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A Few Maclaurin Symmetric Mean Aggregation Operators for Spherical Fuzzy Numbers Based on Schweizer-Sklar Operations and Their Use in Artificial Intelligence

Mehwish Sarfraz

Department of Mathematics, Riphah International University (Lahore Campus), 54000 Lahore, Pakistan

* Correspondence: Mehwish Sarfraz (mehwish.sarfraz.ms@gmail.com)

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Abstract: One significant benefit of the Maclaurin symmetric mean (MSM) is that it is a generalization of many extend operators and can consider the interrelationships among the multi-input arguments, such as multi-attributes or multi-experts in the multi-attribute group decision making (MAGDM). In the information fusion process, the Schweizer-Sklar T-norm (TN) and T-conorm (TCN), an important class of the TN and TCN, have more flexibility. We define SS operational rules of SFNs and extend SSTN, SSTCN to Spherical fuzzy values (SFVs) in order to fully utilize the advantages of SSTN, SSTCN, and MSM. Next, by combining the MSM with SS operational rules, we propose the spherical fuzzy Schweizer-Sklar weighted Maclaurin symmetric mean (SFSSWMSM) and spherical fuzzy Schweizer-Sklar Maclaurin symmetric mean (SFSSMSM) operators. This research examines their advantages and creates a novel approach based on these operators for particular MAGDM issues. Then, by comparing the suggested technique with current approaches in practical settings, its benefits and viability are demonstrated. Lastly, a few real-world examples are provided to demonstrate the applicability and benefits of the suggested approach in comparison to a few other approaches already in use.

Keywords: SFNs; SS; MSM; SFSSMSM; SFSSMSM; MAGDM

1 Introduction

Artificial intelligence has a significant impact on a wide range of fields and aspects of modern life. With the introduction of various AI-based tools, a range of problems can now be addressed with varying degrees of effectiveness. Evaluating the performance and capabilities of these AI tools is essential, especially when it comes to decision-making (DM). However, the DM process is inherently uncertain, and real-world scenarios frequently involve the aggregation of information. To solve these problems, several models have been put forth to enhance decision-making procedures. These models aim to lower uncertainty, improve information aggregation, and provide workable solutions for complex and dynamic real-world scenarios. The continuous development and enhancement of models facilitates the ongoing evolution of decision-making methodologies in the era of artificial intelligence.

Zadeh [1] created the fuzzy set (FS) theory in 9965. In an FS, a membership degree (MD), represented by the symbol μ, stands for all unexplained phenomena or subcategories of human opinion. The non-membership degree (NMD), represented by the symbol η, is obtained by deducting the MD 0 from 1. In light of this theory, Atanassov [2] questioned if FSs could be used to represent ambiguity in human opinion and developed the idea of IFSs, which combine an FS with an incorrect event. Atanassov states that the sum of μ and η may range from 0 to 1. Because it is necessary for their sum to remain within the range [0, 1]. μ and η can never be assigned independently in an IFS since their sum will sometimes be greater than this range. In order to accommodate a decision-maker, Yager [3] expanded the domain for assigning μ and η, abstracting the concept of IFS to Pythagorean FS (PyFS). Even the Pythagorean FS (PyFS) limitations, according to Yager [4], may place limitations on decision-makers in a particular field and cause issues. This leads him to develop the concept of q-rung orthopair FS (q-ROFS), which offers human opinion formation in the face of ambiguity an infinite range.

The abstinence degree (AD) and refusal degree (RD) of an element of PFS are represented by ϑ and δ, respectively. Cuong [5] presented a complex form and structure known as a PFS, which is capable of characterizing four aspects of ambiguous data denoted by μ, ϑ, η, and δ, subject to the requirement that the sum of the three components be contained in [0, 1] (i.e., $sum\,(\mu, \vartheta, \eta) \in [0, 1)$. By extending the use of PFS while taking into account the shortcomings of all the previously mentioned concepts [6]. The SFS's limitation is that, even though the total (μ, ϑ, η) may exceed the unit interval, the sum of their squares, $(\mu^2, \vartheta^2, \eta^2) \in [0, 1]$. Due to this new constraint, there is a larger region in the SFS than the PFS where uncertain parameters can be assigned. When there are three pieces of information, sometimes even squaring all the uncertain parameters is insufficient because the square sum is greater than the unit interval, which causes problems. To address these situations, Mahmood et al. [6] proposed a variant of SFS which has no restrictions on the values assigned to μ, ϑ, and η. SFS now requires the sum $(\mu^2, \vartheta^2, \eta^2) \in [0, 1]$ wheren $\in \mathbb{Z}+$.

1.1 Literature Review

In the year 1960, SS [7] invented the concepts of the Schweizer-Sklar aggregation tools. Owing to the increased flexibility of these aggregation systems, decision-makers can receive reliable information. Based on various fuzzy conditions, a number of mathematicians forecasted several aggregation models using dependable attributes of the SS aggregation tools. For example, utilizing the under IF information of basic Hamacher aggregating tools, Garg et al. [8] presented numerous unique aggregation strategies. The properties of the SS aggregation tools are used to express correlation between independent arguments. By using IF information, Khan et al. [9] classified a class of novel techniques. Zhang [10] devised an original technique for the interval-valued intuitionistic fuzzy (IVIF) information decision-making system. Also, we looked at a few of the robust characteristics and features of the SS aggregation tools across a range of fuzzy settings, as mentioned in references [11–13]. Yu and Xu [14] developed the concept of PAOs for the SF set. This also uses PAOs to identify the data gathering into a singleton set. According to research, limits and equal weighting of standards and decision-makers are necessary for all of the previously provided data to function at its best. Some have concluded different operator types based on different norms when dealing with priority data; however, they cannot use the SS norms because some researchers have developed different operator types that conflict with the priority degree. Assuming a priority degree, all fuzzy set theory organizations face an incredibly difficult task: emerging the system of aggregation operators based on SS t-norm (SSTN) and t-conorm (TCM). SS generalized power aggregation operators were made available to SF sets through [15].

Helmers and Weiss [16] created the concept of using MADM for battery life cycle assessment. Helmers et al. [17] considered the idea of a life cycle assessment for electric cars using actual data derived from MADM. Lundström and Hellström [18] extended the use of an app to evaluate electric vehicles. MADM application chose the electric passenger automobile [19]. Use of traction supply system based on MADM for charging electric vehicles [20, 21]. Evaluation of conventional fuel-powered cars with a greener nature on MADM [22]. In MADM, a sophisticated and delicate study is conducted to choose electric vehicles. The MADM optimization life cycle evaluation application for passenger cars has been expanded [23]. The studies [20, 24] explained how to use the supply system to charge electric vehicles. Hao et al. [25] considered the notion of vehicle emissions from liquefied petroleum gas-powered vehicles. In gasoline exhaust emissions, the hydrocarbons were expanded [26]. Given the notion of life cycle evaluation of mid- range passenger Carson based on the liquid and gaseous [27]. Considering the idea of using a scenario analysis framework to examine passenger cars both now and in the future. Sarfraz et al. [28] gave the concept of prioritization aggregation operator using the IFS. Ullah et al. [29] gave the concept of prioritized Aczel-Alsina using the complex IFS. Berre et al. [30] expanded the theory of Schweizer-Sklar TN and TCN using the Pythagorean fuzzy rough set.

Berre et al. [31] gave the concept of artificial intelligence. Lee et al. [32] developed the theory of artificial intelligence based on technologies. Shan et al. [33] gave the concept of artificial intelligence. Zhou et al. [34] used the application of artificial intelligence. Wan et al. [31] developed the application of artificial intelligence. Lawal et al. [35] developed the concept of artificial intelligence. Wan et al. [36] used the application of artificial intelligence.

1.2 Aim of the Study

Of course, all of the BM and HM operators have this function. However, they can only consider the relationships between two combined arguments. The MSM operator is more suitable for solving actual MAGDM problems because it uses a variable parameter to account for the interrelationships among any multiple arguments. It is necessary to fully consider the interrelationships among multiple aggregated arguments in some practical problems.

1.3 Motivation for the Research

Furthermore, because there is an infinite set of possible parameter values for Schweizer-Sklar operations, the parameters can be adjusted to reflect the different risk attitudes of decision-makers, including risk aversion and preference, which makes the method more flexible and appropriate for real-world MAGDM problems. It is important and useful to extend the MSM operator to SFNs based on the Schweizer-Sklar operations in order to address the MAGDM issues with SFNs. Inspired by these ideas, the goal of this work is to present a new Spherical fuzzy

MSM operator based on these new rules, and some new Schweizer-Sklar operational rules within the Spherical fuzzy environment.

1.4 Contributions

• Some conclusions from the MSM and SS discussed above in the form of the key points are given below.

• All the MSM and SS for the IFS, PYFS, qROFS, PFS and SFS are outdate because these frameworks can extract very limited information from the real-life scenarios. Hence, the decisions makers cannot find the best results due to the involvement of the uncertainty and the information loss. Hence, the advanced AI should be defined for the SFNs.

• Some of the MSM and SS discussed above are failed to compute at some special scenarios. For example, some of the MSM and SS do not provide the decision results due to the division by zero problems. Therefore, to improve the identification ability of the MSM and SS overcome the defects of current MSM with SS, it is very necessary to propose a new operator.

1.5 Organization of the Study

This essay is organized generally as follows. In section 2, we primarily introduce some basic SFSs, combining MS and SS operators, as well as the ideas of SSTCN and SSTN. In section 3, we demonstrate two spherical fuzzy combining MS and SS operators using the SSTN and SSTCN: The spherical fuzzy Schweizer-Sklar weighted MSM (SFSSWMSM) and the spherical fuzzy Schweizer-Sklar MSM (SFSSMSM) operator. In section 4, we use the suggested SFSSWMSM operator to develop a new MAGDM method. And shows how the suggested method works and contrasts it with the ones that are currently in use. In section 5, we offer the paper's conclusions.

2 Preliminaries

To make this paper easier for readers to understand, we will go over the concepts of SFS, Maclaurin symmetric mean (MSM) operator, and Schweizer-Sklar T-norm and T-conorm.

Definition 1: SFS P is a set of a finite set $U = \{\xi_1, \xi_2, \xi_3, \ldots, \xi_t\}$ are specified as [6]:

$$P = \{(\xi_\iota, \mu_p(\xi_\iota), \vartheta_P(\xi_\iota), \eta_P(\xi_\iota)) \in U,\ \iota = 1, 2, 3, \ldots, t\}$$

The $\mu_p(\xi_\iota), \vartheta_P(\xi_\iota)$ and $\eta_P(\xi_\iota)$ represented the MG, AG, and NMG element $\xi_i \in$ U the set P respectively,

$$0 \le \mu_P^2(\xi_\iota) + \vartheta_P^2(\xi_\iota) + \eta_P^2(\xi_\iota) \le 1$$

RG $\delta_p(\xi_\iota)$ as

$$\delta_p(u_\iota) = \sqrt[2]{1 - (\mu_P^2(\xi_\iota) + \vartheta_P^2(\xi_\iota) + \eta_P^2(\xi_\iota))}$$

Definition 2: Suppose $\check{R} = (\mathcal{U}, \vartheta, \beta)$ is a SFN, then the score function $sc(\check{R})$ is defined as [37]

$$sc(\check{R}) = (\mathcal{U}^2 - \vartheta^2 - \beta^2)$$

And the accuracy function $ac(\check{R})$ is defined as

$$ac(\check{R}) = (\mathcal{U}^2 + \vartheta^2 + \beta^2)$$

Let $\check{R}_1 = (\mathcal{U}_1^2, \vartheta_1^2, \beta_1^2)$ and $\check{R}_2 = (\mathcal{U}_2^2, \vartheta_2^2, \beta_2^2)$ be two SFNs, the laws of comparing the two SFNs are as below:

1. If $sc\check{R}_1 \succ sc\check{R}_1$, then $\check{R}_1 \succ \check{R}_1$
2. If $sc\check{R}_1 = sc\check{R}_1$, then
(i). If $ac\check{R}_1 \succ ac\check{R}_1$, then $\check{R}_1 \succ \check{R}_1$
(ii). If $sc\check{R}_1 = sc\check{R}_1$, then $\check{R}_1 = \check{R}_1$

2.1 Schweizer-Sklar T-Norm and T-Conorm

As a particular instance of SSTN, SSTCN is capable of producing operations utilizing the sum and product of Schweizer-Sklar.

Definition 3: Consider $A = (a_A, b_A, c_A)$ and $B = (a_B, b_B, c_{\dot{C}})$ are any two SFNs, the result and total SFSs depending on the $T(\psi, \Lambda)$ [38]:

$$A \cap_{TN,TCN} B = \{\langle \varphi, TN(a_A(\varphi), a_B(\varphi)), a_{\dot{C}}(\varphi), TCN(a_A(\varphi), a_B(\varphi)), a_{\dot{C}}(\varphi) \rangle \varphi \in \varphi\} \quad (1)$$

$$A\cup_{TN,TCN}B=\{\langle\varphi,\ TN\left(a_A\left(\varphi\right),\ a_B\left(\varphi\right)\right),\ a_{\dot{C}}\left(\varphi\right),\ TCN\left(a_A\left(\varphi\right),\ a_B\left(\varphi\right)\right),\ a_{\dot{C}}\left(\varphi\right)\rangle\varphi\in\varphi\}\tag{2}$$

$$TN_{\Delta\Delta},o\left(\psi,\Lambda\right)=\left(\psi^o+\Lambda^o-1\right)^{1/o}\tag{3}$$

$$TCN_{\Delta\Delta},o\left(\psi,\Lambda\right)=1-\left(\left(1-\psi\right)^o+\left(1-\Lambda\right)^o-1\right)^{\frac{1}{o}}\tag{4}$$

So $o<0,\ \psi,\Lambda\in[0,1]$.
where, $o\ =0$, we have $TNo\left(\psi,\Lambda\right)=\ \psi\Lambda$ and $TCNo\ \left(\psi,\Lambda\right)=\ \psi+\Lambda-\psi\Lambda$, which, TN and TC are the algebraic.

The TN $T\left(\psi,\Lambda\right)$ and TCN $\left(\psi,\Lambda\right)$ Schweizer-Sklar operations for SFNs are available.

Definition 4: Suppose $\check{R}_1\ =\left(\mathcal{U}_1^2,\ \vartheta_1^2,\beta_1^2\right)$ and $\check{R}_2\ =\left(\mathcal{U}_2^2,\ \vartheta_2^2,\beta_2^2\right)$ are any two SFNs, then, based on Schweizer-Sklar operations, the product and sum of SFNs are shown as follows [38]

$$\check{R}_1\otimes_{TN,TCN}\check{R}_2=\left(TN\left(\mathcal{U}_1^2,\ \mathcal{U}_2^2\right),\ TCN\left(\vartheta_1^2,\ \vartheta_2^2\right)\right)$$

$$\check{R}_1\oplus_{TN,TCN}\check{R}_2=\left(TN\left(\mathcal{U}_1^2,\ \mathcal{U}_2^2\right),\ TCN\left(\vartheta_1^2,\ \vartheta_2^2\right)\right)$$

We might suggest the Schweizer-Sklar operational norms of SFNs, which are illustrated below, based on formulas $(o<0,\alpha>0)$.

$$\check{R}_1\oplus\Delta\Delta\ \check{R}_2=\left(1-\left(\left(1-\mu_1^2\right)^o\right)+\left(\left(1-\mu_2^2\right)^o-1\right)^{\frac{1}{o}},\ \left(\left(\vartheta_1^2\right)^a+\left(\vartheta_2^2\right)^a-1\right)^{\frac{1}{o}},\ \left(\left(\beta_1^2\right)^a+\left(\beta_2^2\right)^a-1\right)^{\frac{1}{o}}\right)$$

$$\check{R}_1\otimes\Delta\Delta\ \check{R}_2=\left(\left(\left(\mu_1^n\right)^a+\left(\mu_2^n\right)^a-1\right)^{\frac{1}{o}},\ 1-\left(\left(1-\vartheta_1^2\right)^o+\left(1-\vartheta_2^2\right)^o-1\right)^{\frac{1}{o}},\ 1-\left(\left(1-\beta_1^2\right)^o+\left(1-\beta_2^2\right)^o-1\right)^{\frac{1}{o}}\right)$$

$$\alpha\check{R}_1=\left(1-\left(\alpha\left(1-\mu_1^2\right)^o-\left(\alpha-1\right)\right)1/o\right),1-\left(\alpha\left(1-\vartheta_1^2\right)^o-\left(\alpha-1\right)\right)^{1/o},\ \left(\alpha\left(1-\beta_1^2\right)^o-\left(\alpha-1\right)\right)^{1/o}$$

$$\check{R}_1^\alpha=\left(\left(\alpha\left(\mu_1^2\right)^a-\left(\alpha-1\right)\right)^{1/o},\ 1-\left(\alpha\left(1-\vartheta_1^2\right)^o-\left(\alpha-1\right)\right)^{1/o},\ 1-\ \left(\alpha\left(1-\beta_1^2\right)^o-\left(\alpha-1\right)\right)^{1/o}\right)$$

Theorem 1: Let $\check{R}_1=\left(\mathcal{U}_1^2,\ \vartheta_1^2,\beta_1^2\right)$ and $\check{R}_2\ =\left(\ \mathcal{U}_2^2,\ \vartheta_2^2,\beta_2^2\right)$ be any two SFNs, and $o<0$ [39], then

$$(1)\ \check{R}_1\oplus\Delta\Delta\ \check{R}_2=\check{R}_2\oplus\check{R}_2\oplus\Delta\Delta\check{R}_1,$$

$$(2)\ \check{R}_1\otimes\Delta\Delta\ \check{R}_2=\check{R}_2\oplus\check{R}_2\otimes\Delta\Delta\check{R}_1,$$

$$(3)\ \ \alpha\left(\check{R}_1\oplus\Delta\Delta\ \check{R}_2\right)=\alpha\check{R}_1\oplus\Delta\Delta\ \alpha\check{R}_2,\ \alpha\geq0,$$

$$(4)\ \alpha_1\oplus\check{R}_1\Delta\Delta\ \alpha_2\check{R}_1=\left(\alpha_1+\alpha_2\right)\check{R}_1,\ \alpha_1,\alpha_2\geq0,$$

$$(5)\ \check{R}_1^{\alpha_1}\otimes\check{R}_1^{\alpha_2}=\check{R}_1^{\alpha_1+\alpha_2},\ \alpha_1,\ \alpha_2\geq0,$$

$$(6)\ \check{R}_1^\alpha\otimes\Delta\Delta\ \check{R}_2^\alpha=\left(\check{R}_1\otimes\Delta\Delta\ \check{R}_2\right)^\alpha,\ \alpha\geq0.$$

As Theorem 1 is simple to prove, it is not included here.

2.2 Maclaurin Symmetric Mean Operator

In order to take the relationships between various integrated arguments into consideration, Maclaurin created the Maclaurin symmetric mean (MSM) which are follows:

Definition 5 [39]: Consider $a_\imath\ (\imath = 1, 2, \ldots, \alpha)$ and the collection of the real numbers not of the negative, the MSM is written are

$$MSM^{(K)}\left(a_1,\ a_2, \ldots,\ a_\alpha\right) = \left(\frac{\sum_{1\leq \imath_1<\ldots<\imath_q\leq\alpha}\prod_{\ell=1} a_{\imath\ell}}{\dot{\mathrm{C}}_\alpha^K}\right)^{1/K}$$

where, $\dot{\mathrm{C}}_\alpha^q = \frac{\alpha!}{q!(\alpha-q)!}$ is the binomial coefficient, $(\imath_1,\ \imath_2, \ldots,\ \imath_q)$ the combination of $(1, 2, \ldots,\ m)$, and traverses all the k-tuple as, So, $1 \leq q \leq \alpha$. For instance, if $\alpha = 4$ and $q = 3$, then $\sum\limits_{1\leq \imath_1<\ldots<\imath_q\leq\alpha}\prod\limits_{\ell=1}^{q} a_{\imath\ell} = a_1a_2a_3 + a_1a_2a_4 + a_1a_3a_4 + a_2a_3a_4$.

The properties of MSM are following as:

$$MSM^{(q)}\left(0,\ 0,\ \ldots, 0\right) = 0,\ MSM^{(q)}\left(a, a, \ldots, a\right) = a;$$

$$MSM^{(q)}\left(a_1,\ a_2, \ldots, a_\alpha\right) \leq MSM^{(q)}\left(b_1,\ b_2, \ldots, b_\alpha\right),\ \imath f\ a_\imath \leq b_\imath\ for\ all\ \imath;$$

$$\min_\imath\ \{a_\imath\} \leq MSM^{(q)}\left(a_1, a_2, \ldots,\ a_\alpha\right) \leq \max_\imath\ \{a_\imath\}.$$

3 The Operators are SFNs Schweizer-Sklar Maclaurin Symmetric Mean (SSMSM)

This section we can introduces two new operators constants are the SSTT and SFNs. spherical fuzzy Schweizer-Sklar Maclaurin symmetric mean (SFSSMSM) and the spherical fuzzy Schweizer-Sklar weighted Maclaurin symmetric mean (SFSSWMSM).

3.1 Spherical Fuzzy Schweizer-Sklar Maclaurin Symmetric Mean (SFSSMSM) Operator

Definition 6: Consider $\check{R}_\imath\ (\imath = 1, 2, \ldots,\ \alpha)$ is a collection of SFNs, $q = 1, 2, \ldots,\ \alpha$ and SFSSMSM: $\Omega^\alpha \to \Omega$, if

$$SFSSMSM^{(q,o)}\left(\check{R}_1,\ \check{R}_2, \ldots,\ \check{R}_\alpha\right) = \left(\frac{\oplus_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Delta\Delta\otimes_{\ell=1}^{q}\Delta\Delta\ \check{R}_{\imath\ell}}{\dot{\mathrm{C}}_\alpha^q}\right)^{\frac{1}{q}}$$

The collection of SFNs, $\dot{\mathrm{C}}_\alpha^q = \frac{\alpha!}{K!(\alpha-K)!}$ is the binomial coefficient, $(\imath_1,\ \imath_2, \ldots,\ \imath_q)$ the combination of $(1, 2, \ldots,\ \alpha)$ and traverses all the k-tuple. So $1 \leq q \leq \alpha$.

We have aggregation result below, which is presented as in Theorem 3.1.2, on the base of Schweizer-Sklar operational principles of SFNs.

Theorem 2: Suppose $\check{R}_\imath = \left(\mathcal{U}_\imath^2,\ \vartheta_\imath^2, \beta_\imath^2\right)\ (\imath = 1, 2, \ldots, \alpha)$ is a set of SFNs and $o < 0,\ q = 1, 2, \ldots,\ \alpha$, thus the total result remains a SFN, and even

$SFSSMSM^{(q,\ o)}\left(\check{R}_1,\ \check{R}_2, \ldots,\ \check{R}_\alpha\right)$

$$= \left(\begin{array}{c} \left(\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_\alpha^q}\left(\begin{array}{c}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(\mathcal{U}_{\imath\ell}^2\right)^o \\ -(q-1)\end{array}\right)\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}\right)^o, \\ 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_\alpha^q}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(\vartheta_{\imath\ell}^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}\right)^o-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}, \\ 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_\alpha^q}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(\beta_{\imath\ell}^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}\right)^o-\left(\frac{1}{q}-1\right)\right)^{1/o} \end{array}\right)$$

Proof. Firstly, we can calculate $\otimes_{\ell=1}^{q}\Delta\Delta\check{R}_{\imath\ell}$, and get

$$\otimes_{\ell=1}^{q}\Delta\Delta\check{R}_{\imath\ell}=\left(\begin{array}{c}\left(\left(\left(\sum\limits_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}\right)-(q-1)\right)^{\frac{1}{o}}\right),\\ 1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\vartheta_{\imath\ell}^{2}\right)^{o}\right)-(q-1)^{\frac{1}{o}}\right),\\ 1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\beta_{\imath\ell}^{2}\right)^{o}\right)-(q-1)^{\frac{1}{o}}\right)\end{array}\right),$$

And it is SFN.

Then calculate $\oplus_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Delta\Delta\otimes_{\ell=1}^{q}\Delta\Delta\check{R}_{\imath\ell}$, and get

$$\oplus_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Delta\Delta\otimes\Delta\Delta_{\ell=1}^{q}=$$

$$\left(\begin{array}{c}\left(1-\left(\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\sum\limits_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)-\left(\dot{\mathrm{C}}_{\alpha}^{q}-1\right)\right)^{\frac{1}{o}},\\ \left(\left(\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\vartheta_{\imath\ell}^{2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)-\left(\dot{\mathrm{C}}_{\alpha}^{q}-1\right)\right)^{\frac{1}{o}}\right),\\ \left(\left(\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\beta_{\imath\ell}^{2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{\Gamma}\right)-\left(\dot{\mathrm{C}}_{\alpha}^{q}-1\right)\right)^{1/o}\right)\end{array}\right)$$

And it is also SFN.

Further, we can calculate $\frac{\oplus_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Delta\Delta\otimes_{\ell=1}^{q}\Delta\Delta\check{R}_{\imath\ell}}{\dot{\mathrm{C}}_{\alpha}^{q}}$, and get

$$\frac{\bigoplus_{1\leq\imath_1<\cdots<\imath_q\leq\alpha}\Delta\Delta\otimes\Delta\Delta\check{\mathrm{R}}_{\imath\ell}{}_{\ell=1}^{\mathrm{q}}}{\dot{\mathrm{C}}_{\alpha}^{\mathrm{q}}}=$$

$$\left(\begin{array}{c}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\begin{array}{c}\left(\sum\limits_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}\right)\\-(q-1)\end{array}\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)^{\frac{1}{o}},\\ \left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\begin{array}{c}\left(\sum\limits_{\ell=1}^{q}\left(1-\vartheta_{\imath\ell}^{2}\right)^{o}\right)\\-(q-1)\end{array}\right)^{\frac{1}{o}}\right)^{o}\right)\right)^{\frac{1}{o}},\\ \left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\begin{array}{c}\left(\sum\limits_{\ell=1}^{q}\left(1-\beta_{\imath\ell}^{2}\right)^{o}\right)\\-(q-1)\end{array}\right)^{\frac{1}{o}}\right)^{o}\right)\right)^{\frac{1}{o}}\end{array}\right)$$

And it is also SFN.

Finally, we calculate $\left(\frac{\oplus_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Delta\Delta\otimes_{\ell=1}^{q}\Delta\Delta\check{R}_{\imath\ell}}{\dot{\mathrm{C}}_{\alpha}^{q}}\right)^{1/q}$, and get

$$SFSSMSM^{(q,\,o)}\left(\check{R}_1,\check{R}_2,\ldots,\check{R}_\alpha\right)=\left(\frac{\bigoplus_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Delta\Delta\otimes\Delta\Delta\check{\mathrm{R}}_{\imath\ell}{}_{\ell=1}^{\mathrm{q}}}{\dot{\mathrm{C}}_{\alpha}^{\mathrm{q}}}\right)^{\frac{1}{\mathrm{q}}}=$$

$$\left(\begin{array}{c}\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\begin{array}{c}\left(\sum_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}\right)\\-(q-1)\end{array}\right)^{\frac{1}{o}}\right)^{o}\right)\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}},\\ 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\begin{array}{c}\left(\sum_{\ell=1}^{q}\left(1-\vartheta_{\imath\ell}^{2}\right)^{o}\right)\\-(q-1)\end{array}\right)^{\frac{1}{o}}\right)^{o}\right)\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}},\\ 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\begin{array}{c}\left(\sum_{\ell=1}^{q}\left(1-\beta_{\imath\ell}^{2}\right)^{o}\right)\\-(q-1)\end{array}\right)^{\frac{1}{o}}\right)^{o}\right)\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\end{array}\right)$$

And SFN as well.

The SFSSMSM operator has the following qualities, which are simple to demonstrate.

Theorem 3: (Idempotency) suppose $\check{R}_\imath = \left(\mathcal{U}_\imath^2,\ \vartheta_\imath^2, \beta_\imath^2\right)$ $(\imath = 1,\ 2,\ \ldots,\ \alpha)$ is a collection of the SFNs, if $\check{R}_\imath = \check{R} = \left(\mathcal{U}^2,\ \vartheta^2,\ \beta^2\right),\ \imath = 1,\ 2,\ \ldots,\ \alpha$, then

$$SFSSMSM^{(q,\ o)}\left(\check{R}_1,\ \check{R}_2, \ldots, \check{R}_\alpha\right) = \check{R} = \left(\mathcal{U}^2,\ \vartheta^2,\ \beta^2\right)$$

Proof: Since $\check{R}_\imath = \left(\mathcal{U}^2,\ \vartheta^2,\ \beta^2\right)$ $(\imath = 1, 2, \ldots, \alpha)$, then according to formula (22), we have
$SFSSMSM^{(q,\ o)}\left(\check{R}_1,\ \check{R}_2,\ \ldots,\ \check{R}_\alpha\right) =$

$$\left(\begin{array}{c}
\left(\left(\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{C}_\alpha^q}\left(\sum\limits_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(q\left(\mathcal{U}^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)^o\right)\right)\right)\right)^{\frac{1}{o}}\right)^o-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}, \\
1-\left(\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{C}_\alpha^q}\left(\sum\limits_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(1-\vartheta^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}\right)^o-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\right)\right), \\
1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{C}_\alpha^q}\left(\sum\limits_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(1-\beta^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}\right)^o-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\right)
\end{array}\right)$$

$$=\left(\begin{array}{c}
\left(\frac{1}{q}\left(1-\left(1-\left(q\left(\mathcal{U}^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)\right)^o-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}, \\
1-\left(\frac{1}{q}\left(1-\left(1-\left(q\left(1-\vartheta^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)\right)^o-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}, \\
1-\left(\frac{1}{q}\left(1-\left(1-\left(q\left(1-\beta^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)\right)^o-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}
\end{array}\right)$$

$$=\left(\begin{array}{c}
\left(\frac{1}{q}\left(q\left(\mathcal{U}^2\right)^o-(q-1)\right)-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}, \\
1-\left(\frac{1}{q}\left(q\left(1-\vartheta^2\right)^o-(q-1)\right)-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}, \\
1-\left(\frac{1}{q}\left(q\left(1-\beta^2\right)^o-(q-1)\right)-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}
\end{array}\right)$$

$$=\left(\mathcal{U}^2,\ \vartheta^2,\ \beta^2\right)=\check{R}.$$

$$\left(\begin{array}{c}
\left(\frac{1}{q}\left(1-\left(1-\left(q\left(\mathcal{U}^2\right)^o-(q-1)\right)^{\frac{1}{o}}\right)\right)\right)^{\frac{1}{o}}, \\
1-\left(\frac{1}{q}\left(1-\left(1-\left(q\left(1-\vartheta^2\right)^o-\left(\frac{1}{q}-1\right)\right)\right)\right)\right), \\
1-\left(\frac{1}{q}\left(1-\left(1-\left(q\left(1-\beta^2\right)^o-\left(\frac{1}{q}-1\right)\right)\right)\right)\right)
\end{array}\right)$$

Theorem 4: (Monotonicity) suppose $\check{R}_\imath = \left(\mathcal{U}_\imath^2,\ \vartheta_\imath^2, \beta_\imath^2\right)$ and $\check{R}_\imath^{'} = \left(\mathcal{U}_\imath^{2^{'}},\ \vartheta_\imath^{2^{'}},\ \beta_\imath^{2^{'}}\right)$ are two sets of SFNs, if $\mathcal{U}_\imath^2 \ge \mathcal{U}_\imath^{2^{'}}$, $\vartheta_\imath^2 \ge \vartheta_\imath^{2^{'}}$ and $\beta_\imath^2 \ge \beta_\imath^{2^{'}}$ for all $\imath = 1,\ 2,\ \ldots,\ \alpha$, then

$$SFSSMSM^{(q,\ o)}\left(\check{R}_1^{'},\ \check{R}_2^{'},\ \ldots,\ \check{R}_\alpha^{'}\right) \ge SFSSMSM^{(q,\ o)}\left(\check{R}_1^{'},\ \check{R}_2^{'},\ \ldots,\ \check{R}_\alpha^{'}\right)$$

Proof: Since $\mathcal{U}_\imath^2 \ge \mathcal{U}_\imath^{2^{'}}$ for any $\imath$, we have $\sum\limits_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^2\right)^o \le \sum\limits_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2^{'}}\right)^o$ and $\left(\left(\sum\limits_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^2\right)^o\right)-(q-1)\right)^{\frac{1}{o}} \le \left(\left(\sum\limits_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2^{'}}\right)^o\right)-(q-1)\right)^{\frac{1}{o}}$,

Then we have

$$\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^2\right)^o\right)-(q-1)\right)^{\frac{1}{o}}\right)^o \ge \left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2^{'}}\right)^o\right)-(q-1)\right)^{\frac{1}{o}}\right)^o$$

And

$$\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}\right)-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)$$
$$\ge\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2\,\prime}\right)^{o}\right)-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)\right)$$

Further, we have

$$1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}\right)-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)^{\frac{1}{o}}$$
$$\ge 1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2\,\prime}\right)^{o}\right)-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)^{\frac{1}{o}}$$

And

$$\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath_\ell}^{2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}$$
$$\le\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath_\ell}^{\prime 2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}$$

Finally, we have

$$\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}\right)-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)\right)\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$
$$\ge\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2\,\prime}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)\right)\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

Similarly, we have

$$1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\sum_{\ell=1}^{\alpha}\left(\vartheta_{l\ell}^{2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$
$$\le 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\sum_{\ell=1}^{\alpha}\left(\left(\vartheta_{l\ell}^{2}\right)^{\prime}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

$$1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\le\imath_1<\ldots<\imath_q\le\alpha}\left(1-\left(\sum_{\ell=1}^{\alpha}\left(\beta_{l\ell}^{2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

$$\leq 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\left(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\sum_{\ell=1}^{\alpha}\left(\beta_{l\ell}^{2'}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)\right)\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

Let $\check{R}=SFSSMSM^{(q,\,o)}\left(\check{R}_1^{'},\ \check{R}_2^{'},\ldots,\ \check{R}_{\alpha}^{'}\right)$, according to this Equation,

$$\left(\begin{array}{c}\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}= \\ \left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(1-\vartheta_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}- \\ \left(1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\right)- \\ \left(1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(1-\beta_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\right)\geq \\ \left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(\mathcal{U}_{\imath\ell}^{2}{}'\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}- \\ \left(1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(\left(1-\vartheta_{\imath\ell}^{2}\right)'\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\right)-\end{array}\right)-$$

$$\left(1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\sum_{\ell=1}^{\alpha}\left(1-\beta_{\imath\ell}^{2}{}'-(q-1)\ \right)^{o}\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\right)=S\left(\check{R}'\right)$$

We can discussion the two circumstances are necessary.

(1) If $S(\check{R})>S(\check{R}')$, according to Definition 2.2.

$$SFSSMSM^{q,\,o}(\check{R}_1,\ \check{R}_2,\ldots,\ \check{R}_{\alpha})>SFSSMSM^{(q,\,o)}(\check{R}_1',\ \check{R}_2',\ldots,\ \check{R}_{\alpha}').$$

(2) If $S(\check{R})=S(\check{R}')$, because $\mathcal{U}_{\imath\ell}^{2}\geq\mathcal{U}_{\imath\ell}^{2'}\geq 0,\ \vartheta_{\imath\ell}^{2}\geq\vartheta_{\imath\ell}^{2'}\geq 0,\ \beta_{\imath\ell}^{2}\geq\beta_{\imath\ell}^{2'}\geq 0$

$$\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\sum_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

$$=\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\sum_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2\prime}-(q-1)\right)^{o}\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

$$1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{q}\left(1-\vartheta_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

$$=1-\left(\frac{1}{\mathrm{q}}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{\mathrm{q}}}\sum\nolimits_{1\leq\imath_1<\cdots<\imath_{\mathrm{q}}\leq\alpha}\left(1-\left(\left(\sum\nolimits_{\ell=1}^{\mathrm{q}}\left(1-\left(\vartheta_{\imath\ell}^{2}\right)'-(\mathrm{q}-1)\right)^{o}\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{\mathrm{q}}-1\right)\right)^{\frac{1}{o}}$$

$$1-\left(\frac{1}{\mathrm{q}}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{\mathrm{q}}}\sum_{1\leq\imath_1<\cdots<\imath_{\mathrm{q}}\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\mathrm{q}}\left(1-\beta_{\imath\ell}^{2}\right)^{o}-(\mathrm{q}-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{\mathrm{q}}-1\right)\right)^{\frac{1}{o}}$$

$$=1-\left(\frac{1}{\mathrm{q}}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{\mathrm{q}}}\sum_{1\leq\imath_1<\cdots<\imath_{\mathrm{q}}\leq\alpha}\left(1-\left(\left(\sum_{\ell=1}^{\mathrm{q}}\left(1-\beta_{\imath\ell}^{2'}\right)^{o}-(\mathrm{q}-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{\mathrm{q}}-1\right)\right)^{\frac{1}{o}}$$

As a result, using Definition 2, we may obtain

$$H\left(\check{R}\right)=$$

$$\left(\begin{array}{c}\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\\ +1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\vartheta_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}+\\ 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\beta_{\imath\ell}^{2}\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}=\\ \left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(\mathcal{U}_{\imath\ell}^{2}{}'\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}+\\ 1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\left(\vartheta_{\imath\ell}^{2}\right)'\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\\ +1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\sum\limits_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\left(\sum\limits_{\ell=1}^{q}\left(1-\beta_{\imath\ell}^{2}{}'\right)^{o}-(q-1)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}\end{array}\right)$$

$$=H\left(\check{R}'\right).$$

So, according to Definition 2, we can get

$$SFSSMSM^{(q,\,o)}\left(\check{R}_1,\ \check{R}_2,\ldots,\check{R}_\alpha\right)\geq SFSSMSM^{(q,\,o)}\left(\check{R}_1^{'},\ \check{R}_2^{'},\ldots,\check{R}_\alpha^{'}\right).$$

Theorem 5: (Boundedness) suppose $\check{R}_\imath=\left(\mathcal{U}_\imath^2,\ \vartheta_\imath^2,\beta_\imath^2\right)$ is a set of T-SFNs, if $\mathcal{U}^-=\min\limits_{1\leq\imath\leq\alpha}\left\{\mathcal{U}_\imath^2\right\}$, $\mathcal{U}^+=\max\limits_{1\leq\imath\leq\alpha}\left\{\vartheta_\imath^2\right\}$, $\vartheta^-=\min\limits_{1\leq\imath\leq\alpha}\left\{\vartheta_\imath^2\right\}$, $\vartheta^+=\max\limits_{1\leq\imath\leq\alpha}\left\{\vartheta_\imath\right\}$, $\beta^-=\min\limits_{1\leq\imath\leq\alpha}\left\{\beta_\imath^2\right\}$, $\beta^+=\max\limits_{1\leq\imath\leq\alpha}\left\{\beta_\imath^2\right\}$, let $\check{R}^-=(\mathcal{U}^-,\ \vartheta^+,\ \beta^+)$ and $\check{R}^+=(\mathcal{U}^+,\ \vartheta^-,\ \beta^-)$, then

$$\check{R}^-\leq SFSSMSM^{(q,\,o)}\left(\check{R}_1,\ \check{R}_2,\ldots,\ \check{R}_\alpha\right)\leq\check{R}^+$$

Proof: $\left(\mathcal{U}_\imath^2,\ \vartheta_\imath^2,\beta_\imath^2\right)$ $(\imath=1,\ 2,\ \ldots,\ \alpha)$ is a collection of the SFNs, if $\check{R}_\imath=\check{R}=\left(\mathcal{U}^2,\ \vartheta^2,\ \beta^2\right)$

Since $\mathcal{U}^- = \min\limits_{1\le\imath\le\alpha}\{\mathcal{U}_\imath^2\}, \mathcal{U}^+ = \max\limits_{1\le\imath\le\alpha}\{\mathcal{U}_\imath^2\}, \vartheta^- = \min\limits_{1\le\imath\le\alpha}\{\vartheta_\imath^2\}, \mathcal{V}\backslash\vartheta^+ = \max\limits_{1\le\imath\le\alpha}\{\vartheta_\imath^2\}, \beta^- = \min\limits_{1\le\imath\le\alpha}\{\beta_\imath^2\}$, $\beta^+ = \max\limits_{1\le\imath\le\alpha}\{\beta_\imath^2\}$, so $\mathcal{U}^- \le \mathcal{U}_\imath \le \mathcal{U}^+$, $\vartheta^- \le \vartheta_\imath \le \vartheta^+$, $\beta^- \le \beta_\imath^2 \le \beta^+$ base of the Definition 2, $\check{R}^- = \min\limits_{1\le\imath\le\alpha}\{\check{R}_\imath\}, \check{R}^+ = \max\limits_{1\le\imath\le\alpha}\{\check{R}_\imath\}$.

Hence, we can obtain due to idempotency and monotonicity

$$\check{R}^- \le SFSSMSM^{(q,\,o)}\left(\check{R}_1,\ \check{R}_2, \ldots,\ \check{R}_\alpha\right) \le \check{R}^+$$

By assigning various parameter values o and q to the proposed SFSSMSM operator, we can obtain certain specific situations.

(1) If $o = 0$, the $SFSSMSM$ operator are the reduces of spherical fuzzy Maclaurin symmetric mean (SFMSM) operator [40],

$SFSSMSM^{(q,\,o=0)}\left(a_1,\ a_2, \ldots, a_\alpha\right) =$

$$\begin{pmatrix} \left(1 - \left(\prod\limits_{1\le\imath_1<\ldots<\imath_2\le\alpha}\left(1 - \prod\limits_{\ell=1}^{q}\mathcal{U}_{\imath\ell}^2\right)\right)^{\frac{1}{\dot{C}_\alpha^q}}\right), \\ 1 - \left(1 - \left(\prod\limits_{1\le\imath_1<\ldots<\imath_2\le\alpha}\left(1 - \prod\limits_{\ell=1}^{q}\left(1 - \vartheta_{\imath\ell}^2\right)\right)\right)^{\frac{1}{\dot{C}_\alpha^q}}\right)^{\frac{1}{q}}, \\ 1 - \left(1 - \left(\prod\limits_{1\le\imath_1<\ldots<\imath_2\le\alpha}\left(1 - \prod\limits_{\ell=1}^{q}\left(1 - \beta_{\imath\ell}^2\right)\right)\right)^{\frac{1}{\dot{C}_\alpha^q}}\right)^{\frac{1}{q}} \end{pmatrix} = SFMSM^{(q)}\left(a_1,\ a_2, \ldots,\ a_\alpha\right)$$

(2) If $q = 1$, then SFSSMSM operator are reduces in the spherical fuzzy Schweizer-Sklar average (SFSSA) operator.

$$SFSSMSM^{(q=1,\,o)}\left(a_1,\ a_2, \ldots,\ a_\alpha\right) =$$

$$\begin{pmatrix} \left(\left(\frac{1}{1}\left(1 - \left(\frac{1}{\dot{C}_\alpha^1}\left(\sum\limits_{1\le\imath_1\le\alpha}\left(1 - \left(\left(\sum\limits_{\ell=1}^{1}\left(\mathcal{U}_{\imath\ell}^2\right)^o - (1-1)\right)\right)^{\frac{1}{o}}\right)^o\right)\right)^{\frac{1}{o}}\right)\right)^o - \left(\frac{1}{1} - 1\right)\right)^{\frac{1}{o}}, \\ 1 - \left(\frac{1}{1}\left(1 - \left(\frac{1}{\dot{C}_\alpha^1}\left(\sum\limits_{1\le\imath_1\le\alpha}\left(1 - \left(\left(\sum\limits_{\ell=1}^{1}\left(1 - \vartheta_{\imath\ell}^2\right)^o\right) \overrightarrow{\leftarrow} - (1-1)\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}\right)^o\right) - \left(\frac{1}{1} - 1\right)\right)^{\frac{1}{o}}, \\ 1 - \left(\frac{1}{1}\left(1 - \left(\frac{1}{\dot{C}_\alpha^1}\left(\sum\limits_{1\le\imath_1\le\alpha}\left(1 - \left(\left(\sum\limits_{\ell=1}^{1}\left(1 - \beta_{\imath\ell}^2\right)^o\right) \overrightarrow{\leftarrow} - (1-1)\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}\right)^o\right) - \left(\frac{1}{1} - 1\right)\right)^{\frac{1}{o}} \end{pmatrix}$$

$$\begin{pmatrix} 1 - \left(\left(1 - \left(\frac{1}{\alpha}\sum\limits_{1\le\imath_1\le\alpha}\left(\mathcal{U}_{\imath 1}^2\right)^o\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}, \left(\left(1 - \left(\frac{1}{\alpha}\sum\limits_{1\le\imath_1\le\alpha}\left(1 - \vartheta_{\imath 1}^2\right)^o\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}}, \\ \left(\left(1 - \left(\frac{1}{\alpha}\sum\limits_{1\le\imath_1\le\alpha}\left(1 - \beta_{\imath 1}^2\right)^o\right)^{\frac{1}{o}}\right)^o\right)^{\frac{1}{o}} \end{pmatrix}$$

$$(let\ \imath_\ell = \ell) = \left(1 - \left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(1 - \mathcal{U}_\ell^2\right)^o\right)^{\frac{1}{o}}, \left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(\vartheta_\ell^2\right)^o\right)^{\frac{1}{o}}, \left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(\beta_\ell^2\right)^o\right)^{\frac{1}{o}}\right)$$

(3) If $q = 2$, then $SFSSMSM$ operator are reduces on the Spherical fuzzy Schweizer-Sklar Bonferroni mean (SFSSBM) operator,

$$SFSSMSM^{(q=2,\,o)}\left(a_1,\ a_2, \ldots, a_\alpha\right) = \pi$$

$$
= \left(\left(\frac{1}{2} \left(1 - \left(\frac{1}{\dot{C}_{\alpha}^{2}} \left(\sum_{1 \le \imath_1 < \ldots < \imath_2 \le \alpha} \left(\begin{array}{c} \left(\sum_{\ell=1}^{2} \left(\mathcal{U}_{\imath\ell}^{2} \right)^{o} \right)^{\frac{1}{o}} \\ -(2-1) \end{array} \right)^{o} \right) \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{2} - 1 \right) \right)^{\frac{1}{o}},
$$

$$
1 - \left(\frac{1}{2} \left(1 - \left(\frac{1}{\dot{C}_{\alpha}^{2}} \left(\sum_{1 \le \imath_1 < \ldots < \imath_2 \le \alpha} \left(1 - \left(\begin{array}{c} \left(\sum_{\ell=1}^{2} \left(1 - \vartheta_{\imath\ell} \right)^{o} \right)^{\frac{1}{o}} \\ -(2-1) \end{array} \right)^{o} \right) \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{2} - 1 \right) \right)^{\frac{1}{o}} \right),
$$

$$
1 - \left(\frac{1}{2} \left(1 - \left(\frac{1}{\dot{C}_{\alpha}^{2}} \left(\sum_{1 \le \imath_1 < \ldots < \imath_2 \le \alpha} \left(1 - \left(\begin{array}{c} \left(\sum_{\ell=1}^{2} \left(1 - \beta_{\imath\ell}^{2} \right)^{o} \right)^{\frac{1}{o}} \\ -(2-1) \end{array} \right)^{o} \right) \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{2} - 1 \right) \right)^{\frac{1}{o}} \right)
$$

$$
= \left(\begin{array}{c} \left(\frac{1}{2} \left(1 - \left(\frac{2}{\alpha(\alpha-1)} \left(\sum_{1 \le \imath_1 < \imath_2 \le \alpha} \left(1 - \left(\left(\mathcal{U}_{\imath 1}^{2} \right)^{o} + \left(\mathcal{U}_{\imath 2}^{2} \right)^{o} - 1 \right)^{\frac{1}{o}} \right)^{o} \right) \right)^{\frac{1}{o}} \right)^{o} + \frac{1}{2} \right)^{\frac{1}{o}}, \\ 1 - \left(\frac{1}{2} \left(1 - \left(\frac{2}{\alpha(\alpha-1)} \sum_{1 \le \imath_1 < \imath_2 \le \alpha} \left(1 - \left(\begin{array}{c} \left(1 - \vartheta_{\imath\ell}^{2} \right)^{o} + \\ \left(1 - \vartheta_{\imath\ell}^{2} \right)^{o} - 1 \end{array} \right)^{\frac{1}{o}} \right)^{o} \right)^{\frac{1}{o}} \right)^{o} + \frac{1}{2} \right)^{\frac{1}{o}} \\ 1 - \left(\frac{1}{2} \left(1 - \left(\frac{2}{\alpha(\alpha-1)} \sum_{1 \le \imath_1 < \imath_2 \le \alpha} \left(1 - \left(\begin{array}{c} \left(1 - \beta_{\imath\ell}^{2} \right)^{o} + \\ \left(1 - \beta_{\imath\ell}^{2} \right)^{o} - 1 \end{array} \right)^{\frac{1}{o}} \right)^{o} \right)^{\frac{1}{o}} \right)^{o} + \frac{1}{2} \right)^{\frac{1}{o}} \end{array} \right)
$$

$$
= SFSSBM^{1,1} \left(a_1, a_2, \ldots, a_{\alpha} \right).
$$

(4) If $q = \alpha$, then $SFSSMSM$ operator are the reduces of the spherical fuzzy Schweizer –Sklar geometric (SFSSG) operator

$$
SFSSNSM^{(\mathrm{q}=\alpha, o)} \left(a_1, a_2, \ldots, a_{\alpha} \right) =
$$

$$
\left(\begin{array}{c} \left(\left(\frac{1}{\alpha} \left(1 - \left(\frac{1}{\dot{C}_{\alpha}^{\alpha}} \left(\sum_{1 \le \imath_1 < \ldots < \iota_{\alpha} \le \alpha} \left(1 - \left(\begin{array}{c} \left(\sum_{\ell=1}^{\alpha} \left(\mathcal{U}_{\imath\ell}^{2} \right)^{o} \right) \\ -(\alpha-1) \end{array} \right)^{\frac{1}{o}} \right)^{o} \right) \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{\alpha} - 1 \right) \right)^{\frac{1}{o}}, \\ 1 - \left(\frac{1}{\alpha} \left(1 - \left(\frac{1}{\dot{C}_{\alpha}^{\alpha}} \left(\sum_{1 \le \imath_1 < \ldots < \iota_{\alpha} \le \alpha} \left(1 - \left(\begin{array}{c} \left(\sum_{\ell=1}^{\alpha} \left(\mathcal{U}_{\imath\ell}^{2} \right)^{o} \right) \\ -(\alpha-1) \end{array} \right)^{\frac{1}{o}} \right)^{o} \right) \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{\alpha} - 1 \right) \right)^{\frac{1}{o}}, \\ 1 - \left(\frac{1}{\alpha} \left(1 - \left(\frac{1}{\dot{C}\alpha} \left(\sum_{1 \le \imath_1 < \ldots < \iota_{\alpha} \le \alpha} \left(1 - \left(\left(\begin{array}{c} \sum_{\ell=1}^{\alpha} \left(1 - \vartheta_{\imath\ell}^{2} \right)^{o} \right)^{\frac{1}{o}} \\ -(\alpha-1) \end{array} \right) \right)^{o} \right) \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{\alpha} - 1 \right) \right)^{\frac{1}{o}} \end{array} \right)
$$

$$
= \left(\left(\frac{1}{\alpha} \left(1 - \left(\left(1 - \left(\left(\sum_{\ell=1}^{\alpha} \mathcal{U}_{\imath\ell}^{2} \right)^{o} \right) - (\alpha-1) \right)^{\frac{1}{o}} \right)^{o} \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{\alpha} - 1 \right) \right)^{\frac{1}{o}},
$$

$$
1 - \left(\frac{1}{\alpha} \left(1 - \left(\left(1 - \left(\begin{array}{c} \left(\sum_{\ell=1}^{\alpha} \left(1 - \vartheta_{\imath\ell}^{2} \right)^{o} \right) \\ -(\alpha-1) \end{array} \right)^{\frac{1}{o}} \right)^{o} \right)^{\frac{1}{o}} \right)^{o} - \left(\frac{1}{\alpha} - 1 \right) \right)^{\frac{1}{o}},
$$

$$1-\left(\frac{1}{\alpha}\left(1-\left(\left(1-\left(\begin{array}{c}\left(\sum_{\ell=1}^{\alpha}\left(1-\beta_{\imath\ell}^{2}\right)^{o}\right)\\-(\alpha-1)\end{array}\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{\alpha}-1\right)\right)^{\frac{1}{o}} \;(\,let\;\imath_{\ell}=\ell)$$

$$=\left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(\mu_{\ell}^{o}\right)^{\frac{1}{o}},1-\left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(1-\vartheta_{\ell}^{2}\right)^{o}\right)^{\frac{1}{o}},1-\left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(1-\beta_{\ell}^{2}\right)^{o}\right)^{\frac{1}{o}}\right).$$

3.2 Spherical Fuzzy Schweizer-Sklar Weighted Maclaurin Symmetric Mean (SFSSWMSM) Operator

Even though the operator accounts the relationships between many aggregated, it does not take into account how important each aggregated argument is on its own. To address this flaw, the spherical fuzzy Schweizer-Sklar weighted Maclaurin symmetric mean (SFSSWMSM) operator is defined in this sections:

Definition 6: Suppose $\check{R}_{\imath}$ $(\imath=1,2,\ldots,\alpha)$ is a collection of SFNs, $q=1,2,\ldots,\alpha$, and SFSSWMS: $\Omega^{\alpha}\rightarrow\Omega$, if

$$SFSSWMSM^{(q,o)}\left(\check{R}_1,\ \check{R}_2,\ldots,\check{R}_{\alpha}\right)=\left(\frac{1\leq\imath_1<\ldots^{\oplus_{SS}}<\imath_q\leq\alpha_{\ell=1}^{q}\otimes SS\left(\delta_{\imath_\ell}\check{R}_{\imath_\ell}\right)}{\frac{q}{\alpha}}\right)^{\frac{1}{q}}$$

where, Ω is the set of all SFNs, and $(\delta=\delta_1,\ \delta_2,\ldots,\delta_{\alpha})$ are the vector weight $\check{R}=\left(\check{R}_1,\check{R}_2,\ldots,\check{R}_{\alpha}\right)$, $\dot{C}_{\alpha}^{q}=\frac{\alpha!}{q!(\alpha-q)!}$ is the binomial coefficient, $(\imath_1,\imath_2,\ldots,\imath_q)$ are the combination of $(1,2,\ldots,\alpha)$ and traverses all the k-tuple. SFSSWMSM is termed of spherical fuzzy Schweizer-Sklar weighted Maclaurin symmetric mean operator.

We have following aggregation result, denoted the Theorem 6, and based on the Schweizer-Sklar operational principles of SFNs.

Theorem 6: Suppose $\check{R}_{\imath}=\left(\left(\mathcal{U}_{\imath}^{2},\ \vartheta_{\imath}^{2},\beta_{\imath}^{2}\right)\right)$ are the collection of SFNs, and $q=1,2,\ldots,\alpha$, then result is still a SFN, and even

$$SFSSWMSm^{(q,o)}\left(\check{R}_1,\check{R}_2,\ldots,\check{R}_{\alpha}\right)=$$

$$\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{C}_{\alpha}^{q}}\left(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\left(1-\left(\sum_{\imath=1}^{q}\left(1-\left(\begin{array}{c}\delta_{\imath\ell}\left(1-\mathcal{U}_{\imath\ell}^{2}\right)^{o}\\-\left(\delta_{\imath\ell}-1\right)\end{array}\right)^{\frac{1}{o}}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{q}},$$

$$1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{C}_{\alpha}^{q}}\left(\sum_{1\leq\imath_1<\ldots<\iota_q\leq\alpha}\left(1-\left(\sum_{\ell=1}^{q}\left(1-\left(\begin{array}{c}\delta_{\ell\ell}\left(\vartheta_{\imath\ell}^{2}\right)^{o}-)^{\frac{1}{o}}\\\left(\delta_{\imath\ell}-1\right)\end{array}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}},$$

$$1-\left(\frac{1}{q}\left(1-\left(\frac{1}{\dot{C}_{\alpha}^{q}}\left(\sum_{1\leq\imath_1<\ldots<\iota_q\leq\alpha}\left(1-\left(\sum_{\ell=1}^{q}\left(1-\left(\begin{array}{c}\delta_{\imath\ell}\left(\beta_{\imath\ell}^{2}\right)^{o}-\\\left(\delta_{\imath\ell}-1\right)\end{array}\right)^{\frac{1}{o}}\right)^{o}-(q-1)\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{q}-1\right)\right)^{\frac{1}{o}}$$

$$\text{SFSSWMSM}\,M^{(q)}\left(\check{R}_1,\check{R}_2,\ldots,\check{R}_{\alpha}\right)=\left(\frac{\left(\ell_{\ell=1}^{q}\otimes SS\left(\delta_{\iota_\ell}\check{R}_{\iota_\ell}\right)\right)_{1\leq\imath_1<\ldots\oplus_{SS}<\imath_q\leq\alpha}}{\dot{C}_{\alpha}^{q}}\right)^{\frac{1}{q}}$$

$$\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\mathrm{C}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\left(\begin{matrix}\delta_{\imath\ell}\left(1-\mathcal{U}_{\imath\ell}^{2}\right)^{o}\\-\left(\delta_{\imath\ell}-1\right)\end{matrix}\right)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg),$$

$$1-\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\overline{\mathrm{C}}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\left(\begin{matrix}\delta_{\imath\ell}\left(\vartheta_{\imath\ell}^{2}\right)^{o}-)^{\frac{1}{o}}\\\left(\delta_{\imath\ell}-1\right)\end{matrix}\right)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)^{1/o}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)^{\frac{1}{o}},$$

$$1-\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\mathrm{C}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\left(\begin{matrix}\delta_{\imath\ell}\left(\beta_{\imath\ell}^{2}\right)^{o}-\\\left(\delta_{\imath\ell}-1\right)\end{matrix}\right)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)^{1/o}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)^{\frac{1}{o}}$$

So it is also SFN.

The straightforward to verify the SFSSWMSM operator following the properties.

Definition 7: (Monotonicity) Consider $\check{R}_{\imath}=\left(\left(\mathcal{U}_{\imath}^{2},\ \vartheta_{\imath}^{2},\beta_{\imath}^{2}\right)\right)\ a\alpha d\ \check{R}_{\imath}^{'}=\left(\mathcal{U}_{\imath}^{2^{'}},\ \vartheta_{\imath}^{2^{'}},\beta_{\imath\ell}^{2^{'}}\right)$ are two sets of SFNs, if $\mathcal{U}_{\imath}^{2}\geq\mathcal{U}_{\imath}^{2^{'}},\ \vartheta_{\imath}^{2}\geq\vartheta_{\imath}^{2^{'}}$ and $\beta_{\imath}^{2}\geq\beta_{\imath\ell}^{2^{'}}$ for all $\imath=1,2,\ldots,\alpha$, then

$$SFSSWMSM^{(q,o)}\left(\check{R}_{1},\ \check{R}_{2},\ldots,\check{R}_{\alpha}\right)\geq SFSSWMSM^{(q,o)}\left(\check{R}_{1}^{'},\ \check{R}_{2}^{'},\ldots,\check{R}_{\alpha}^{'}\right)$$

Theorem 7: (Boundedness) suppose $\check{R}_{\imath}=\left(\left(\mathcal{U}_{\imath}^{2},\ \vartheta_{\imath}^{2},\beta_{\imath}^{2}\right)\right)$ are the set of SFNs and $\check{R}^{-}=\left(\mu_{min}^{2-},\vartheta_{max}^{2-},\ \beta_{max}^{2-}\right)$, $\check{R}^{+}=\left(\mu_{max}^{2+},\vartheta_{min}^{2+},\ \beta_{min}^{2+}\right)$

$\mu_{min}^{2-}=$

$$\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\Bigg(\delta_{\imath\ell}\Bigg(1-\min_{1\leq\imath\leq\alpha}\left\{\mathcal{U}_{\imath}^{2}\right\}\Bigg)^{o}-\left(\delta_{\imath\ell}-1\right)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)^{\frac{1}{o}}$$

$\vartheta_{max}^{2-}=$

$$1-\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\Bigg(\delta_{\imath\ell}\Bigg(\max_{1\leq\imath\leq\alpha}\left\{\vartheta_{\imath}^{2}\right\}\Bigg)^{o}-\left(\delta_{\imath\ell}-1\right)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)^{\frac{1}{o}}$$

$\beta_{max}^{2-}=$

$$1-\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\Bigg(\delta_{\imath\ell}\Bigg(\min_{1\leq\imath\leq\alpha}\left\{\beta_{\imath}^{2}\right\}\Bigg)^{o}-\left(\delta_{\imath\ell}-1\right)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)^{\frac{1}{o}}$$

$\mu_{max}^{2+}=$

$$1-\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\Bigg(\delta_{\imath\ell}\Bigg(\min_{1\leq\imath\leq\alpha}\left\{\mathcal{U}_{\imath}^{2}\right\}\Bigg)^{o}-\left(\delta_{\imath\ell}-1\right)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)^{\frac{1}{o}}$$

$\vartheta_{min}^{2+}=$

$$\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\Bigg(\delta_{\imath\ell}\Bigg(1-\min_{1\leq\imath\leq\alpha}\left\{\vartheta_{\imath}^{2}\right\}\Bigg)^{o}-\left(\delta_{\imath\ell}-1\right)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)^{\frac{1}{o}}$$

$\beta_{min}^{2+}=$

$$\Bigg(\Bigg(\frac{1}{q}\Bigg(1-\Bigg(\frac{1}{\dot{\mathrm{C}}_{\alpha}^{q}}\Bigg(\sum_{1\leq\imath_1<\ldots<\imath_q\leq\alpha}\Bigg(1-\Bigg(\sum_{\ell=1}^{q}\Bigg(1-\left(\begin{matrix}\delta_{\imath\ell}\Bigg(1-\min\limits_{1\leq\imath\leq\alpha}\left\{\beta_{\imath}^{2}\right\}\Bigg)^{o}\\-\left(\delta_{\imath\ell}-1\right)\end{matrix}\right)^{\frac{1}{o}}\Bigg)^{o}-(q-1)\Bigg)^{\frac{1}{o}}\Bigg)^{o}\Bigg)\Bigg)\Bigg)^{\frac{1}{o}}\Bigg)^{o}-\left(\frac{1}{q}-1\right)\Bigg)\Bigg)$$

Then, $\check{R}^{-} \leq SFSSWMSM^{(q,o)}\left(\check{R}_1, \check{R}_2, \ldots, \check{R}_\alpha\right) \leq \check{R}^{+}$.

By assigning various parameter values for o and q, we can obtain some specific cases of the proposed SFSSWMSM operator in a similar manner.

(1) If $o = 0$, the $SFSSWMSM$operator of the spherical fuzzy weighted Maclaurin symmetric mean SFWMSM operator [40].

$$SFSSWMSM^{(q,o=0)}\left(\check{R}_1, \check{R}_2, \ldots, \check{R}_\alpha\right) = \left(\begin{array}{c} \left(\left(1-\left(\prod_{1\leq\imath_1,<\imath_2\leq\alpha}\left(1-\prod_{\ell=1}^{q}\left(1-\left(1-\mathcal{U}_{\imath\ell}^2\right)^{\delta_{\imath\ell}}\right)\right)\right)^{\frac{1}{C_\alpha^q}}\right)^{\frac{1}{q}},\right) \\ 1-\left(1-\left(\prod_{1\leq\imath_1,<\imath_2\leq\alpha}\left(1-\prod_{\ell=1}^{q}\left(1-\left(\vartheta_{\imath\ell}^2\right)^{\delta_{\imath\ell}}\right)\right)\right)^{\frac{1}{C_\alpha^q}}\right)^{\frac{1}{q}}, \\ 1-\left(1-\left(\prod_{1\leq\imath_1,<\imath_2\leq\alpha}\left(1-\prod_{\ell=1}^{q}\left(1-\left(\beta_{\imath\ell}^2\right)^{\delta_{\imath\ell}}\right)\right)\right)^{\frac{1}{C_\alpha^q}}\right)^{\frac{1}{q}} \end{array}\right)$$

$$= SFWMSM^{(q)}\left(\check{R}_1, \check{R}_2, \ldots, \check{R}_\alpha\right).$$

(2) If $q = 1$, the $SSSWMSM$ operator are,

$$SFSSWMSM^{(q=1,o)}\left(\check{R}_1, \check{R}_2, \ldots, \check{R}_\alpha\right)$$

$$= \left(\begin{array}{l} \left(\left(1-\left(\frac{1}{\alpha}\sum_{1\leq\imath_1\leq\alpha}\delta_{\imath\ell}\left(1-\mathcal{U}_{\imath\ell}^2\right)^{o}-\left(\delta_{\imath\ell}-1\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}, \\ 1-\left(\left(1-\left(\frac{1}{\alpha}\sum_{1\leq\imath_1\leq\alpha}\left(\omega_{\imath\ell}\left(\vartheta_{\imath\ell}^2\right)^{o}-\left(\omega_{\imath\ell}-1\right)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}, \\ 1-\left(\left(1-\left(\frac{1}{\alpha}\sum_{1\leq\imath_1\leq\alpha}\left(\omega_{\imath\ell}\left(\beta_{\imath\ell}^2\right)^{o}-\left(\omega_{\imath\ell}-1\right)\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\ (\text{let } \imath_\ell = \ell) \end{array}\right)$$

$$= \left(\begin{array}{l} 1-\left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(\delta_\ell\left(1-\mathcal{U}_\ell^2\right)^{o}-\left(\delta_\ell-1\right)\right)\right)^{\frac{1}{o}}, \\ \left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(\delta_\ell\left(\vartheta_\ell^2\right)^{o}-\left(\delta_\ell-1\right)\right)\right)^{\frac{1}{o}}, \\ \left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(\delta_\ell\left(\beta_\ell^2\right)^{o}-\left(\delta_\ell-1\right)\right)\right)^{\frac{1}{o}} \end{array}\right)$$

(3) If $q = 2$, the SFSSWMSM operator is the reduces of spherical fuzzy Schweizer-Sklar weighted Bonferroni SFSSWBM $(p = q = 1)$ operator,

$$SFSSWMSM^{(q=2,\ o)}\left(\check{R}_1,\ \check{R}_2, \ldots, \check{R}_\alpha\right) =$$

$$\left(\begin{array}{c} \left(\frac{1}{2}\left(1-\left(\frac{2}{\alpha(\alpha-1)}\left(\sum_{1\leq\imath_1<\imath_2\leq\alpha}\left(1-\left(1-\left(\sum_{\ell=1}^{2}\left(1-\left(\begin{array}{c}\delta_{\imath\ell}\left(\mathcal{U}_{\imath\ell}^2\right)^{o}- \\ \left(\delta_{\imath\ell}-1\right)\end{array}\right)^{\frac{1}{o}}\right)^{o}-1\right)^{\frac{1}{o}}-1\right)^{\frac{1}{o}}\right)^{o}\right)\right)\right)^{\frac{1}{o}}\right)^{o}+\frac{1}{2}\right)^{\frac{1}{o}}, \\ 1-\left(\frac{1}{2}\left(1-\left(\frac{2}{\alpha(\alpha-2)}\sum_{1\leq\imath_1<\imath_2\leq\alpha}\left(1-\left(\sum_{\ell=1}^{2}\left(1-\left(\begin{array}{c}\delta_{\imath\ell}\left(\vartheta_{\imath\ell}^2\right)^{o}- \\ \left(\delta_{\imath\ell}-1\right)\end{array}\right)^{\frac{1}{o}}\right)^{o}-1\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}+\frac{1}{2}\right)^{\frac{1}{2}}, \\ 1-\left(\frac{1}{2}\left(1-\left(\frac{2}{\alpha(\alpha-2)}\sum_{1\leq\imath_1<\imath_2\leq\alpha}\left(1-\left(\sum_{\ell=1}^{2}\left(1-\left(\begin{array}{c}\delta_{\imath\ell}\left(\beta_{\imath\ell}^2\right)^{o}- \\ \left(\delta_{\imath\ell}-1\right)\end{array}\right)^{\frac{1}{o}}\right)^{o}-1\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}+\frac{1}{2}\right)^{\frac{1}{2}} \end{array}\right)$$

$$= SFSSWBM^{1,1}\left(\check{R}_1, \check{R}_2, \ldots, \check{R}_\alpha\right).$$

(4) If $q = \alpha$, the SFSSWMSM operator is

$$SFSSWMSM^{(q=\alpha,o)}\left(\check{R}_1, \check{R}_2, \ldots, \check{R}_\alpha\right) =$$

$$\begin{pmatrix} \left(\frac{1}{\alpha}\left(1-\left(\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(1-\begin{pmatrix}\delta_{\imath\ell}\left(1-\mathcal{U}_{\imath\ell}^{2}\right)^{o}\\-\left(\delta_{\imath\ell}-1\right)\end{pmatrix}^{\frac{1}{o}}\right)^{o}\right)-(\alpha-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{\alpha}-1\right)\right)^{\frac{1}{o}}, \\ 1-\left(\frac{1}{\alpha}\left(1-\left(\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(1-\begin{pmatrix}\delta_{\imath\ell}\left(\vartheta_{\imath\ell}^{2}\right)^{o}-\\\left(\delta_{\imath\ell}-1\right)\end{pmatrix}^{\frac{1}{o}}\right)^{o}\right)-(\alpha-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{\alpha}-1\right)\right)^{\frac{1}{o}}, \\ 1-\left(\frac{1}{\alpha}\left(1-\left(\left(1-\left(\left(\sum_{\ell=1}^{\alpha}\left(1-\begin{pmatrix}\delta_{\imath\ell}\left(\beta_{\imath\ell}^{2}\right)^{o}-\\\left(\delta_{\imath\ell}-1\right)\end{pmatrix}^{\frac{1}{o}}\right)^{o}\right)-(\alpha-1)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}\right)^{o}-\left(\frac{1}{\alpha}-1\right)\right)^{\frac{1}{o}} \end{pmatrix}$$

$$(let\ \imath_\ell = \ell)$$

$$= \begin{pmatrix} \frac{1}{\alpha}\left(\sum_{\ell=1}^{\alpha}\left(1-\left(\delta_{\ell}\left(1-\mathcal{U}_{\ell}^{2}\right)^{o}-\left(\delta_{\ell}-1\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{\frac{1}{o}}, \\ 1-\left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(1-\left(\delta_{\ell}\left(\vartheta_{\ell}^{2}\right)^{o}-\left(\delta_{\ell}-1\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{o}, \\ 1-\left(\frac{1}{\alpha}\sum_{\ell=1}^{\alpha}\left(1-\left(\delta_{\ell}\left(\beta_{\ell}^{2}\right)^{o}-\left(\delta_{\ell}-1\right)\right)^{\frac{1}{o}}\right)^{o}\right)^{o} \end{pmatrix}$$

4 MAGDM Based on the SFSSWMSM Operator

The MAGDM difficulties will be solved using the SFSSWMSM operator in this part. Let $\mathcal{L} = \{\mathcal{L}_1, \mathcal{L}_2, \ldots, \mathcal{L}_m\}$ are the set of alternatives and $P = \{P_1,\ P_2, \ldots, P_\alpha\}$ the set of attributes. The weight vector is $\delta = (\delta_1, \delta_2, \ldots, \delta_\alpha)^T$ with $\delta_\imath \geq 0$, $\imath = 1, 2, \ldots, \alpha$, and $\sum_{\imath=1}^{\alpha} \delta_\imath = 1$. Further, let $\{D_1, D_2, \ldots, D_t\}$ set of experts and $\omega = (\omega_1, \omega_2, \ldots, \omega_t)$ weight vector with $\omega_s \geq 0\,(s = 1, 2, \ldots, t)$, and $\sum_{s=1}^{t} \delta_s = 1$. The decision matrices of this MAGDM problem are expressed by $\check{R}^s = [r_{\imath\ell}^{\ s}]_{m\times\alpha}$, where $r_{\imath\ell}^{\ s} = (a_{\imath\ell}^s,\ b_{\imath\ell}^s)$ is the evaluation information of alternative $\mathcal{L}_\imath$ concerning the attribute P_ℓ given by the decision maker D_s, which is expressed by the SFNs. Then alternatives are required. We will provide the detailed decision-making process based on the suggested SFSSWMSM operator, which is illustrated as follows.

Step 1. Information can be normalized are used the make decisions.

Benefit and cost attributes two main categories of the characteristics. Normalizing the decision matrix, $\check{R}^s = [r_{\imath\ell}^s]_{m\times\alpha}$we eliminate effects of various attribute types prior to integrating the evaluated attribute values. The normalized decision matrix is given as $\check{R}^s = [r_{\imath\ell}^s]_{m\times\alpha}$ after converting the cost attribute values to benefits ones, $(\imath = 1,\ 2,\ \ldots, m; \ell = 1,\ 2, \ldots, \alpha)$,
where,

$$\check{R}_{\imath\ell}^s = \begin{cases} (a_{\imath\ell}^s,\ b_{\imath\ell}^s)\ \ for\ be\alpha efit\ attribute\ p_\ell \\ (b_{\imath\ell}^s,\ a_{\imath\ell}^s)\ \ for\ cost\ attribute\ p_\ell \end{cases}$$

Step 2. By use the theorem 3.2 get the group evaluation values $r_{\imath\ell}\,(\imath = 1,\ 2, \ldots, m; \ell = 1,\ 2, \ldots, \alpha)$ are the introduced $SFSSWMSM$ operator

$$r_{\imath\ell} = SFSSWMSM^{(q,\ o)}\left(r_{\imath\ell}^1,\ r_{\imath\ell}^2, \ldots, r_{\imath\ell}^t\right)$$

Step 3. By the presented SFSSWMSM operator expressed in theorem 3, are comprehensive the values

$$\varphi_\imath\left(r_{\imath 1},\ r_{\imath 2}, \ldots, r_{\imath\alpha}\right),$$

shown as follows.

$$\varphi_\imath = SFSSWMSM^{(q,\ o)}\left(r_{\imath 1},\ r_{\imath 2}, \ldots, r_{\imath\alpha}\right)$$

Step 4. The Definitions 2 and 3, we evaluate the SVs $S(\varphi_\imath)$ of every alternative $\varphi_\imath\,(\imath = 1,\ 2, \ldots, m)$, are equal, to calculate the important AVs $H(\varphi_\imath)\ \ \varphi_\imath\,(\imath = 1, 2, \ldots, m)$.

Step 5. Prioritize the options. Definition 2 are used the best ranking.

4.1 An Illustrative the Example

Artificial Intelligence (AI) is used in many different fields to improve productivity and creativity. AI helps with drug discovery, personalized medicine, and diagnostics in the healthcare industry. It optimizes trading strategies and risk management in the financial domain. AI powers driverless cars for safer and more effective mobility in the transportation sector. Personalized learning platforms are beneficial for education, and AI-driven catboats are used for customer service. AI has a wider impact on manufacturing through predictive maintenance, and it also maximizes crop yields in agriculture. All things considered, the versatility of AI revolutionizes industries through task automation, data-driven decision-making, and the promotion of technological and problem-solving breakthroughs. Consider the problem of the assessment of some artificial intelligence tools based on some attributes $\{\mathcal{L}1, \mathcal{L}2, \mathcal{L}3, \mathcal{L}4\}$. Three experts D_s $(s = 1, 2, 3)$ and the weight vector are $\omega = (0.35, 0.40, 0.25)\,T)$ are invited the decision problem by the SFNs based on four basic indexes are weight vector is $\delta = (0.2, 0.1, 0.3, 0.4))$: the four alternative P_s $(s = 1, 2, 3)$ machine learning $(P1)$, natural language processing $(P2)$, expert systems$(P3)$ and the robotics $(P4)$, and four attributes learning $\mathcal{L}_1$, reasoning $\mathcal{L}_2$, perception $\mathcal{L}_3$ and problem-solving $\mathcal{L}_4$. then the three decision matrices $\check{R}_s = [r^s_{\imath\ell}]_{5\times 4}$ $(s = 1, 2, 3)$ are constructed and the objective is to choose the finest company for investment out of those given in Table 1, Table 2, Table 3 and Table 4. These tables contain only SFNs for all of the data for $q = 3$.

The following process is illustrated for the suggested method:

Step 1. Normalizing the information used for decision-making $\check{R}^s$. We don't need the decision matrix because in this step all attribute values are all of the benefit.

Step 2. By the introduced SFSSWMSM operator expressed, we obtain the group evaluation values $r_{\imath\ell}(l = 1, 2, 3, 4; \ell = 1, 2, 3, 4\,)$ (consider q $= 2$ and $o = -6$).

Step 3. By the introduced SFSSWMSM operator expressed in theorem 3.2 we can obtain the comprehensive evaluation values $\varphi_\imath(\imath = 1, 2, 3, 4)$ (suppose $q = 2$ and $o = -6$).

$$\varphi_1 = (0.6260,\ 0.7220,\ 0.5440)\,,\ \varphi_2 = (0.3510,\ 0.5400,\ 0.2010)\,,\ \varphi_3 = (0.4530,\ 0.5122,\ 0.5441)\,,$$

$$\varphi_4 = (0.4020,\ 0.3320,\ 0.2450)$$

Step 4. Considering Definition 2 we evaluate the SVs $S(\varphi_\imath)$ of each alternative $\varphi_\imath$ $(\imath = 1, 2, 3, 4)$, and get

$$S(\varphi_1) = 0.5332,\ S(\varphi_2) = 0.7255,\ S(\varphi_3) = 0.7044, S(\varphi_4) = 0.3245$$

Step 5. Rank the alternatives.
Considering Definition 2.2 and the values of $S(\varphi_\imath)$, we obtain the ranking alternatives as follows:

$$\mathcal{L}_2 > \mathcal{L}_3 > \mathcal{L}_1 > \mathcal{L}_4$$

Table 1. $\check{\mathrm{R}}^1$ Decision maker D_1

	P_1			P_2			P_3			P_4		
$\mathcal{L}_1$	0.33	0.44	0.72	0.33	0.53	0.68	0.22	0.32	0.42	0.51	0.53	0.22
$\mathcal{L}_2$	0.62	0.78	0.32	0.34	0.74	0.55	0.52	0.33	0.52	0.62	0.42	0.45
$\mathcal{L}_3$	0.52	0.29	0.71	0.34	0.54	0.44	0.52	0.56	0.43	0.32	0.61	0.75
$\mathcal{L}_4$	0.51	0.33	0.22	0.44	0.23	0.35	0.11	0.29	0.32	0.44	0.24	0.21

Table 2. $\check{\mathrm{R}}^2$ the Decision maker D_2

	P_1			P_2			P_3			P_4		
$\mathcal{L}_1$	0.26	0.19	0.35	0.54	0.23	0.18	0.35	0.21	0.25	0.32	0.44	0.22
$\mathcal{L}_2$	0.42	0.34	0.12	0.25	0.25	0.32	0.16	0.31	0.22	0.32	0.32	0.45
$\mathcal{L}_3$	0.25	0.35	0.12	0.23	0.24	0.34	0.23	0.12	0.36	0.34	0.43	0.33
$\mathcal{L}_4$	0.55	0.22	0.33	0.24	0.32	0.25	0.27	0.27	0.21	0.32	0.13	0.22

So, the best alternative is $\mathcal{L}_2$. The finest decision is $\mathcal{L}_2$ according to the theory of the SFSSWMSM operator. Additionally, we are comparing the suggested work with the current operator while taking into account the aforementioned numerical examples in order to demonstrate the efficacy and dependability of the derived theory.

Table 3. $\check{R}^3$ the Decision maker D_3

	P_1			P_2			P_3			P_4		
$\mathcal{L}_1$	0.53	0.21	0.18	0.25	0.31	0.14	0.23	0.22	0.15	0.22	0.65	0.11
$\mathcal{L}_2$	0.28	0.16	0.32	0.33	0.14	0.24	0.22	0.11	0.22	0.34	0.82	0.24
$\mathcal{L}_3$	0.12	0.45	0.18	0.36	0.12	0.11	0.15	0.31	0.33	0.22	0.36	0.22
$\mathcal{L}_4$	0.23	0.15	0.26	0.16	0.22	0.36	0.41	0.22	0.36	0.22	0.25	0.21

Table 4. Results of ranking based on various parameter values of o

	P_1			P_2			P_3			P_4		
$\mathcal{L}_1$	0.4282	0.5182	0.7346	0.4243	0.7229	0.7533	0.4022	0.7677	0.5670	0.4362	0.3642	0.5564
$\mathcal{L}_2$	0.6322	0.8190	0.7101	0.3124	0.4941	0.4312	0.5500	0.5762	0.3095	0.3017	0.2264	0.5432
$\mathcal{L}_3$	0.6283	0.3588	0.6205	0.4188	0.5974	0.6241	0.5322	0.4354	0.6826	0.462	0.6828	0.6455
$\mathcal{L}_4$	0.4107	0.5503	0.520	0.4243	0.6244	0.3430	0.44	0.52	0.5521	0.5322	0.3253	0.5233

4.2 Comparative Analysis

In order to improve the strength and value of the novel operators, we focus on drawing comparisons between the derived work and other existing works. To do this, we attempt to gather a variety of existing data, including Le Berre et al. [31] derived the theory of AOs for Artificial Intelligence, Lee and Yoon [32] exposed the theory of AO for Artificial Intelligence and finally, Lundström and Hellström [18] examined the theory of AO for electric car. Additionally, Hussain and Pamucar [30] derived the idea of PAOs for PyFSs. Under the presence of the information in Table 1, the comparative information is available in Table 5.

Table 5. Comparison information matrix

Methods	Ranking Values
SFSSWMSM	$\mathcal{L}_2 > \mathcal{L}_1 > \mathcal{L}_4 > \mathcal{L}_3$
Le Berre et al. [31]	$\mathcal{L}_2 > \mathcal{L}_1 > \mathcal{L}_4 > \mathcal{L}_3$
Lee and Yoon [32]	$\mathcal{L}_2 > \mathcal{L}_1 > \mathcal{L}_4 > \mathcal{L}_3$
Hussain and Pamucar [30]	$\mathcal{L}_2 > \mathcal{L}_1 > \mathcal{L}_4 > \mathcal{L}_3$
Lundström and Hellström [18]	$\mathcal{L}_2 > \mathcal{L}_1 > \mathcal{L}_4 > \mathcal{L}_3$

5 Conclusion

This study of, algebraic operations are less flexible than Schweizer-Sklar operations, which have a variable and infinite parameter. For tackling information fusion problems, the Maclaurin symmetric mean (MSM) turns out to be an invaluable tool. For spherical fuzzy number sets (SFNSs), this paper presents two new Schweizer-Sklar Maclaurin mean operators: the SFSSMSM operator and the SFSSWMSM operator. Based on the SFSSWMSM operator, a novel method for resolving Multi-Attribute Group Decision Making (MAGDM) problems is created, improving versatility with the help of the parameters α. In addition to taking into account the relationships between the two independent integrated arguments, this method also takes decision-makers' risk preferences into account by using the parameter gamma. The suggested approach contributes to the understanding of fuzzy decision-making strategies and offers a better way to handle difficult MAGDM scenarios. This novel approach is anticipated to be beneficial for future applications in fields such as consensus models, expert allocation, T-spherical Hami mean and supply selection.

Author Contributions

Mehwish Sarfaraz: Conceptualization, writing, review.

Data Availability

The data used to support the research findings are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that none of the work reported in this paper could have been influenced by any known competing financial interests or personal relationships.

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