th>
<th>The Name of the Criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Potential reduce in CO₂ (in kilotonnes)</td>
</tr>
<tr>
<td>C2</td>
<td>Possibility/rationality of short term applications</td>
</tr>
<tr>
<td>C3</td>
<td>Availability for users</td>
</tr>
<tr>
<td>C4</td>
<td>Change in modal split-sustainable mobility</td>
</tr>
<tr>
<td>C5</td>
<td>Public acceptability</td>
</tr>
</tbody>
</table>

Let’s assume that the criteria in Table 4 are stacked according to significance, so that criterion C1 is the most significant, and criterion C5 is the least significant. Let the decision maker define the following relationships between the criteria, as well as in existing research [60]:

\[
\theta_{1,2} = \frac{\omega_1}{\omega_2} = 1; \theta_{2,3} = \frac{\omega_2}{\omega_3} = 1; \theta_{3,4} = \frac{\omega_3}{\omega_4} = 1; \theta_{4,5} = \frac{\omega_4}{\omega_5} = 1
\]

And let him be 50% sure of the given claims, that is, let his degree of conviction be 0.5. By translating the decision maker’s statements into fuzzy numbers using Eq. (2), the following fuzzified values of the relationship are obtained:

\[
\hat{\theta}_{1,2} = \frac{\tilde{\omega}_1}{\tilde{\omega}_2} = \frac{\tilde{\omega}_2}{\tilde{\omega}_3} = \frac{\tilde{\omega}_3}{\tilde{\omega}_4} = \frac{\tilde{\omega}_4}{\tilde{\omega}_5} = (1, 1.5)
\]

By applying Eqs. (7)-(11), the following values of the weight coefficients of the criteria are obtained (Table 5):

**Table 5.** The values of the weight coefficients of the criteria using fuzzy DIBR II method with \( \rho = 0.5 \)

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Criterion Weight (( \omega ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>0.254</td>
</tr>
<tr>
<td>C₂</td>
<td>0.216</td>
</tr>
<tr>
<td>C₃</td>
<td>0.191</td>
</tr>
<tr>
<td>C₄</td>
<td>0.175</td>
</tr>
<tr>
<td>C₅</td>
<td>0.164</td>
</tr>
</tbody>
</table>

Given that the decision-maker is partially unsure of his claims, let us assume that he defines the relationship between the first-ranked and the last-ranked criteria as \( \theta_{1,5} = \frac{\omega_1}{\omega_5} = 1.4 \) that is, \( \hat{\theta}_{1,5} = \frac{\tilde{\omega}_1}{\tilde{\omega}_5} = (1, 1.4, 2.1) \). We come to the fact that the relationships between the criteria are well defined and that \( \omega_5 = 0.181 \) and \( V_5 = 0.096 \). Based on the obtained results and the results obtained in the existing research [60], it can be concluded that it is necessary to take into consideration the degree of conviction of the decision-maker (expert) when defining the weight coefficients of the criteria. In the first case [60], all criteria have equal importance and equally affect the final decision, while using the proposed methodology, knowledge of the research problem area and the certainty of the decision maker (expert) in the defined relationships between the criteria change the importance of each of the criteria, i.e., its influence on the final choice of the optimal alternative.

The calculated values of criteria weights using the fuzzy DIBR II method represent the input data for choosing the optimal alternative from the set of admissible ones, in this case, the fuzzy COPRAS method [60, 61].

In the subject research [60], the linguistic scale \( L \in [1, 2, ..., 5] \) was used to evaluate the criteria, where the confidence interval of each fuzzy number is defined as follows (Eq. (16)):

\[
\tilde{F} = (f_1, f_2, f_3) = \begin{cases} 
  f_1 = f_2 - 0.5, f_1 \leq f_2, f_1 \geq 1, \\
  f_2 = f_2 \\
  f_3 = f_2 + 0.5, f_3 \geq f_2 \\
  \text{if } f_2 = 1 \text{ then } f_2 = f_3 = 1
\end{cases}
\]

The initial step of the fuzzy COPRAS method is the formation of the initial decision matrix, presented in Table 6.
By applying the steps of the fuzzy COPRAS method, presented in studies [60, 61], and implementing the obtained criteria weights (Table 5), the ranking of alternatives of the proposed methodology is obtained, and a comparative view of the proposed and the existing one is shown in Figure 3.

By analyzing the results presented in Figure 3, it can be concluded that the rankings of the alternatives differ in these two studies, so that the impact of defining the significance of each of the criteria on the final ranking of the alternatives is noticeable. In particular, alternative A2 is optimal in the proposed methodology, while in the existing one [60]; it is in tenth place. Also, the A20 alternative was ranked second in this research, while it was ranked first in the existing one. Several last-ranked alternatives in both studies are at the very bottom of the scale, that is, definitively, alternatives A17–A19 and A24–A26, cannot be optimal in any case.

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### Table 6. Initial fuzzy decision matrix

<table>
<thead>
<tr>
<th></th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
<th>C5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>(2.5,3,3.5)</td>
<td>(2.5,3,3.5)</td>
<td>(3.5,4,4.5)</td>
<td>(3.5,4,4.5)</td>
<td>(2.5,3,3.5)</td>
</tr>
<tr>
<td>A2</td>
<td>(3.5,4,4.5)</td>
<td>(3.5,4,4.5)</td>
<td>(3.5,4,4.5)</td>
<td>(3.5,4,4.5)</td>
<td>(3.5,4,4.5)</td>
</tr>
<tr>
<td>A3</td>
<td>(2.5,3,3.5)</td>
<td>(2.5,3,3.5)</td>
<td>(2.5,3,3.5)</td>
<td>(2.5,3,3.5)</td>
<td>(2.5,3,3.5)</td>
</tr>
<tr>
<td>A4</td>
<td>(1.5,2,2.5)</td>
<td>(3.5,4,4.5)</td>
<td>(2.5,3,3.5)</td>
<td>(1.5,2,2.5)</td>
<td>(2.5,3,3.5)</td>
</tr>
<tr>
<td>A5</td>
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Figure 3. Final ranking of alternatives proposed methodology and existing research

Note: This figure has been prepared by the authors
6 Conclusions

In the context of contemporary challenges of MCDM, this paper investigates the importance of applying fuzzy theory to the quantification of qualitative data and the simulation of existing reality. The main goal is to make the methods for dealing with uncertainty and incompleteness in MCDM better by using the new DIBR II method that uses triangular fuzzy numbers and confidence intervals that can change.

The theoretical contribution of this research is reflected in the analysis and improvement of the effectiveness of the existing MCDM method using fuzzy theory in dealing with uncertainty in decision-making. This theory, which has already proven to be a useful tool in modeling uncertainty in decision-making, finds further application in the context of MCDM, where complex and multiple criteria need to be dealt with. The improved DIBR II method, which uses triangular fuzzy numbers with a variable confidence interval, represents an innovative step towards improving accuracy and flexibility in this context. The empirical analysis shows that the improved DIBR II method is clearly better than the original DIBR II. This provides solid evidence for the benefits of triangular fuzzy numbers when dealing with uncertainty. Numerical examples demonstrate how this method effectively balances accuracy and adaptability, allowing analysts and decision-makers to effectively deal with complex and uncertain situations.

The practical application of the improved DIBR II method in the evaluation of sustainable mobility measures additionally confirms its importance in solving concrete challenges in the real world. Integration with the fuzzy COPRAS methodology provides a comprehensive approach to the analysis of sustainable mobility, considering a wide range of factors and criteria. These results not only confirm the theoretical strength of fuzzy theory in MCDM but also demonstrate its real contribution to solving key decision-making problems.

In conclusion, this research represents a significant step towards the advancement of MCDM methods, offering an effective tool for dealing with inaccuracies and incompleteness in real decision-making. The improved DIBR II method with triangular fuzzy numbers with variable confidence intervals not only confirms the theoretical basis of fuzzy theory but also opens the door to the application of this method in various domains of complex decisions, promising to contribute to the improvement of the decision-making process in the future.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References


