Journal of Operational and Strategic Analytics
https://www.acadlore.com/journals/JOSA

Undesirable Input in Production Process: A DEA-Based Approach

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Received: 05-04-2023 **Revised:** 06-06-2023 **Accepted:** 06-12-2023

Citation: M. Maghbouli and A. P. Yekta, "Undesirable input in production process: A DEA-based approach," *J. Oper. Strateg Anal.*, vol. 1, no. 2, pp. 46–54, 2023. https://doi.org/10.56578/josa010201.

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Abstract: The prevalent economic principle of weak disposability has been the foundation for studies in environmental assessment using Data Envelopment Analysis (DEA). Recently, a shift from classical free disposability to weak disposability has been observed as an emerging trend for treating undesirable factors in research. Weak disposability is perceived to have significant analytical power in measuring the efficiency of Decision-Making Units (DMUs). Addressing the increment of undesirable inputs, a non-radial model grounded on a non-uniform augment factor is presented. The application of this proposed model anticipates a suitable quantity for the increment of undesirable inputs. Concurrently, the model ensures a corresponding reduction in desirable inputs. Numerical instances illuminate the practicality and robustness of the proposed model and demonstrate its superior performance over its original counterpart.

Keywords: Undesirable inputs; Data Envelopment Analysis (DEA); Decision Making Unit (DMU); Weak disposability; Environmental assessment

1 Introduction

Over the past decade, there has been an increasing interest in the efficiency and productivity management of undesirable outputs and inputs. In production theory, both parametric and non-parametric techniques offer the advantage of imposing the "weak-disposability" assumption on the functional form of the underlying technology. Data Envelopment Analysis (DEA), first introduced by Charnes et al. [1] and later extended by Banker et al. [2], has recently made significant contributions to the analysis of undesirable variables. The modeling of undesirable factors has gained considerable attention not only for measuring efficiency and productivity but also for estimating pollution factors. This issue has been investigated in various research studies, with early contributions from Hailu and Veeman [3], Färe and Grosskopf [4], Hailu [5], and Kuosmanen [6].

The concept of reducing undesirable outputs by decreasing the level of production activity (output weak disposability axiom) was first proposed by Shepard [7]. The author applied a uniform abatement factor to all observed activities in the sample. Subsequently, Kuosmanen [6] argued that using a uniform abatement factor is inconsistent with the conventional wisdom of focusing abatement factors on firms with lower abatement costs. Podinovski and Kuosmanen [8] developed two additional technologies for modeling weak-disposability under relaxed convexity assumptions. A methodological contribution of such DEA-based studies, in alignment with the output weak disposability definition, posits that a proportional reduction in the level of undesirable outputs can be achieved if accompanied by a reduction in desirable outputs in the same proportion.

Considering the importance of recycling processes, both desirable and undesirable inputs may be employed in some real-world situations. Due to the scarcity of natural resources and the need to preserve them by using renewable resources, the concept of undesirable inputs has gained prominence in recent studies. One of the main examples of undesirable inputs is grey water. In light of global warming and water resource scarcity, the reconstitution and recycling of grey water into the production process have become crucial.

Several researchers have proposed methods to address undesirable inputs. Färe and Grosskopf [4] introduced an alternative approach called input weak disposability, based on the concept of weak disposability. Jahanshahloo et al. [9] presented a non-radial DEA-based model for managing both undesirable inputs and outputs, aiming to decrease undesirable outputs and increase undesirable inputs simultaneously. Eyni and Maghbouli [10] transformed undesirable inputs into desirable inputs, and then derived a common set of weights in the presence of these transformed inputs. Liu et al. [11] treated undesirable inputs and outputs as desirable outputs and inputs, respectively, while assuming the standard strong disposability assumption using a non-radial model applying Russell measure or slack-based DEA models.

A review of the DEA literature reveals numerous DEA models for modeling undesirable inputs using the concept of weak disposability. Roshdi et al. [12] introduced a new concept of exponential weak disposability assumption for undesired outputs, allowing for different types of trade-offs between desirable and undesirable outputs. By satisfying three axioms (concavity, linearity, and convexity), a piecewise Cobb-Douglas environmental technology was derived. Based on this technology, radial and non-radial functions were extracted to measure environmental performance. Mehdiloozad and Podinovski [13] noted that Shepard's technology modification for increasing undesirable input with a single scaling factor can cause problematic side effects, such as congestion measurement issues. To address this deficiency, the authors developed an appropriate technology that incorporates weak input disposability. Then, based on progressively relaxed convexity assumptions, various ranges of technologies were also investigated. Mehdiloo and Podinovski [14] argued that the disposability assumption may not be suitable and could lead to meaningless proportions when inputs or outputs are overlapping or strongly correlated. To address this issue, they developed a production technology in which groups of closely related inputs and outputs are only jointly weakly disposable. Pham and Zelenyuk [15] discussed the use of single or multiple scaling factors in different scenarios and revealed the link between various returns to scale and weak disposability of desirable and undesirable outputs. Another contribution of their study was the construction of a comprehensive taxonomy of reference technology sets for activity analysis models with various return to scale assumptions. Fouladvand et al. [16] developed a linear model to investigate congestion in the presence of undesirable outputs, employing the concept of output weak disposability. Li et al. [17] proposed a model based on a circular economy structure to analyze waste treatment efficacy for solid waste during the 11th and 12th Five-Year Plans from 2011 to 2015. The research showed that efficiency in pollution and disposal of solid waste improved during these periods. Monzeli et al. [18] determined efficiency measurements in the presence of undesirable inputs and outputs using a three-step approach: first, an appropriate production possibility set was defined based on problem assumptions; second, the undesirable effects in DMUs were modeled by considering the weak disposability assumption; and third, the efficiency of DMUs was calculated using a radial DEA model. Kordrostami et al. [19] expanded the classical definition of weak disposability to accommodate undesirable inputs and introduced a linear formulation. By implementing the concept of simultaneous proportional reduction in desirable and undesirable outputs, along with proportional expansion in favorable and unfavorable inputs, a linear model was applied to efficiency analysis. Further information can be found in recent studies by Hua and Wang [20] and Yu and Rakshit [21]. Although each approach in the literature has its merits, the application of the weak disposability axiom in activity analysis continues to elicit questions.

In the current context, the utilization of recycled materials is becoming increasingly crucial as it addresses environmental concerns and promotes the incorporation of waste and recycled materials into production systems. According to the World Commission on Environment and Development (WCED), pollution levels continue to rise while resource scarcity persists. As a result, there is growing interest in the recycling industry and efficiency management, which takes undesirable inputs into account. From a computational standpoint, it is rational to consider a suitable amount for the extended use of undesirable inputs, necessitating the development of an optimization-based approach to address this issue.

With respect to weak disposable technology, minimal attention has been given to the potential increase in undesirable inputs. To gain a deeper understanding of the concept of weak input disposability, this study examines the use of a scaling factor for undesirable inputs within the context of Shepard technology. In other words, an extension factor is identified to manage both desirable and undesirable inputs. The focus of this research is on the assumption of weak input disposability, which may yield more realistic results in terms of economic development.

The remainder of this study is organized as follows: Section 2 provides a brief overview of weak input disposability axioms, followed by a redefinition of weak input disposability in Section 3. Finally, a conclusion is presented to summarize the findings.

2 Weak Disposability of Inputs

In recent years, the modeling of undesirable inputs, such as plastic waste, grey water, or rotten fruits, has garnered significant attention among researchers. The increasing levels of pollution and the depletion of natural resources have intensified the demand for cleaner sources. Recycling industries are at the forefront of addressing this challenge, as recycling processes not only reduce energy consumption and environmental pollution but also convert waste into new materials for the sake of environmental protection. To the best of our knowledge, limited attention has been directed towards undesirable inputs.

Mehdiloozad and Podinovski [13] focused on the weak disposability of inputs, developing a convex technology that mitigates the bias of non-convex Shepard technology in congestion evaluation. Several production technologies accounting for the weak disposability of inputs have also been investigated. Färe and Grosskopf [4] proposed using a monotone scaling factor instead of weak disposability, terming it an expansion factor. Kordrostami et al. [19] defined weak disposability for input and corresponding technology.

Assuming there are K Decision-Making Units (DMUs), the data for DMUk on vectors of desirable inputs, undesirable inputs, and outputs are represented as $x_k\left(x_{1k},\ldots,x_{Nk}\right)\geq 0,\quad v_k\left(v_{1k},\ldots,v_{Mk}\right)\geq 0$ and $w_k\left(w_{1k},\ldots,w_{Jk}\right)\geq 0$, respectively. Furthermore, it is assumed that $x_k\neq 0, v_k\neq 0$ and $w_k\neq 0$. The production technology can be defined as follows:

$$P(w)=\left\{(x,v,w)\mid (x,v)\text{ can produce }w, w\in R_J^+\right\}$$

Definition 1: Inputs (desirable and undesirable) are weakly disposable if and only if $(x,v)\in P(w)$ and $\varphi\geq 1$ imply that $(\varphi x,\varphi v)\in P(w), w\in R_J^+$.

The multiplier φ in the definition above refers to the profit factor, which allows for proportional increases in inputs according to the conditions $\varphi\geq 1$ presented. Kordrostami et al. [19] defined the non-linear minimal technology in terms of the profit factor φ.

$$\begin{aligned}P(w)=\{(x,v)\Bigg|&\sum_{k=1}^{K}\varphi^k z^k x_n^k\leq x_n,\quad n=1,\ldots,N\\&\sum_{k=1}^{K}\varphi^k z^k v_m^k=v_m,\quad m=1,\ldots,M\\&\sum_{k=1}^{K}z^k w_j^k\geq w_j,\quad j=1,\ldots,J\\&\sum_{k=1}^{K}z^k=1\\&z^k\geq 0\\&\varphi^k\geq 1\}\end{aligned}\tag{1}$$

The variable $z=\left(z^1,\ldots,z^k\right)$ is referred to as the intensity variable. The profit factor $\varphi\geq 1$ in the first and second constraints enables the simultaneous increase of undesirable inputs. The last constraint also accounts for the growth of consumption by recycled units. Kordrostami et al. [19] stated that the non-linear technology above can be expressed in its equivalent linear form by substituting the intensity variable $z^k=\lambda^k-\mu^k$, where $\lambda^k=\phi^k z^k$ and $\mu^k=\left(\phi^k-1\right)z^k$. The linear technology is as follows:

$$\begin{aligned}P(w)=\{(x,v)\Bigg|&\sum_{k=1}^{K}\lambda^k x_n^k\leq x_n, n=1,...,N\\&\sum_{k=1}^{K}\lambda^k v_m^k=v_m,\ m=1,...,M\\&\sum_{k=1}^{K}(\lambda^k-\mu^k)w_j^k\geq w_j,\ j=1,...,J\\&\sum_{k=1}^{K}(\lambda^k-\mu^k)=1, k=1,...,K\\&\lambda^k,\mu^k\geq 0\}\end{aligned}\tag{2}$$

This technology is linear with respect to the unknown variables λ and μ. In evaluating the efficiency of DMU_o, attention is given to the radial measure. The linear model can be formatted as follows:

$$
\begin{aligned}
& Max\ \rho \\
& s.t. \\
& \qquad \sum_{k=1}^{K} \lambda^k x_n^k \leq x_n^o \ , n = 1, ..., N \\
& \qquad \sum_{k=1}^{K} \lambda^k v_m^k = \rho v_m^o ,\ m = 1, ..., M \\
& \qquad \sum_{k=1}^{K} (\lambda^k - \mu^k) w_j^k \geq w_j^o ,\ j = 1, ..., J \\
& \qquad \sum_{k=1}^{K} (\lambda^k - \mu^k) = 1, k = 1, ..., K \\
& \qquad \lambda^k, \mu^k \geq 0
\end{aligned}
\tag{3}
$$

The objective function aims to maximize the proportional profit factor for all undesirable inputs while preserving the current level of desirable inputs and all outputs. It is evident that Model (3) is feasible. Moreover, if the evaluated unit has an efficiency score of unity, it is considered an efficient unit.

3 Modifying Weak Disposability of Inputs

The profit factor φ, as discussed in the previously mentioned technologies, belongs to the infinite interval $[1, +\infty)$. In real-world scenarios, cases may arise where it is impossible to completely disregard undesirable factors, such as the increasing need for recycling to protect the environment. Moreover, the profit factor φ cannot attain unbounded quality, and its usage has inherent limitations. To obtain reliable results and improve applicability, a modification appears warranted.

Model (3), discussed in the previous section, and solely focuses on increasing undesirable inputs. This perspective may lead to different efficiency measures and, in some cases, deviate from reality. In fact, in all Data Envelopment Analysis (DEA) applications, input reduction is desired and expected. In other words, the model should consider both perspectives simultaneously. Consequently, it is logical to modify the model to not only support the increase of undesirable factors but also encourage the reduction of desirable inputs. This modification may develop approaches aimed at addressing the problem in the presence of undesirable inputs.

Considering the concept of input weak disposability, dual points are replaced in Model (3). To achieve this, a linear model is applied to expect the simultaneous reduction of desirable inputs and increase of undesirable inputs. The concept of slack variables is modified to be used in the first constraint related to desirable input, ensuring the reduction of desirable inputs. Applying the modified constraint, based on the idea of weak disposability of undesirable inputs, may lead to the reduction of desirable inputs, as expected in the production process, and increase the quantity of undesirable input, as the underlying technology intended.

Assuming there are K DMUs, the data for DMUk on vectors of desirable inputs, undesirable inputs, and outputs are represented $x_k\,(x_{1k}, \ldots, x_{Nk}) \geq 0, v_k\,(v_{1k}, \ldots, v_{Mk}) \geq 0$ and $w_k\,(w_{1k}, \ldots, w_{Jk}) \geq 0$, respectively. Furthermore, it is assumed that $x_k \neq 0, v_k \neq 0$ and $w_k \neq 0$. The production technology can be defined as follows:

$$P(w) = \{(x, v, w) \,|(x, v) \text{ can produce } w, w \in R_J^+\}$$

To evaluate the efficiency of DMUs with the above proposition, the production technology of Kuosmanen [6] is considered, and Model (3) can be modified accordingly:

$$
\begin{aligned}
& Max \sum_{m=1}^{M} \rho_m + \frac{\varepsilon}{N} \sum_{n=1}^{N} \frac{s_n}{x_n^o} \\
& s.t. \\
& \qquad \sum_{k=1}^{K} \lambda^k x_n^k \le x_n^o \ - s_n, n=1,...,N \\
& \qquad \sum_{k=1}^{K} \lambda^k v_m^k = \rho_m v_m^o,\ m=1,...,M \\
& \qquad \sum_{k=1}^{K} (\lambda^k + \mu^k) w_j^k \ge w_j^o,\ j=1,...,J \\
& \qquad \sum_{k=1}^{K} (\lambda^k + \mu^k) = 1, k=1,...,K \\
& \qquad 1 \le \rho_m \\
& \qquad x_n^o \ - s_n \ge 0 \\
& \qquad s_n, \lambda^k, \mu^k \ge 0
\end{aligned}
\tag{4}
$$

Upon close examination, all constraints within the modified model support the idea of DEA weak input disposability. The profit factor $\rho_m \ge 1$ ensures the increase of undesirable input and leads to its increment. The first constraint is modified as $\sum_{k=1}^{K} \lambda^k x_n^k \le x_n^o - s_n, n = 1, \ldots, N$ and refers to the proportional desirable input reduction, admitting that the remaining increase can be traced back to desirable inputs. The requirement for dominance constraints, $1 \le \rho_m$ intends the proportional increase determined for undesirable inputs. The objective function, defined as $\sum_{m=1}^{M} \rho_m + \frac{\varepsilon}{N} \sum_{n=1}^{N} \frac{s_n}{x_n^o}$, secures the contribution of both desirable and undesirable inputs in the production process. The first term indicates the increase of undesirable input as the first priority, and the second term guarantees the share of desirable input reduction. The existence of non-Archimedean $\mathcal{E}$ in the second term confirms the priority of desirable input reduction concerning the increase of undesirable inputs.

In summary, the modified Model (1) not only emphasizes the increment of undesirable input as expected but also seeks the simultaneous reduction of desirable inputs. The main characteristic of the modified Model (1) is supporting the weak disposability axiom by imposing the constraint $\sum_{k=1}^{K} \left(\lambda^k + \mu^k\right) = 1$ that stems from the transformation of non-uniform increase factor for all units. As stated earlier, the intensity variable λ^k is associated with the part of the input that remains active, and the variable μ^k is related to the part of the input that has increased due to the growth in activity level. It can be easily demonstrated that Model (4) is always feasible.

Theorem 1: Model (4) is always feasible.

Proof: Refer to Kordrostami et al. [19].

When evaluating using Model (4), the unit being assessed, DMU_o is considered efficient if the efficiency measure equals one. The efficiency of an inefficient unit is greater than unity. Overall, Model (4) not only avoids overestimating the efficiency of DMUs but also provides relatively better discrimination among DMUs compared to existing models.

4 Numerical Example

4.1 Example 1

The applicability of the proposed approach is demonstrated using a real data set consisting of thirty units. To investigate the effect of temperature on chemical instances, each unit employs two sets of inputs: desirable and undesirable, to produce two categories of outputs. The desirable inputs include ionic liquid and metal materials, while the undesirable inputs consist of temperature. Notably, higher temperatures appear more acceptable during experiments. The outputs are characterized by time (measured in minutes) and the percentage of yielded material. Table 1 provides a summary of the data set.

The results of Model (3) and Model (4) are presented in Table 2.

As Table 2 indicates, Model (3) identifies nine efficient units, whereas Model (4) considers seven out of thirty units as efficient. Additionally, the augmentation factor in Model (4) is lower than that in Model (3). The slack variables in Model (4) are all zero, except for the first slack for DMUs #26, #27, and #28. It is revealed that the first desirable input needs to be reduced for these three DMUs. In DMU #28, the profit factor obtained by Model (3) is 13.66, while it decreases to 6.27 in Model (4). It seems that constraint $\sum_{k=1}^{K} \left(\lambda^k + \mu^k\right) = 1$ in Model (4) limits the profit factor to an achievable level for a unit. Concurrently, the reduction of desirable input is observed, as demonstrated by the fourth column of Table 2. Accordingly, these three units may reduce their first desirable inputs to the quantities 393.79, 317.4, and 257.6, respectively. As Model (4) suggests, the second desirable input remains unchanged in the production process. Statistically, the average efficiency scores in Model (4) are lower than those in Model (3). Moreover, the last row of Table 2 shows that the variance of efficiencies obtained from Model (4) is significantly lower than that of Model (3) with values of 2.640012 and 9.763928, respectively.

Table 1. The data set

DMU	Undesirable Input	Desirable Input 1	Desirable Input 2	Output 1	Output 2
1	665	437	1438	2015	14667
2	491	884	1061	3296	1162
3	417	1160	9171	2276	1819
4	302	626	10151	1640	555
5	229	374	8416	9564	3287
6	1083	597	3038	5409	1833
7	1053	870	3342	1651	754
8	740	685	9984	4787	1625
9	845	582	8877	3521	1667
10	517	763	2829	2629	1158
11	664	689	6057	6286	1763
12	313	355	1609	20512	9482
13	1206	851	2352	12654	3786
14	377	926	1222	3188	1087
15	792	203	9698	6477	2121
16	524	1109	7141	7613	4565
17	307	861	4391	1539	763
18	1449	249	7856	1205	496
19	1131	652	3173	8957	1819
20	826	364	3314	16195	3515
21	1357	670	5422	1538	363
22	1089	1023	4388	3099	848
23	652	1049	3665	4412	1516
24	999	1164	8549	2530	985
25	526	1012	5162	5165	1702
26	218	464	10504	3142	1131
27	1339	406	9365	2120	847
28	231	1132	9958	1320	488
29	1431	593	3552	5807	2503
30	965	262	6211	6977	3757

4.2 Example 2

This example examines seven chemical instances during a laboratory experiment. The data set comprises two sets of inputs, desirable and undesirable, along with two categories of outputs. Table 3 presents the data set, which is derived from Eyni and Maghbouli [10].

By applying Model (3) and Model (4), Table 4 displays the efficiency scores of Model (3) and the proposed Model (4).

As the results indicate, Model (4) identifies four efficient units out of seven, while Model (3) yields an unbounded solution. It is observed that, in the presence of undesirable input, the objective function of the augmentation factor for undesirable input may lead to unrealistic results. Conversely, Model (4) provides more realistic scores. Although there are no differences between efficient units in Model (4), additional information is detected in the presence of undesirable inputs. As Table 3 indicates, the first desirable input can be reduced to the quantities 9.67 and 9.70 for units #5 and #7. In summary, Model (4) outperforms Model (3) in this experimental example.

Table 2. Efficiency assessment for thirty chemical data

DMU	**Model 3**	**Model 4**	S_1	S_2	$x_1 - s_1$
1	1	1	0	0	437
2	1	1.11	0	0	884
3	7.42	3.45	0	0	1160
4	7.88	4.79	0	0	626
5	6.33	4.73	0	0	374
6	1.17	1.18	0	0	597
7	1.45	1.32	0	0	870
8	3.29	1.94	0	0	685
9	2.51	1.70	0	0	582
10	2.52	2.51	0	0	763
11	2.88	2.13	0	0	689
12	1	1	0	0	355
13	1	1	0	0	851
14	1.54	1.66	0	0	926
15	1	1.38	0	0	203
16	5.22	2.39	0	0	1109
17	6	4.67	0	0	861
18	1	1	0	0	249
19	1.16	1.15	0	0	652
20	1	1	0	0	364
21	1.33	1.06	0	0	670
22	1.76	1.32	0	0	1023
23	2.64	2.20	0	0	1049
24	3.02	1.45	0	0	1164
25	4.11	2.73	0	0	1012
26	9.81	6.61	70.21	0	393.79
27	1.42	1.08	88.60	0	317.4
28	13.66	6.27	874.40	0	257.6
29	1	1	0	0	593
30	1	1	0	0	437
Average	3.204	2.194333	—-	—-	—-
Variance	9.763928	2.640012	—-	—-	—-

Table 3. The chemical data set

DMU	**Undesirable Input**	**Desirable Input**	**Output 1**	**Output 2**
1	40	0	240	56
2	25	5	75	65
3	40	3	30	75
4	40	10	10	96
5	25	10	40	89
6	55	10	8	90
7	40	15	15	88

Table 4. The result of Model (4)

DMU	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Model 4	1	1.9	1	1	1.77	1	1.36
s_1	0	0	0	0	1.33	0	5.30
$x_1 - s_1$	0	5	3	10	9.67	10	9.70

5 Conclusions

In light of global warming and the scarcity of natural resources, the importance of recycling and the utilization of renewable energy resources have garnered increased attention from researchers. In the realm of efficiency and productivity analysis, various approaches have been developed within DEA research to address the concept of undesirable inputs. One favored approach involves substituting the axiom of weak input disposability with the free disposability assumption. Despite numerous advancements in this area, the debate on weak disposability persists.

The present study introduces a non-radial alternative model based on a non-uniform augmentation factor for undesirable inputs, addressing two critical issues. Firstly, the proposed model offers a reasonable amount for the increment of undesirable outputs, yielding different results from existing models in the literature. Secondly, the model achieves a simultaneous reduction in desirable outputs, a feature that can be justified in real-world applications. The applicability and strength of the proposed model are demonstrated through two examples.

By developing this non-radial alternative model, the study contributes to the ongoing discourse on weak disposability and expands the range of available techniques for handling undesirable inputs in efficiency and productivity analysis. Future research could explore other non-uniform augmentation factors, further refining the proposed model and its applicability to various industrial and environmental contexts. Additionally, comparisons with other existing models could provide valuable insights into the strengths and limitations of the proposed approach, facilitating the development of even more robust methods for addressing the challenges posed by undesirable inputs in efficiency and productivity analysis.

Data Availability

All references of data set are addressed in the manuscript.

Acknowledgements

The authors would like to appreciate Professor SEYYED AHMAD EDALATPANAH for his motivation and support.

Conflicts of Interest

This manuscript has not been submitted to, or nor is under review at another journal or other publishing venue. The authors have no affiliation with any organization with a direct or indirect financial interest in the subject matter discussed in the manuscript. The corresponding author will complete and submit the manuscript via Editorial Manger or to the Journal's Editorial Office, on behalf of all authors. I certify that there is no actual or potential conflict of interest in relation to this study.

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