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Performance Assessment of DMUs Using Intuitionistic Fuzzy DEA with Complete Ranking and Benchmarking Capabilities

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Abstract: Assessing the performance of decision-making units (DMUs) under intuitionistic fuzzy conditions has emerged as an essential area of investigation in today's performance evaluation studies. The framework demonstrated in Intuitionistic Fuzzy Data Envelopment Analysis (IFDEA) is a way to assess the relative performance of DMUs when the observed data are notably expressed as ambiguity or uncertainty in the inputs and outputs represented by intuitionistic fuzzy numbers (IFN). When the situations define the conditions to use models with traditional input-output distinctions, traditional models are not less applicable when the parameters are vague, thus prompting the need for a set of more flexible tools. In this work, a ranking procedure is utilized that uses the centroid of triangular intuitionistic fuzzy numbers (TIFNs) to address the IFDEA model that defined input and output variable through TIFNs, it allows to calculate the efficiency status of each unit and to differentiate the DMUs between efficient and inefficient groups. An intuitionistic super-efficiency (IFSE) model is provided to obtain a complete ranking of DMUs that identified as efficient. To help decision makers, a reference-set-oriented benchmarking strategy is created to identify relevant peer units of the DMUs identified as inefficient to assist in improving their performance. To demonstrate the strength and practical applicability of the proposed framework, two examples of application are presented, as well as discussed, the technical differences of comparing the outcomes of analysis with the ranking proposals existing in the literature.

Keywords: Data envelopment analysis; Intuitionistic fuzzy numbers; Efficiency assessment; Complete ranking; Benchmarking approach

1 Introduction

The facts in practical scenarios, such as engineering, social sciences, economics, computer science, decision making, and medical diagnosis, frequently exhibit uncertainty, imprecision, ambiguity, or lack of clarity. This data is inherently imprecise and uncertain due to its fuzzy and ambiguous character. The idea of fuzzy sets (FS) was first established by Zadeh [1] to characterize information that is inherently vague and imprecise, in which every element is characterized by its memberships value that shows the degree of belonging. It is clear that there were too many problems when only one memberships value was available and in many of those situations, there is evidence in support of the fuzzy information and some evidence against it. Atanassov [2] observed this and developed the intuitionistic fuzzy set (IFS). An IFS also contains a degree of membership to a FS as well as a degree of non-membership to those FS. An IFS offers a richer representation of uncertainty since both a degree of membership and degree of non-membership are needed to represent hesitation or uncertainty reflect the complexities inherent in our decision-making processes and uncertainties in a complex environment. An IFS quantifies our degree of acceptance and rejection plus the hesitation to decide between the two. These features allow an IFS to better represent fuzzy information in a system. IFS has received tremendous attention from academic and application practitioners. The methodological advantage of the IFS has stimulated applications in logic programming, medical diagnostics, pattern recognition, machine learning, robotics, and forecasting. At a rapid pace, the applications for IFS have evolved significantly so research is continuing to build the theoretical context and make it relevant in real-world decisions. Therefore, increased relevance is placed on decision models built upon intuitionistic fuzzy theory [3–5].

Evaluating the performance of DMUs is not straightforward when the information available is uncertain, fuzzy, and/or incomplete. Often in real-world situations, the decision-makers do not possess detailed knowledge of the input and output parameters. The traditional DEA methodologies suppose that the input and output values are known exactly, which restricts these approaches in dealing with ambiguity. In this endeavor, the IFDEA model has been adopted in the literature. This model incorporates both membership and non-membership information in the evaluation of efficiency. Therefore, this model is better suited for an ambiguous and hesitant environment where both elements are considerations of choice. For IFDEA, the values of inputs and outputs, are typically described by triangular intuitionistic fuzzy numbers (TIFNs). TIFNs provide a means in which to derive a level of uncertainty because they incorporate three key concepts: memberships degree, non-memberships degree, and hesitation. However, it is never easy to compare TIFNs and to rank them correctly. Accurate ranking is important since the results of efficiency will depend on how the fuzzy numbers are ranked and interpreted. Recent studies indicate that there are new ranking functions that allow to increase the discriminatory power of IFDEA models.

Driven by this need, the current study presents a novel and more effective centroid-based ranking function for TIFNs which allows for clearer and more reliable differentiation between TIFNs. Building on this improved ranking function, an IFDEA model is developed to assess the relative efficiency of DMUs operating under intuitionistic fuzzy conditions. An IFSE model is provided to generate a complete rank ordering of all DMUs, to compensate for standard efficiency scores that provide no rank ordering of the efficient units. The study also presents a benchmarking approach that delineates peer units for each inefficient DMU, which can be useful for decision makers to better understand where improvements are needed and inefficient DMUs can learn from more efficient DMUs. Therefore, the combination of efficiency scores, super-efficiency scores, and benchmarking information now presents a more complete perspective on performance. To demonstrate the reliability and usefulness of the proposed models, two numerical case studies are provide. The two case studies illustrate the approach's ability to handle uncertainty and provide useful and interpretable performance rankings.

The remaining portions of this study are organized as follows: Section 2 provides a comprehensive literature analysis on the progress of the IFDEA model and compares it with the suggested technique for solving the IFDEA model. Section 3 presents some essential concepts of TIFS, centroid point and the ranking for TIFN. Section 4 presents framework for the IFDEA model. Section 5 presents the intuitionistic super-efficiency model, which provides a complete ranking of the DMUs and benchmarking units for peer evaluation of inefficient DMUs, helping to identify areas for improvement to become an efficient DMU. Section 6 provides algorithm and step wise solution procedure for efficiency evaluation and complete ranking technique for DMUs in an intuitionistic fuzzy environment. Section 7 provides two numerical examples to demonstrate the validity and usefulness of the proposed method. Additionally, the efficiency scores of the DMUs in the suggested approach are compared with those of the existing approach. Finally, Section 8 focuses about the conclusion and the direction of future study.

2 Literature Review

Data Envelopment Analysis (DEA), is a mathematical approach that uses linear programming (LP) techniques to evaluate the relative efficiency of a group of similar units with multiple inputs and outputs. It is non-parametric, meaning it does not make assumptions about the underlying data, and unit invariant, meaning it is not affected by the scale or size of the units being evaluated. Since 1978, with the introduction of the first DEA model is called as CCR model [6] under the assumption of constant return scale, the DEA has received significant attention in both theoretical and practical applications. The BCC model was established by Banker et al. [7] with the incorporation of the convexity criterion into the CCR model. These two DEA models are widely employed to address many challenges in business and economics, engineering, and industries such as telecommunications, manufacturing, production, transportation, energy, finance, and marketing [8]. The conventional DEA models require accurate data for both inputs and outputs. However, the actual world generally includes a certain level of uncertainty. The field of optimisation under uncertainty has become a very intriguing subject that may be explored through several approaches, such as probabilistic, stochastic, interval, fuzzy, etc. Sengupta [9] was the first to include fuzzy inputs and outputs in DEA to evaluate the performance of DMUs. Subsequently, other authors, researchers, and academicians have proposed various methodologies to address fuzzy DEA (FDEA) models, as evidenced by the bibliometric review studies [10, 11].

The IFDEA model is a mathematical tool used by decision makers to assess the performance of DMUs while considering intuitionistic fuzzy inputs and outputs. In their initial study, Gandotra et al. [12] employed a weighted entropy technique to address the IFDEA model and evaluate the efficiency of suppliers for a software company. Hajiagha et al. [13] employed a logarithmic approach to address the IF-CCR and IF-BCC models and evaluated the efficiency of the 20 divisions within the finance and credit organization. Puri and Yadav [14] employed the expected value approach to solve the Optimistic and Pessimistic IFDEA model. This approach was utilized to assess the efficiency of the 16 bank branches of SBOP in Amritsar district and determine their complete ranking using the super efficiency model. The (α, β)-cut approach is employed to solve the intuitionistic fuzzy CCR, dual CCR, SBM, and Super SBM models for assessing the efficiency of 16 public hospitals in Meerut [15]. Arya and Yadav [16] used expected value approach for solving IF-BCC and IF-Super BCC model for assessing and complete ranking of hospitals. Singh [17] used expected value approach for solving IFDEA/AR model to measure manufacturing industry. The IFDEA model is solved using an aggregate ranking approach to evaluate the efficiency score of the DMUs [18]. Shakouri et al. [19] used a parametric approach to address the parallel and series network IFDEA model to evaluate the relative efficiencies of the DMUs. Javaherian et al. [20] used an expected value approach to solve a two-stage network IFBCC model in order to assess the relative efficiency of the DMUs. Arteaga et al. [21] used an alphabetical technique approach for solving IFCCR model. Sahil et al. [22] employed a parametric methodology to address the IFDEA model by integrating parabolic IFN as both input and output data. Rasoulzadeh et al. [23] employed a genetic algorithm to address the combined Markowitz and cross DEA model, aiming to determine the best portfolio of assets in Tehran Stock exchange. Ardakani et al. [24] utilized the (α, β)-cut approach to solve the two-stage network IFDEA model with TrIFN-inputs and outputs. This approach was used to measure the efficiency of the DMUs in interval form. Recently, Mohanta and Sharanappa [25] used possibility mean approach for solving IFDEA model in order to assess the agricultural performance of different states in India. Mahmoodirad et al. [26] generalized the classical CCR model by designing an input-oriented IFDEA model grounded with TrIFN and an entirely new possibility-driven to find better flexibility and uncertainty to model a healthcare challenge. Furthermore, building on the incorporation of preferences, Gao et al. [27] developed an interactive IFDEA model that allows decision-makers to incorporate a feasibility-based preference and then develop algebraic rules for the IFN, resulting in a solvable linear model presented with comparative advantages. Following this work, Xin et al. [28] furthered the IFDEA work with three-way decision principles along with hesitancy-based correlation measures so that decision-makers could evaluate optimism-neutrality-pessimism-based conclusions with ambiguous multi-input-output situations if desired. Rasoulzade et al. [29] applied a combination of Markowitz's portfolio theory with network DEA along with cross-efficiency analysis under TrIFNs to develop a four-objective portfolio optimization model that provided better portfolio performance. Lastly, Sahil and Lohani [30] also want to develop a two-stage network IFDEA incorporating parabolic IFN in order to provide for higher-order uncertainty. This was validated using data from a public-sector banking application. Most recently, Peykani et al. [31] developed an Adjustable Intuitionistic Fuzzy Network DEA framework that unites IFSs, AFCCP, ME measures, and AED based NDEA in evaluating two-stage systems. This framework serves as a linear, decomposable, and high-discrimination tool verified to work in real-life banking applications. Moradi and Meybodi [32] proposed a series-parallel IFDEA model integrating optimistic and pessimistic perspectives to provide balanced efficiency evaluations across multi-stage systems under diverse uncertainty structures. Researchers have created different variations of DEA by merging advanced FS theories with it to better account for uncertainty and imprecision in input–output data. Some proposed frameworks based on extended fuzzy set theories include DEA models based on bipolar fuzzy sets [33, 34], Fermetean fuzzy sets [35], Pythagorean fuzzy sets [36, 37], neutrosophic sets [38–40], picture fuzzy sets [41, 42] and spherical fuzzy set [43, 44]. Some other combinations, such as fuzzy soft sets, probabilistic fuzzy sets, and interval-valued fuzzy sets, have been optimized with DEA to provide even more flexibility and decision making reliability. Each of these extensions allows for a more realistic efficacy measurement and benchmarking framework for DMUs operating in uncertain and imprecise situations. A comparison was made between the suggested approach and existing techniques, which is presented in Table 1.

Table 1. Development of intuitionistic fuzzy DEA models

Source	Concept / Approach	Data	Models	Benchmarking	Application
[12]	Weighted entropy	SVTIFS	CCR	×	Suppler selection
[13]	Logarithm	IFN	CCR and BCC	×	Finance and credit institution
[14]	Expected value	TIFN	CCR	×	Banking sector in India
[45]	MCDM	TIFN	Hybrid AHP-DEA	×	Healthcare institutions
[17]	Expected value	TIFN	DEA/AR	×	Manufacturing system
[46]	MCDM	IFPR	CCR cross-efficiency	×	Supply chain industry
[15]	α, β-cut	TIFN	CCR & dual CCR	×	Health care sector
[18]	Ranking	TIFN	CCR	×	Numerical example
[16]	Expected value	TIFN	BCC & Supper BCC	×	Health sector
[19]	Parametric	TrIFN	Network DEA	×	Health sector
[47]	MCDM	IFN	CCR	×	Energy sector
[21]	Alphabetical	TIFN	CCR	×	Numerical example
[20]	Expected value	TIFN	Two stage network DEA	×	Numerical example
[22]	α, β-cut	PIFN	CCR	×	Existing Example
[23]	Expected value	TrIFN	Hybrid markowitz-cross-efficiency DEA	×	Portfolio section
[24]	α, β-cut	TrIFN	Two stage network DEA	×	Numerical example
[25]	Possibility mean	TIFN	CCR	×	Agriculture sector
This Work	Ranking	TIFN	CCR and super efficiency	✓	Numerical example

3 Preliminaries

This section presents the fundamental mathematical notions of FS and IFS, which are the theoretical basis of the IFDEA model paradigm. These constructs justify the design of the efficiency evaluation model, complete ranking model, and benchmarking procedures for DMUs.

Definition1. [1] Let U denote a universal space. A FS (A) defined on U is written as

$$A = \{(u, \alpha_A(u)), |, u \in U\}, \tag{1}$$

where, $\alpha_A : U \to [0, 1]$ is the memberships function assigning to each element (u) a numerical grade representing how strongly (u) is associated with the set (A).

Definition2. [2] An IFS (B) on the universe (U) is expressed as

$$B = \{(u, \alpha_B(u), \beta_B(u))|u \in U\}, \tag{2}$$

where, $\alpha_B(u)$ indicates the memberships degree of u, and $\beta_B(u)$ represents the non-memberships degree of u. These functions satisfy the constraint $0 \le \alpha_B(u) + \beta_B(u) \le 1, \forall u \in U$, ensuring that the remaining value $\pi_B(u) = 1 - \alpha_B(u) - \beta_B(u)$ captures the hesitation or uncertainty associated with the element (u).

Definition3. [48] Trapezoidal intuitionistic fuzzy number (TrIFN) is denoted by $A = \langle a^{t_1}, a^{t_2}, a^{t_3}, a^{t_4}; a^{f_1}, a^{f_2}, a^{f_3}, a^{f_4}\rangle$, the memberships and non-memberships degrees of $u \in \mathbb{R}$ is expressed as:

$$\alpha_A(u) = \begin{cases} g_A^L(u) = \dfrac{u - a^{t_1}}{a^{t_2} - a^{t1}}, & a^{t_1} \le u \le a^{t_2} \\ 1, & a^{t_2} \le u \le a^{t_3} \\ g_A^U(u) = \dfrac{a^{t_4} - u}{a^{t_4} - a^{t_3}}, & a^{t_3} \le u \le a^{t_4} \\ 0, & otherwise \end{cases} \qquad \beta_A(u) = \begin{cases} h_A^L(u) = \dfrac{a^{f_2} - u}{a^{f_2} - a^{f_1}}, & a^{f_1} \le u \le a^{f_2} \\ 0, & a^{f_2} \le u \le a^{f_3} \\ h_A^U(u) = \dfrac{u - a^{f_3}}{a^{f_4} - a^{f_3}}, & a^{f_3} \le u \le a^{f_4} \\ 1, & otherwise \end{cases}$$

where, $0 \le \alpha_A(u) + \beta_A(u) \le 1,\ u \in \mathbb{R}$.

Definition4. [48] A TIFN is denoted by $A = \langle a^{t_1}, a^{t_2}, a^{t_3}; a^{f_1}, a^{f_2}, a^{f_3}\rangle$, the memberships and non-memberships degrees of $u \in \mathbb{R}$ is expressed as:

$$\alpha_A(u) = \begin{cases} g_A^L(u) = \dfrac{u - a^{t_1}}{a^{t_2} - a^{t1}}, & a^{t_1} \le u \le a^{t_2} \\ 1, & x = a^{t_2} \\ g_A^U(u) = \dfrac{a^{t_3} - x}{a^{t_3} - a^{t_2}}, & a^{t_2} \le u \le a^{t_3} \\ 0, & otherwise \end{cases} \qquad \beta_A(u) = \begin{cases} h_A^L(u) = \dfrac{a^{f_2} - u}{a^{f_2} - a^{f_1}}, & a^{f_1} \le u \le a^{f_2} \\ 0, & x = a^{f_2} \\ h_A^U(u) = \dfrac{u - a^{f_2}}{a^{f_3} - a^{f_2}}, & a^{f_2} \le u \le a^{f_3} \\ 1, & otherwise \end{cases}$$

where, $0 \le \alpha_A(u) + \beta_A(u) \le 1,\ u \in \mathbb{R}$. It is represented in Figure 1.

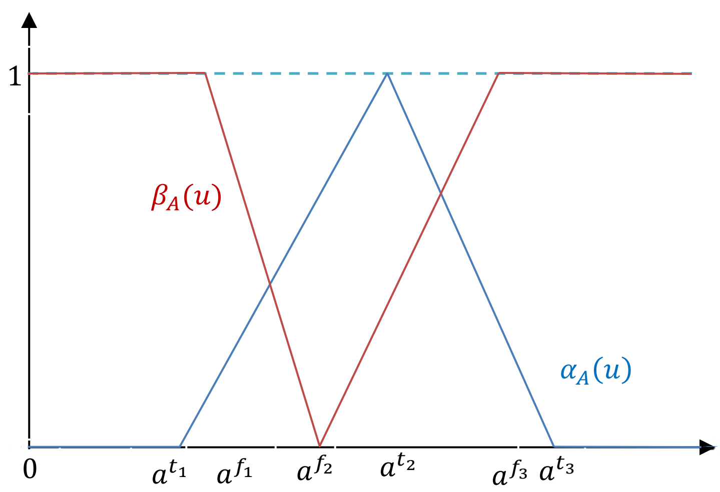

Figure 1. The membership and non-membership degree of TIFN

Definition5. [49] Let $g_A^L(x):\mathbb{R}\to[0,1]$, $g_A^U(x):\mathbb{R}\to[0,1]$, $h_A^L(x):\mathbb{R}\to[0,1]$, and $h_A^U(x):\mathbb{R}\to[0,1]$ are called the sides of the IFN where g_A^L and h_A^U are increasing, and g_A^U and h_A^L are decreasing functions. Therefore, the inverse functions of g_A^L, g_A^U, h_A^L and h_A^U exists which are also same nature. Let $g_A^{L^{-1}}, g_A^{U^{-1}}, h_A^{L^{-1}}$ and $h_A^{U^{-1}}$ are the inverse functions of g_A^L, g_A^U, h_A^L and h_A^R respectively. The above inverse functions are analytically expressed as

$$
\begin{aligned}
g_A^{L^{-1}}(u) &= a^{t_1} + \left(a^{t_2} - a^{t_1}\right) u \\
g_A^{U^{-1}}(u) &= a^{t_4} + \left(a^{t_3} - a^{t_4}\right) u \\
h_A^{L^{-1}}(u) &= a^{f_2} + \left(a^{f_1} - a^{f_2}\right) u \\
h_A^{U^{-1}}(u) &= a^{f_3} + \left(a^{f_4} - a^{f_3}\right) u
\end{aligned}
$$

Definition6. [49] The centroid point $\Big((u_\alpha(A), u_\beta(A)), (v_\alpha(A), v_\beta(A))\Big)$ of the TrIFNs A is determined as follows:

$$
\begin{aligned}
u_\alpha(A) &= \frac{\int_{a^{t_1}}^{a^{t_2}} u g_A^L(u)du + \int_{a^{t_2}}^{a^{t_3}} u du + \int_{a^{t_3}}^{a^{t_4}} u g_A^U(u)du}{\int_{a^{t_1}}^{a^{t_2}} g_A^L(u)du + \int_{a^{t_2}}^{a^{t_3}} du + \int_{a^{t_3}}^{a^{t_4}} g_A^U(u)du} \\
&= \frac{1}{3}\left[\frac{(a^{t_3})^2 + (a^{t_4})^2 - (a^{t_1})^2 - (a^{t_2})^2 - a^{t_1}a^{t_2} + a^{t_3}a^{t_4}}{a^{t_4} + a^{t_3} - a^{t_2} - a^{t_1}}\right] \\
u_\nu(A) &= \frac{\int_{a^{t_1}}^{a^{t_2}} u h_A^L(u)du + \int_{a^{t_2}}^{a^{t_3}} u du + \int_{a^{t_3}}^{a^{t_4}} u h_A^U(u)du}{\int_{a^{t_1}}^{a^{t_2}} h_A^L(u)du + \int_{a^{t_2}}^{a^{t_3}} du + \int_{a^{t_3}}^{a^{t_4}} h_A^U(u)du} \\
&= \frac{1}{3}\left[\frac{2(a^{f_4})^2 - 2(a^{f_1})^2 + 2(a^{f_2})^2 + 2(a^{f_3})^2 + a^{f_1}a^{f_2} - a^{f_3}a^{f_4}}{a^{f_3} + a^{f_4} - a^{f_1} - a^{f_2}}\right] \\
v_\alpha(A) &= \frac{\int_0^1 v g_A^{L^{-1}}(v)dv - \int_0^1 v g_A^{U^{-1}}(v)dv}{\int_0^1 g_A^{L^{-1}}(v)dv - \int_0^1 g_A^{U^{-1}}(v)dv} = \frac{1}{3}\left[\frac{a^{t_1} + 2a^{t_2} - 2a^{t_3} - a^{t_4}}{a^{t_1} + a^{t_2} - a^{t_3} - a^{t_4}}\right] \\
v_\beta(A) &= \frac{\int_0^1 v h_A^{L^{-1}}(v)dv - \int_0^1 v h_A^{U^{-1}}(v)dv}{\int_0^1 h_A^{L^{-1}}(v)dv - \int_0^1 h_A^{U^{-1}}(v)dv} = \frac{1}{3}\left[\frac{2a^{f_1} + a^{f_2} - a^{f_3} - 2a^{f_4}}{a^{f_1} + a^{f_2} - a^{f_3} - a^{f_4}}\right]
\end{aligned}
$$

Definition7. [49] The centroid point $\Big((u_\alpha(A), u_beta(A)), (v_\alpha(A), v_\beta(A))\Big)$ of the TIFNs A is determined as follows:

$$
u_\alpha(A) = \frac{a^{t_1} + a^{t_2} + a^{t_3}}{3};\ u_\beta(A) = \frac{2a^{f_1} - a^{f_2} + 2a^{f_3}}{3}
$$
$$
v_\alpha(A) = \frac{1}{3};\ v_\beta(A) = \frac{2}{3}
$$

Definition8. [49] The ranking function of the TrIFN (or TIFN) A is expressed by

$$
\Re(A) = \sqrt{\frac{1}{2}\Bigg([u_\alpha(A) - v_\alpha(A)]^2 + [u_\beta(A) - v_\beta(A)]^2\Bigg)} \tag{3}
$$

4 Mathematical Formulation of the IFDEA Model

Let there be N DMUs, each transforming P input resources into Q output products. For the j^{th} DMU, the input and output quantities are represented by the vectors $\boldsymbol{a}_j \in \mathbb{R}_+^P$ and $\boldsymbol{b}_j \in \mathbb{R}_+^Q$, respectively. The complete input and output datasets may be written as $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_N] \in \mathbb{R}_+^{P\times N}$, $\boldsymbol{B} = [\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_N,] \in \mathbb{R}_+^{Q\times N}$. Assuming all entries in $\boldsymbol{A}$ and $\boldsymbol{B}$ are strictly positive, the classical DEA framework evaluates the performance of a specific unit, say DMU$_o$, by solving the following optimization model:

$$
\begin{aligned}
\max_{\boldsymbol{\alpha},\boldsymbol{\beta}}\ \ & \eta_o = \frac{\sum_{k=1}^{Q} \alpha_k b_{ko}}{\sum_{i=1}^{P} \beta_i a_{io}} \qquad (4) \\
\text{subject to}\ \ & \sum_{k=1}^{Q} \alpha_k b_{kj} \le \sum_{i=1}^{P} \beta_i a_{ij}, \qquad j = 1, 2, \ldots, N, \\
\text{and}\ \ & \alpha_k \ge 0, \qquad k = 1, 2, \ldots, Q, \\
& \beta_i \ge 0, \qquad i = 1, 2, \ldots, P.
\end{aligned}
$$

where, α_k denotes the weight assigned to the k^{th} output, β_i represents the weight associated with the i^{th}th input, and η_o gives the efficiency score of DMU$_o$. The corresponding linear program (LP_o) is,

$$\begin{aligned} \max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \ & \eta_o = \sum_{k=1}^{Q} \alpha_k b_{ko} \\ \text{subject to} \ & \sum_{i=1}^{P} \beta_i a_{io} = 1 \\ & \sum_{k=1}^{Q} \alpha_k b_{kj} \le \sum_{i=1}^{P} \beta_i a_{ij}, \quad j = 1, 2, \ldots, N, \\ \text{and} \ & \alpha_k \ge 0, \qquad k = 1, 2, \ldots, Q, \\ & \beta_i \ge 0, \qquad i = 1, 2, \ldots, P. \end{aligned} \tag{5}$$

The traditional DEA model, sometimes referred to as the CCR model, can produce misleading assessments of DMU performance as a result of defective, imprecise, or confusing observational data. Moreover, when a DMU demonstrates outstanding performance, it might become an unreliable benchmark for other inefficient DMUs. To tackle such situations, a strong and effective method is needed, utilizing the principles of IFS theory. The construction of an IFDEA model entails employing TIFN as input-output parameters, and the methodology for solving it is described below in the next section.

Let us consider inputs-outputs are TIFNs while the weights $u_r,\ v_i \in \mathbb{R}$. Therefore, the formulation IFDEA model is expressed as follows:

$$\begin{aligned} \max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \ & \eta_o = \sum_{k=1}^{Q} \alpha_k \widehat{b_{ko}} \\ \text{subject to} \ & \sum_{i=1}^{P} \beta_i \widehat{a_{io}} = 1 \\ & \sum_{k=1}^{Q} \alpha_k \widehat{b_{kj}} \le \sum_{i=1}^{P} \beta_i \widehat{a_{ij}}, j = 1, 2, \ldots, N, \\ \text{and} \ & \alpha_k \ge 0, \quad k = 1, 2, \ldots, Q, \\ & \beta_i \ge 0, \quad i = 1, 2, \ldots, P. \end{aligned} \tag{6}$$

where, $\widehat{a_{ij}}$ and $\widehat{b_{kj}}$, are the IFN, for $i = 1, 2, \ldots, P,\ j = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, Q$.

Applying ranking function to each input and output which convert it into corresponding crisp input and output. Then the above Eq. (6) becomes

$$\begin{aligned} \max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \ & \eta_o = \sum_{k=1}^{Q} \alpha_k \Re\left(\widehat{b_{ko}}\right) \\ \text{subject to} \ & \sum_{i=1}^{P} \beta_i \Re\left(\widehat{a_{io}}\right) = 1 \\ & \sum_{k=1}^{Q} \alpha_k \Re\left(\widehat{b_{kj}}\right) \le \sum_{i=1}^{P} \beta_i \Re\left(\widehat{a_{ij}}\right), \quad j = 1, 2, \ldots, N, \\ \text{and} \ & \alpha_k \ge 0, \qquad k = 1, 2, \ldots, Q, \\ & \beta_i \ge 0, \qquad i = 1, 2, \ldots, P. \end{aligned} \tag{7}$$

The optimal solution of crisp LP model, given Eq. (7), is the corresponding optimal solution of the IFDEA model (see Eq. (6)).

Definition9. An efficiency score of 1 indicates that a DMU is efficient, while any lower score classifies it as inefficient.

5 Complete Ranking and Benchmarking Techniques

5.1 Intuitionistic Fuzzy Super Efficiency Model

We present an IFSE model to enable a comprehensive ranking of all DMUs, especially those found to be efficient. In order to handle intuitionistic fuzzy data, this method expands the traditional super-efficiency model [50]. The following is the mathematical formulation:

$$
\begin{aligned}
\max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \ \eta_o &= \sum_{k=1}^{Q} \alpha_k \widehat{b_{ko}} \\
\text{subject to} \quad & \sum_{i=1}^{P} \beta_i \widehat{a_{io}} = 1 \\
& \sum_{k=1}^{Q} \alpha_k \widehat{b_{kj}} \leq \sum_{i=1}^{P} \beta_i \widehat{a_{ij}}, \quad j = 1,2,\ldots,N,\ j \neq o \\
\text{and} \quad & \alpha_k \geq 0, \qquad k = 1,2,\ldots,Q, \\
& \beta_i \geq 0, \qquad i = 1,2,\ldots,P.
\end{aligned} \tag{8}
$$

where, $\widehat{a_{ij}}$ and $\widehat{b_{kj}}$, are the intuitionistic fuzzy numbers (IFN), for $i = 1,2,\ldots,P,\ j = 1,2,\ldots,N$ and $k = 1,2,\ldots,Q$. The corresponding crisp LP model is obtained by using the ranking function to each TIFN-inputs and outputs. The above IFSE model becomes

$$
\begin{aligned}
\max_{\boldsymbol{\alpha},\boldsymbol{\beta}} \ \eta_o &= \sum_{k=1}^{Q} \alpha_k \Re\left(\widehat{b_{ko}}\right) \\
\text{subject to} \quad & \sum_{i=1}^{P} \beta_i \Re\left(\widehat{a_{io}}\right) = 1 \\
& \sum_{k=1}^{Q} \alpha_k \Re\left(\widehat{b_{kj}}\right) \leq \sum_{i=1}^{P} \beta_i \Re\left(\widehat{a_{ij}}\right), \quad j = 1,2,\ldots,N,\ j \neq o \\
\text{and} \quad & \alpha_k \geq 0, \quad k = 1,2,\ldots,Q, \\
& \beta_i \geq 0, \quad i = 1,2,\ldots,P.
\end{aligned} \tag{9}
$$

The optimal solution of Eq. (9), is the corresponding optimal solution of the IFSE model, given in Eq. (8).

5.2 Benchmarking Technique

In traditional DEA, benchmarking methods are employed to assess the performance of DMUs and to identify opportunities to improve DMUs by evaluating their comparison to efficient frontiers. Fuzzy DEA and IFDEA expand traditional DEA techniques to scenarios associated with uncertain or imprecise settings, by representing data using fuzzy sets and intuitionistic fuzzy sets, respectively, which offer greater ability to represent uncertainty and hesitation than traditional fuzzy DEA models. In IFDEA, benchmarking observations focus on reference units for each DMU. Performance targets for a DMU are given fuzzy numbers which consider both the input and output data of the DMU, where the DMUs, the input and output data are gleaned with fuzzy nature of intuitionistic uncertainty. The available methods in IFDEA allows decision makers to generate meaningful and realistic improvement paths for inefficient DMUs, while keeping the fuzzy nature of uncertainty and hesitation in the input and output data. The inefficient DMU might enhance its efficiency score by locating its relevant peers or benchmarking units. The benchmarking units constitute the components of the reference set. The reference set for each DMU may be determined using the provided Eq. (10).

$$
E_o = \left\{ DMU_j : \frac{\sum_{k=1}^{Q} \alpha_k^* \widehat{b_{kj}}}{\sum_{i=1}^{P} \beta_i^* \widehat{a_{ij}}} = 1, j = 1,2,\cdots,N \right\} \tag{10}
$$

and its corresponding crisp one is

$$
E_o = \left\{ DMU_j : \frac{\sum_{k=1}^{Q} \alpha_k^* \Re\left(\widehat{b_{kj}}\right)}{\sum_{i=1}^{P} \beta_i^* \Re\left(\widehat{a_{ij}}\right)} = 1, j = 1,2,\cdots,N \right\} \tag{11}
$$

where, α_k^* and β_i^* are the optimal output and input weights for DMU_o, obtained by solving Eq. (7). The benchmarking units for each DMU are evaluated using the reference set presented in Eq. (11), which assist the decision makers in identifying peers in order to pinpoint the DMU's inefficiencies. The weights (λ) are derived from the envelopment form of the IFDEA model, which is described from the production possibility set (PPS), and serve as intensity variables in the Dual LPP to find the efficient reference point for each inefficient DMU along the production frontier and to help identify where an inefficient unit needs to improve its efficiency. Therefore, the DMU can follow the peer DMUs identified by the $\lambda \geq 0$ weights as a clear target for improving their efficiency.

6 Algorithm for Solving IFDEA Model

The solution technique for IFDEA is given in the flowchart as shown in Figure 2. The process we have utilized to evaluate the performance of the DMUs has been summarized as follows:

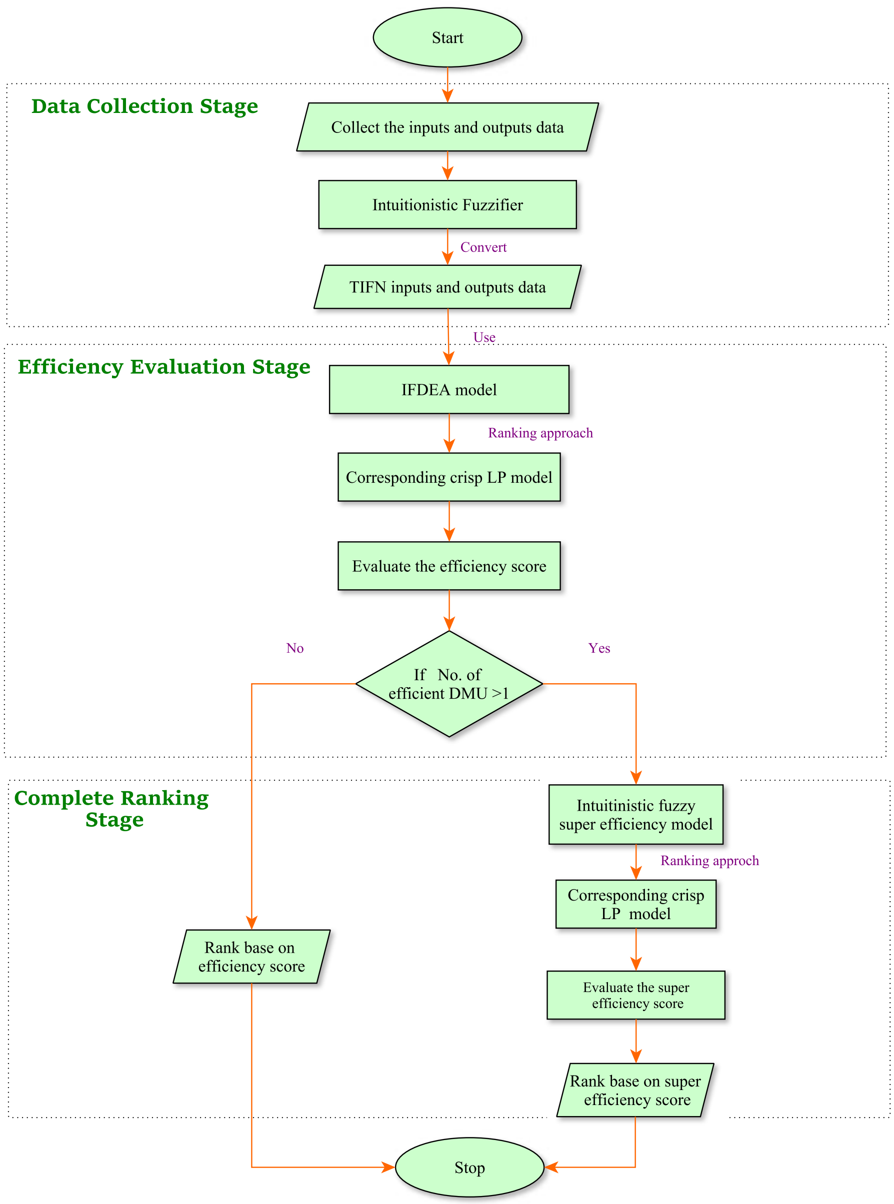

Figure 2. Methodology for evaluating performance of the DMUs using the suggested approach

6.1 Efficiency Calculation

1. The observed input-output data are first converted into their intuitionistic fuzzy counterparts through the chosen fuzzification procedure.

2. The standard DEA formulation is then expressed in the intuitionistic fuzzy setting by representing all inputs and outputs as TIFNs, as described in Eq. (6).
3. A ranking approach is applied to each intuitionistic fuzzy value, allowing the IFDEA model to be converted into an equivalent crisp linear program specified in Eq. (7).
4. The resulting crisp model is solved for every DMU to obtain the corresponding optimal value η_o; $o = 1, 2, \cdots, N$, which reflects its efficiency within the proposed method.
5. Based on the computed efficiency measures, the DMUs are categorized into appropriate performance groups in accordance with the criteria of Definition 9.

6.2 Complete Ranking and Benchmarkinng of DMUs

Ranking all the DMUs in a decision-making problem is an essential component. However, it is a challenge for decision makers to prioritise DMUs when several DMUs belong to an inefficient category. Then the following steps are follow to overcome this issue.

1. To evaluate the efficiency score and the DMUs are categorized into two groups: the efficient group (E_1) and the inefficient group (E_2).
2. Thus, $|E_1| + |E_2| = n > 1$ (n= No. of DMUs).
   (a) If $|E_1| = 1$, then the units are directly ranked using their efficiency score. The unit achieving the highest score placed at the top of the ranking and the unit with the lowest score positioned last.
   (b) If $|E_1| > 1$, then it is difficult to assign which DMUs as top because the efficient DMUs have equal efficiency score (i.e., 1). The following steps should be follow to completely rank the DMUs.
3. The super-efficiency value for each efficient DMU is computed by solving the proposed IFSE model given in Eq. (8).
4. To solve IFSE model by following the step 1 to step 4 of subsection 6.1. The corresponding crisp LP model, provided in Eq. (9), is solved to determine the DMUs' super efficiency score.
5. A DMU's super efficiency score determines its overall ranking. Each DMU is rank according to its super efficiency score; the highest-scoring DMU is at the top, while the lowest-scoring DMU is at the bottom.
6. The reference set in Eq. (11) is used to generate the benchmarking units for each DMU, which aid decision makers in identifying areas where the DMUs could be enhanced their efficiency.

Algorithm 1 is an integrated computational framework that allows for the use of IFDEA model to be turned into a series of steps that can provide efficiency scores, rank entities on a discriminative basis, and establish benchmarks. This framework can be easily utilized within the realm of performance analysis, resource allocation, and decision-making systems by providing a systematic approach for applications in a variety of industries, including supply chain management, healthcare efficiency, and analyzing financial portfolios.

Algorithm 1 IFDEA Model: Efficiency Calculation, Complete Ranking, and Benchmarking

Input: Observed crisp input–output data for all DMUs $j = 1, 2, \ldots, N$.

Phase 1: Initial Efficiency Calculation and Grouping
Step 1.1: Convert all crisp input–output data into Triangular Intuitionistic Fuzzy Numbers (TIFNs) using the chosen fuzzification procedure.
Step 1.2: Formulate the standard Intuitionistic Fuzzy DEA (IFDEA) model as given in Eq. (6).
Step 1.3: Apply the chosen ranking method to convert the IFDEA model into an equivalent crisp Linear Program (LP) as per Eq. (7).
Step 1.4: Solve the crisp LP for each DMU o to obtain its optimal efficiency score η_o.
Step 1.5: Categorize each DMU into group E_1 (efficient, $\eta_o = 1$) or E_2 (inefficient, $\eta_o < 1$) based on Definition 9.
Phase 2: Complete Ranking and Benchmarking
Step 2.1: **if** $|E_1| = 1$ **then**
Rank all DMUs directly by their efficiency scores η_o in descending order.
else **(i.e.,** $|E_1| > 1$ **)**
Step 2.2: For each DMU $k \in E_1$ (efficient group):
    a. Formulate the Intuitionistic Fuzzy Super-Efficiency (IFSE) model for DMU k as in Eq. (8).
    b. Convert the IFSE model to its equivalent crisp LP using the ranking method (Eq. (9)).
    c. Solve the crisp LP to obtain the super-efficiency score η_k^* for DMU k.
Step 2.3: Assign a final composite score $\tilde{\eta}_j$ to each DMU j: $\tilde{\eta}_j = \begin{cases} \eta_j^* \ (\geq 1) & \text{if } j \in E_1 \\ \eta_j \ (< 1) & \text{if } j \in E_2 \end{cases}$
Step 2.4: Rank all DMUs in descending order based on their composite score $\tilde{\eta}_j$.
Step 2.5: For each DMU j, identify its benchmarking units using the reference set defined in Eq. (11).
end if
Output:
    - Efficiency score (η_j) and group (E_1 or E_2) for each DMU.
    - Super-efficiency scores (η_k^*) for DMUs in E_1.
    - Complete ranking of all DMUs.
    - Benchmarking units for each DMU.

7 Numerical Examples

Two numerical examples have been utilised to showcase the practicality and effectiveness of the suggested solution strategies. These examples have been compared with the present results for validation.

7.1 Example 1

In order to illustrate the practicality and effectiveness of the suggested methodology, a numerical example taken from Arya and Yadav [15] is considered. The data set contains five DMUs, each having two input variables and two output variables. The intuitionistic fuzzy representations of the input and output variables are found in Table 2.

Table 2. The intuitionistic fuzzy inputs and outputs data [15]

DMU	Input (I1)	Input (I2)	Output (O1)	Output (O2)
D1	$\langle 3.5, 4, 4.5;\ 3.2, 4, 4.7 \rangle$	$\langle 1.9, 2.1, 2.3;\ 1.7, 2.1, 2.5 \rangle$	$\langle 2.4, 2.6, 2.8; 2.2, 2.6, 3 \rangle$	$\langle 3.8, 4.1, 4.4; 3.6, 4.1, 4.6 \rangle$
D2	$\langle 2.9, 2.9, 2.9; 2.9, 2.9, 2.9 \rangle$	$\langle 1.4, 1.5, 1.6;\ 1.3, 1.5, 1.8 \rangle$	$\langle 2.2, 2.2, 2.2;\ 2.2, 2.2, 2.2 \rangle$	$\langle 3.3, 3.5, 3.7; 3.1, 3.5, 3.9 \rangle$
D3	$\langle 4.4, 4.9, 5.4;\ 4.2, 4.9, 5.6 \rangle$	$\langle 2.2, 2.6, 3;\ 2.1, 2.6, 3.2 \rangle$	$\langle 2.7, 3.2, 3.7;\ 2.5, 3.2, 3.9 \rangle$	$\langle 4.3, 5.1, 5.9; 4.1, 5.1, 6.2 \rangle$
D4	$\langle 3.4, 4.1, 4.8;\ 3.1, 4.1, 4.9 \rangle$	$\langle 2.2, 2.3, 2.4;\ 2.1, 2.3, 2.6 \rangle$	$\langle 2.5, 2.9, 3.3; 2.4, 2.9, 3.6 \rangle$	$\langle 5.5, 5.7, 5.9; 5.3, 5.7, 6.1 \rangle$
D5	$\langle 5.9, 6.5, 7.1;\ 5.6, 6.5, 7.2 \rangle$	$\langle 3.6, 4.1, 4.6;\ 3.5, 4.1, 4.7 \rangle$	$\langle 4.4, 5.1, 5.8;\ 4.2, 5.1, 6.6 \rangle$	$\langle 6.5, 7.4, 8.3; 5.6, 7.4, 9.2 \rangle$

The efficiency values of all DMUs using the IFDEA approach were calculated using the crisp linear programming formulation that is given in Eq. (7). The efficiency scores are shown in Table 3 and serve to evaluate the performance of the units under consideration. Out of the five DMUs represented in this case study, it is noteworthy that three of the DMUs achieve an efficiency value of one indicating that these units fall on the efficient frontier and other DMUs are treated as inefficient. Hence, the ranking of DMUs relates to their efficiency or ability to achieve the most output from the input, as follows: "D2 = D4 = D5 > D3 > D1". Hence, D2, D4, D5, and D2 reach efficient DMUs, while D1 is the least efficient. However, as the efficient DMUs could not be totally differentiated at that point, the DMUs were further ranked according to an IFSE model. The super-efficiency results generated from our suggested ranking method can be found in Table 3, which provides an ideal complete ordering of all DMUs. Given these results, DMU D4 has the highest super-efficiency values, making it the top-ranked DMU. Conversely, DMU D1 has the lowest super-efficiency values and thus is placed last in the ranking. For this reason, the overall ranking for all DMUs is as follows: D4 > D2 > D5 > D3 > D1.

Table 3. Efficiency score comparisons with existing approach [15]

DMU	Proposed Approach			Arya and Yadav Approach	
	Efficiency Score	**Super-Efficiency Score**	**Ranking**	**Efficiency Score**	**Ranking**
D1	0.8565	0.8565	5	0.3569	4
D2	1	1.1543	2	0.38	2
D3	0.8602	0.8602	4	0.3587	3
D4	1	1.1566	1	0.455	1
D5	1	1.0471	3	0.3435	5

The efficiency scores derived from the proposed method are also shown in Figure 3. This figure depicts the efficiency computed through the traditional model and demonstrates the differences in performance evaluations between both techniques. In the suggested approach three DMUs are efficient which works as a benchmark for the other two inefficient DMUs. But in the existing approach it is not possible to determine which DMUs are inefficient or not because all the DMUs efficiency score is less than 1, so we can't find the peer group.

Table 4 provides the benchmarking results of the DMUs with the efficiency coding scores across the five DMUs assessed, three (D2, D4, and D5) were determined to be efficient with an efficiency score of 1. Consequently, these three DMUs represent the best-performing frontier within the analysis and act as reference benchmarks for the inefficient units. The remaining two units (D1 and D3) were determined to be inefficient with efficiency scores of 0.8565 and 0.8602, respectively. This shows that both D1 and D3 are under the efficiency frontier and potentially could improve their performance. That is, the DMUs D1 and D3 would be able to improve their performance by using the efficient DMUs (D2, D4, or D5) as benchmarking standards. The inputs and outputs of both D1 and D3 could be proportionately optimized to reach efficiency based upon the configuration of the DMUs within the benchmarking analysis. The summarization of the benchmarking analysis identifies a clear difference between an efficient category DMU and an inefficient category DMU. The efficient category DMUs can be thought of as performance standards, while inefficient DMUs can be provided guidance into reaching the highest level of efficiency possible through hierarchical performance benchmarking of the best-performing DMUs.

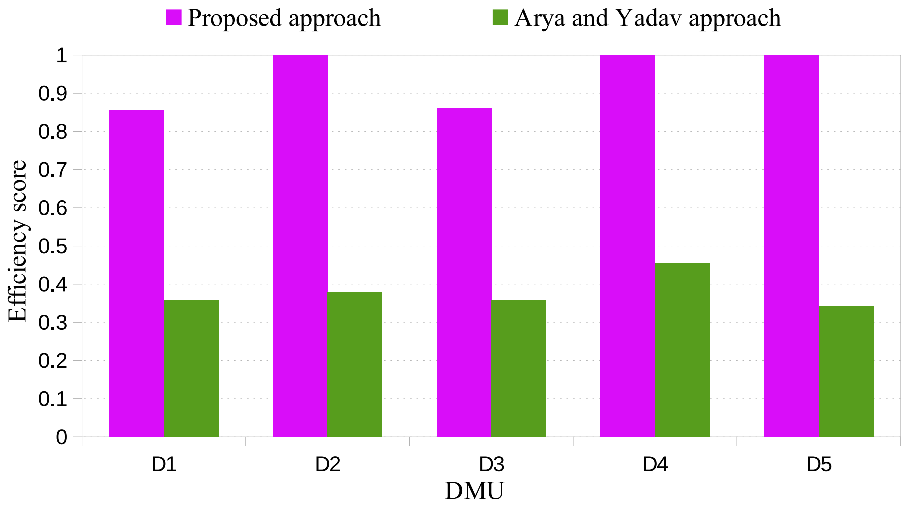

Figure 3. Comparative analysis of DMU efficiency scores under the proposed method versus the existing approach [15]

Table 4. Benchmarking of the DMUs

DMUs	Type (Efficiency Score)	Benchmarking Units	Weights
D1	Inefficient (0.8565)	D2, D4, D5	$\lambda_{D2} = 0.9810,\ \lambda_{D4} = 0.0222,\ \lambda_{D5} = 0.0784$
D2	Efficient (1)	D2	1
D3	Inefficient (0.8602)	D2, D4, D5	$\lambda_{D2} = 0.9794,\ \lambda_{D4} = 0.1261,\ \lambda_{D5} = 0.1502$
D4	Efficient (1)	D4	1
D5	Efficient (1)	D5	1

Also, the reference units listed in Table 4 are examples and targets for all of the inefficient DMUs to learn from. That is, DMU D1 has an efficiency score of 0.8565, while its benchmark (reference) units are D2, D4, and D5. The weightings (λ-values) assigned to each of the reference DMUs indicate how much each of the benchmark units contributes to developing an improvement target for DMU D1. The higher the weight of a benchmark unit, the greater the impact that unit has on developing DMU D1's improvement target. Therefore, since D2 has the highest weight of all three DMUs, DMU D1 should modify its inputs and outputs to more closely match those of D2. Again, DMU D3 also has DMU D2 and DMU D4's and DMU D5's combination used to create its improvement target. Thus, the benchmarking process creates a clear measurable direction for inefficient DMUs to follow in order to become efficient DMUs through weighting the combination of their efficient counterparts.

7.2 Example 2

To show the applicability and robustness of the proposed methodology, the benchmark used here comes from existing literature [14]. There are ten DMUs with two inputs and two outputs in the study.

The evaluation framework provided in Subsection 6.1 is employed to evaluate the operational performance of the DMU. The inputs and outputs consider the TIFNs, to formulate and solve the respective crisp linear programming models. Subsequently, we generate efficiency measures for each DMU based on the proposed approaches and are noted in Table 5. Among all DMUs, D1, D2, and D3 have an efficiency score equal to 1, meaning these DMUs are fully efficient while the remaining DMUs are inefficient. In fact, the DMU D5 has the lowest efficiency score of 0.7592, making it the least efficient unit. Because the only fully efficient DMUs are D1, D2, and D3, we cannot rank the efficient DMUs based on relative efficiency. To gain a complete ranking for all DMUs, we also use the super-efficiency potential described in Subsection 6.2. The super-efficiency scores were obtained from the IFSE model (Table 5) and form the basis for ranking DMUs. The DMU with the largest super-efficiency score assigned the one being replaced as rank 1, while the DMU with the smallest value scored the last rank. Consequently, the overall ranking is: D1> D2> D4> D12> D9 >D11> D7> D3> D10> D8> D5. Thus, the DMUs are completely ranked by using IFSE model.

The efficiency scores obtained in the suggested approach and existing approach are compared in Figure 4. We have found that in the proposed approach three DMUs (i.e., D1, D2, D4) are efficient, whereas in the existing approach their are four DMUs (i.e., D1, D2, D4, D12) are efficient. The DMU D5 is consistently ranked last in both approaches, with the lowest efficiency score.

Table 5. Efficiency score comparisons with existing approach [14]

DMU	Proposed Approach			Puri and Yadav Approach		
	Efficiency Score	**Super Efficiency Score**	**Ranking**	**Efficiency Score**	**Super Efficiency Score**	**Ranking**
D1	1	1.4101	1	1	1.448	1
D2	1	1.1833	2	1	1.1806	2
D3	0.8829	0.8829	8	0.9243	0.9243	8
D4	1	1.0167	3	1	1.0267	4
D5	0.7592	0.7592	12	0.7726	0.7726	12
D6	0.8338	0.8338	10	0.8394	0.8394	10
D7	0.8916	0.8916	7	0.9502	0.9502	7
D8	0.7966	0.7966	11	0.8151	0.8151	11
D9	0.9619	0.9619	5	0.983	0.983	5
D10	0.872	0.872	9	0.877	0.877	9
D11	0.9528	0.9528	6	0.9623	0.9623	6
D12	0.9679	0.9679	4	1	1.1106	3

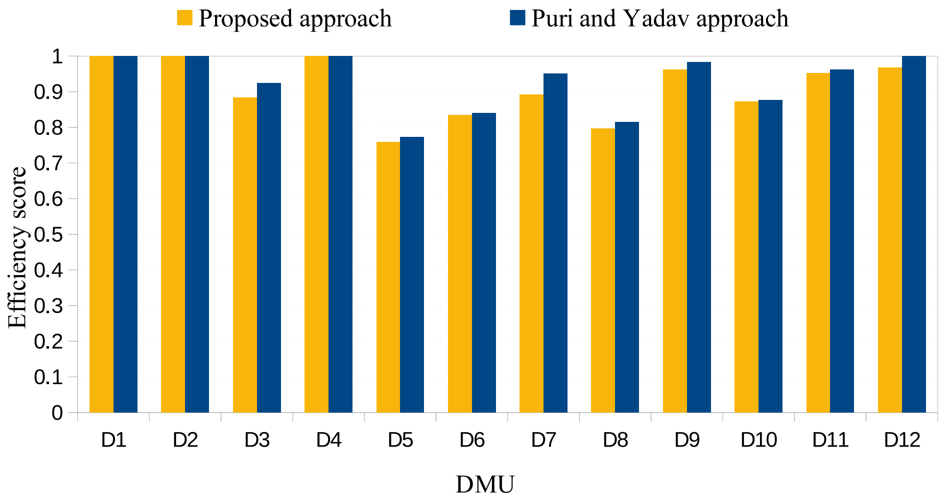

Figure 4. Comparison of the efficiency scores obtained in proposed method versus existing method [14]

Table 6 illustrates the benchmarking results of the DMUs with respect to the efficiency scores reached in the assessment. Among the efficiency scores, three DMUs (D1, D2, and D4) are considered efficient, with an efficiency score equal to one (1). This means that DMUs D1, D2, and D4 operate on the efficient frontier for their benchmarking group, and are therefore classed as reference DMUs for their benchmarking group of DMUs. The remaining nine DMUs are inefficient DMUs, with efficiency scores that range from 0.7592 to 0.9679. This means that inefficient DMUs can improve their performance by utilizing input-output combinations based on the benchmark DMUs. Furthermore, the three DMUs that are efficient (D1, D2, and D4) served as reference DMUs to many of the inefficient DMUs. For example, the DMUs D3, D6, and D7 made comparisons to both D2 and D4, while D5, D8, D10, and D11 made comparisons to D1 and D4. The significant overlap with D1, D2, and D4 would indicate that those DMUs are relative to optimal DMUs that are successfully implementing efficient operational practices that can be replicated under similar input-output usage. Two other DMUs (D9 and D12) had relatively higher efficiency scores (0.9619 and 0.9679, respectively), meaning these also approach the efficient frontier but have minor changes to be fully efficient. All benchmark DMUs made by the inefficient DMUs suggest enhancements in the DMUs resource allocation and operational practices could lead to the inefficient DMUs moving along to a higher performance level.

Also, it helps in demonstrating how inefficient DMUs may enhance themselves by employing efficient DMUs as comparison groups. Reference DMUs act as models. For each inefficient DMU, DEA finds associated efficient DMUs and assigns an amount (λ) to determine how much of each reference creates the ideal situation for that specific DMU. All weighted configurations result in creating a “virtual efficient unit” that indicates what the inefficient DMU needs to attain as the efficient score to become fully functional. For instance, DMU3 utilizes D2 and D4 as benchmarks with weightings of 0.9443 for D2 and 0.1026 for D4; therefore, D2 has a more substantial impact on D3 than D4. Similarly, DMUs D5, D8, D10 and D11 primarily utilize D1 and D4. As these units have determined which reference(s) to use, the inefficiencies of each unit are clearly documented regarding whether to reduce or increase either the input or output into the DMU and to what degree in order to become efficient.

Table 6. Benchmarking units for each DMUs

DMUs	Type (Efficiency Score)	Benchmarking Units	Weights
D1	Efficient (1)	D1	1
D2	Efficient (1)	D2	1
D3	Inefficient (0.8829)	D2, D4	$\lambda_{D2} = 0.9443,\ \lambda_{D4} = 0.1026$
D4	Efficient(1)	D4	1
D5	Inefficient (0.7592)	D1, D4	$\lambda_{D1} = 0.5515,\ \lambda_{D4} = 0.2146$
D6	Inefficient (0.8338)	D2, D4	$\lambda_{D2} = 0.0547,\ \lambda_{D4} = 1.2251$
D7	Inefficient (0.8916)	D1, D2, D4	$\lambda_{D1} = 0.1918,\ \lambda_{D2} = 1.0171,\ \lambda_{D4} = 0.2681$
D8	Inefficient (0.7966)	D1, D4	$\lambda_{D1} = 0.3968,\ \lambda_{D4} = 0.6205$
D9	Inefficient (0.9619)	D1, D2	$\lambda_{D1} = 0.6256,\ \lambda_{D2} = 0.8529$
D10	Inefficient (0.872)	D1, D4	$\lambda_{D1} = 0.0329,\ \lambda_{D4} = 1.3651$
D11	Inefficient (0.9528)	D1, D4	$\lambda_{D1} = 0.8563,\ \lambda_{D4} = 0.9637$
D12	Inefficient (0.9679)	D1, D2	$\lambda_{D1} = 0.6510,\ \lambda_{D2} = 1.2465$

8 Conclusions and Future Direction

DEA is a well-established non-parametric method for the evaluation of the relative efficiency of DMUs that use several inputs to produce several outputs. However, in practice, the data available to conduct such evaluations often contain uncertainty, imprecision, or partially missing information, which renders the traditional DEA method unusable for reliable performance evaluation. To address this shortcoming, several extensions of DEA have been proposed, including FDEA models that employ fuzzy logic to represent to uncertainty and vagueness in input–output data. Even so, fuzzy sets are sometimes not able to fully capture the variety of uncertainties existing in practical datasets that are often encountered in real-world problems when there is some hesitancy and ambiguity concerning their degrees of membership and non-membership. So that, this article proposes an IFDEA, which is able to accurately handle uncertainty of TIFNs. Within the confines of the conventional DEA framework, the proposed model builds on the properties of IFS, which simultaneously representing degrees of membership, non-membership and hesitancy can be considered together, which provides a realistic, comprehensive representation of uncertainty.

The IFDEA can evaluate the efficiency of the DMUs and identify them as being either efficient or inefficient and establishes a full ranking of the DMUs through a super-efficiency model. This would make it possible to separate units that achieve the same efficiency score under the focus on the non-parametric DEA method. In addition to evaluating efficiency, the study also presents the benchmarking process to identify peer DMUs/units for the inefficient units. The basis of the benchmarking process is to guide the Decision-Making Units (DMUs) to units they should target their performance based on, which would then assist the inefficient units in their quest to improve their operational efficiency and move toward the efficient frontier. The usability and robustness of the proposed IFDEA model were confirmed using two previously established numerical examples. The reported computational results illustrate ease of use as well as validity and effectiveness of the suggested methodology. One of the key benefits of the proposed IFDEA approach method is that it offers an utterly comprehensive and efficient way to model and inform the decision-making process by representing and processing the intuitionistic fuzzy information.

The aforementioned methodology introduced is easily extensible to many sophisticated DEA models to favor better usability and analytical reliability. Future studies can focus on the development of intuitionistic fuzzy BCC, SBM, Undesirable DEA, Network DEA, Dynamic DEA, and other sophisticated DEA models. These can be used to analyze efficiencies under varying returns to scale, including undesirable outputs and the analysis of multi-stage or time-subjective systems under ambiguity. Additionally, the proposed methodology can be extended to MCDM methods, such as AHP, TOPSIS, VIKOR, and related MCDM methods, and developed intuitionistic fuzzy framework for MCDM. Including the methodologies developed for ranking and weighting into the MCDM approach would ultimately diversify, in a positive manner, the utility functions available for decision-making and will add realism and flexibility in making decisions under ambiguity.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The author declares no conflict of interest.

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