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An Enhanced AHP Group Decision-Making Model Employing Neutrosophic Trapezoidal Numbers

Shahid Ahmad Bhat*

LUT Business School, LUT University, P.O. Box 20, FIN-53851 Lappeenranta, Finland

* Correspondence: Shahid Ahmad Bhat (shahid.bhat@lut.fi)

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Abstract: This study emphasizes the limitations observed in the prevailing neutrosophic AHP group decision-making model. To address these limitations, an augmented neutrosophic AHP group decision-making model has been established, leveraging the potential of neutrosophic trapezoidal numbers. A comprehensive exploration of a key property of the neutrosophic trapezoidal pairwise comparison matrix is performed in this research, revealing that the current model inadequately maintains the reciprocal property of the neutrosophic trapezoidal pairwise comparison matrix. A real-world decision-making problem is resolved utilizing the introduced model, and a comparative analysis is furnished between the pre-existing neutrosophic AHP group decision-making model and the revised version. The results unequivocally demonstrate the superiority of the enhanced model.

Keywords: AHP; Neutrosophic trapezoidal number; Group decision-making; Pairwise comparison matrices

1 Introduction

In the intricate domain of multi-criteria decision-making (MCDM), identifying an optimal choice from a selection of alternatives based on various parameters can present substantial challenges due to the complexity inherent in real-world decision-making dilemmas. The analytic hierarchy process (AHP) is a pivotal technique, developed by Saaty [1], which has demonstrated proficiency in navigating such intricate decision-making scenarios [2]. AHP simplifies the complexities by transforming them into a hierarchy with multiple layers, including the objective level, criteria level, sub-criteria level, and alternative level. The procedure mandates the decision maker to engage in pairwise comparisons of the objects at each level, utilizing a fundamental 1-9 scale and incorporating the findings into a pairwise comparison matrix [2, 3].

Subsequently, the priority weights of the alternatives, in relation to each criterion, are calculated, alongside the priority weights of the criteria with respect to the objective of the issue at hand. The synthesis of global priority weights then permits the ranking of the available alternatives [4].

The canonical version of AHP has undergone several extensions and adaptions, particularly to encompass fuzzy and intuitionistic fuzzy environments [3–8]. With a similar goal in mind, Abdel-Basset et al [9] expanded AHP into the realm of the neutrosophic environment, employing trapezoidal neutrosophic numbers to accomplish this. Abdel-Basset et al [9] identified that the construction of pairwise comparison matrices with $\frac{n\times(n-1)}{2}$ judgments may lead to inconsistent judgments owing to the sizable value of n. To address this, a restriction of judgments to $(n-1)$ was proposed. Further, they posited that the traditional 1-9 scale of AHP is not without flaws and, in response, proposed a new scale [0, 1] and offered a process to verify the consistency of trapezoidal neutrosophic pairwise comparison matrices [9].

However, an in-depth exploration has revealed certain limitations inherent in Abdel-Basset et al.'s methodology [9] when applied to a neutrosophic environment. This document seeks to articulate these shortcomings and offer a modified approach that addresses these limitations. A real-life decision-making problem, which has previously been resolved by Abdel-Basset et al., is presented, and an exact solution is determined using the proposed modified method.

Through this proposed method, an in-depth investigation will take place, challenging existing models and expanding the understanding of complex decision-making processes in the neutrosophic environment. The ultimate goal is to further enhance the scientific community's knowledge and practical capabilities when encountering MCDM problems. The profound influence of AHP and its numerous extensions is acknowledged, while simultaneously offering this updated method to confront the evolution of decision-making scenarios. Thus, this work hopes to contribute an additional dimension to the ongoing scientific dialogue on MCDM and AHP, fostering future research and application of these methodologies.

2 Preliminaries

In this section, basic concepts of SVNSs, single valued trapezoidal neutrosophic numbers, and their operational laws are presented, focusing on the universal set $X = \{x_1, x_2, \ldots, x_n\}$ comprising n objects.

Definition 2.1 A single valued trapezoidal neutrosophic number $\tilde{N}$ is denoted as $\widetilde{N} = \langle (L, M, M', U)\,; T, I, F\rangle$ and defined as [10]:

$$T_{\tilde{N}}(x) = \begin{cases} T_{\tilde{N}}\left(\frac{x-L}{M-L}\right), & L < x \leq M \\ T_{\tilde{N}}, & M < x \leq M' \\ T_{\tilde{N}}\left(\frac{U-x}{U-M'}\right), & M' \leq x < U \\ 0, & \text{otherwise.} \end{cases};$$

$$I_{\tilde{N}}(x) = \begin{cases} \frac{\left(M-x+I_{\tilde{N}}(x-L)\right)}{(M-L)}, & L < x \leq M \\ I_{\tilde{N}}, & M \leq x \leq M' \\ \frac{\left(x-M'+I_{\tilde{N}}(U-x)\right)}{(U-M')}, & M' \leq x < U \\ 0, & \text{otherwise.} \end{cases}$$

$$F_{\tilde{N}}(x) = \begin{cases} \frac{\left(M-x+F_{\tilde{N}}(x-L)\right)}{(M-L)}, & L < x \leq M \\ F_{\tilde{N}}, & M \leq x \leq M' \\ \frac{\left(x-M'+F_{\tilde{N}}(U-x)\right)}{(U-M')}, & M' \leq x < U \\ 1, & \text{otherwise.} \end{cases}$$

where, $T_{\widetilde{N}}(x), I_{\widetilde{N}}(x)$ and $F_{\widetilde{N}}(x)$ represents the truth membership degree, indeterminacy-membership degree and falsity-membership degree respectively. Moreover, $T, I, F \in [0, 1]$ such that $0 \leq T + I + F \leq 3$ and $L, M, M', U \in \mathbb{R}$ such that $L \leq M \leq M' \leq U$.

Definition 2.2 Arithmetic operational laws

Let $\widetilde{N}_1 = \langle (L_1, M_1, M_1', U_1)\,; T_1, I_1, F_1\rangle$ and $\widetilde{N}_2 = \langle (L_2, M_2, M_2', U_2)\,; T_2, I_2, F_2\rangle$ be any two single valued trapezoidal neutrosophic numbers then:

1. $\widetilde{N}_1 + \widetilde{N}_2 = \langle (L_1 + L_2, M_1 + M_2, M_1' + M_2', U_1 + U_2)\,; T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2\rangle$
2. $\widetilde{N}_1 - \widetilde{N}_2 = \langle (L_1 - U_2, M_1 - M_2', M_1' - M_2, U_1 + L_2)\,; T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2\rangle$
3. $\widetilde{N}_1 \times \widetilde{N}_2 = \begin{cases} \langle (L_1L_2, M_1M_2, M_1'M_2', U_1U_2)\,; T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2\rangle \text{ if } U_1 > 0, U_2 > 0 \\ \langle (L_1U_2, M_1M_2', M_1'M_2, U_1L_2)\,; T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2\rangle \text{ if } U_1 < 0, U_2 > 0 \\ \langle (U_1U_2, M_1'M_2', M_1M_2, L_1L_2)\,; T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2\rangle \text{ if } U_1 < 0, U_2 < 0 \end{cases}$
4. $\dfrac{\widetilde{N}_1}{\widetilde{N}_2} = \begin{cases} \left\langle \left(\frac{L_1}{U_2}, \frac{M_1}{M_2'}, \frac{M_1'}{M_2}, \frac{U_1}{L_2}\right); T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2 \right\rangle \text{ if } U_1 > 0, U_2 > 0 \\ \left\langle \left(\frac{U_1}{U_2}, \frac{M_1'}{M_2'}, \frac{M_1}{M_2}, \frac{L_1}{L_2}\right); T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2 \right\rangle \text{ if } U_1 < 0, U_2 > 0 \\ \left\langle \left(\frac{U_1}{L_2}, \frac{M_1'}{M_2}, \frac{M_1}{M_2'}, \frac{L_1}{U_2}\right); T_1 \wedge T_2, I_1 \vee I_2, F_1 \vee F_2 \right\rangle \text{ if } U_1 < 0, U_2 > 0 \end{cases}$
5. $\widetilde{N}_1^{-1} = \left\langle \left(\dfrac{1}{U_1}, \dfrac{1}{M_1'}, \dfrac{1}{M_1}, \dfrac{1}{L_1}\right); T_1, I_1, F_1 \right\rangle; L_1, M_1, M_1', U_1 \neq 0.$
6. $k\widetilde{N}_1 = \begin{cases} \langle (kL_1, kM_1, kM_1', kU_1)\,; T_1, I_1, F_1\rangle, & \text{if } k > 0 \\ \langle (kU_1, kM_1', kM_1, kL_1)\,; T_1, I_1, F_1\rangle, & \text{if } k < 0 \end{cases}$

3 A Brief Overview of Abdel-Basset et al.'s Approach

The primary objective of this study is to identify the shortcomings in the existing method [9] and propose a modified approach to address these limitations. Consequently, a discussion of Abdel-Basset et al.'s approach [9] is necessary. This section presents a brief overview of their methodology.

The steps of Abdel-Basset et al.'s approach are outlined as follows:

Step 1: The goal of the problem, which depends on the criteria, sub-criteria, and alternatives, is initially identified. A hierarchical structure for the given MCDM problem is then constructed, and information provided by experts is gathered.

Step 2: The single valued trapezoidal neutrosophic number pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ of (n-1) judgments, rather than $\frac{n\times(n-1)}{2}$, is constructed. This matrix, $\tilde{p}_{ij} = \left\langle \left(L_{a_{ij}}, M_{a_{ij}}, M'_{a_{ij}} U_{a_{ij}} \right); T_{a_{ij}}, I_{a_{ij}}, F_{a_{ij}} \right\rangle i, j = 1, 2, \ldots, n$, encompasses the criterion with respect to the goal of the problem. A similar process is applied to the alternatives with respect to the corresponding criteria.

Step 3: The construction of the single valued trapezoidal neutrosophic pairwise comparison matrix is completed with $\frac{n\times(n-1)}{2}$ entries, utilizing Theorem 1 from the study [9] and either expression (1) or (2), depending on whether $U_{a_{ij}} > 1$ or $L_{a_{ij}} < 0$, respectively.

$$p'_{ij} = \frac{p_{ij} + k_x}{1 + 2k_x}; k_x = \max\{u_{ij} - 1, 0 - l_{ij}\} \text{ for every } i, j = 1, 2, \ldots, n \tag{1}$$

$$p'_{ij} = \frac{-p_{ij} + k_x}{1 + 2k_x}; k_x = \max\{u_{ij} - 1, 0 - l_{ij}\} \text{ for every } i, j = 1, 2, \ldots, n \tag{2}$$

Step 4: After the construct single valued trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \left(\left\langle \left(L_{a_{ij}}, M_{a_{ij}}, M'_{a_{ij}} U_{a_{ij}} \right); T_{a_{ij}}, I_{a_{ij}}, F_{a_{ij}} \right\rangle\right)_{n\times n}$. In order to check that the $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ is consistent or not. The trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ is transformed into the four crisp pairwise comparison matrices by using the following expressions:

$$L = (L_{ij})_{n\times n} = \begin{cases} L_{a_{ij}}, & i < j \\ 0.5, & i = j \\ U_{a_{ij}}, & i > j \end{cases}; = (U_{ij})_{n\times n} = \begin{cases} U_{a_{ij}}, & i < j \\ 0.5, & i = j \\ L_{a_{ij}}, & i > j \end{cases}; \tag{3}$$

$$M = (M_{ij})_{n\times n} = M_{a_{ij}} \text{ and } M' = \left(M'_{ij}\right)_{n\times n} = M'_{a_{ij}}. \tag{4}$$

Using the relation $p_{ik} + p_{kj} + p_{ji} = \frac{1}{2}$, for every i, j, k if all the four matrices $L = (L_{ij})_{n\times n}, U = (U_{ij})_{n\times n}, M = (M_{ij})_{n\times n}$ and $M' = \left(M'_{ij}\right)_{n\times n}$ are additive approximate consistent i.e., satisfying the condition $p_{ik} + p_{kj} + p_{ji} = \frac{1}{2}$ [9] then the corresponding trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ is also consistent, otherwise $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ is inconsistent.

Step 5: If the trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ is consistent, then transform the trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ into the corresponding crisp deterministic pairwise comparison matrix $P = \{p_{ij}\}_{n\times n}$, by using the expression (5) and (6)

$$S_1\left(\langle (L, M, M', U); T, I, F \rangle\right) = \frac{1}{16}\left[(L + M + M' + U) \times (2 + T - I - F)\right] \tag{5}$$

$$S_2\left(\langle (L, M, M', U); T, I, F \rangle\right) = \frac{1}{16}\left[(L + M + M' + U) \times (2 + T - I - F)\right] \tag{6}$$

Step (6a): Transform the matrix, $P = [p_{ij}]_{n\times n}$ into the matrix, $P' = [p'{}_{ij}]_{n\times n}$, where $p'_{ij} = \frac{p_{ij}}{\sum_{k=1}^{n} p_{kj}}, i, j = 1, 2, \ldots, n$.

Step (6b): Find a column matrix, $= [w_{i1}]_{n\times 1}$, where, $w_{i1} = \frac{\sum_{k=1}^{n} p'_{ik}}{n}; i = 1, 2, \ldots, n$.

Step 7: Check that $W(A_i) > W(A_j)$ or $W(A_i) < W(A_j)$ or $W(A_i) = W(A_j)$.

Case (i): If $W(A_i) = W(A_j)$ then $A_i = A_j$,

Case (ii): If $W(A)_i > W(A_j)$ then $A_i > A_j$,

Case (iii): If $W(A_i) < W(A_j)$ then $A_i < A_j$.

4 Shortcomings of Abdel-Basset et al.'s Approach

Basset et al. highlighted that constructing pairwise comparison matrices with $\frac{n\times(n-1)}{2}$ judgments could result in inconsistent expert judgments due to the large value of n. To address this limitation, they employed $(n-1)$ restricted judgments instead of $\frac{n\times(n-1)}{2}$ judgments. Furthermore, they noted that the traditional 1-9 scale of AHP has certain drawbacks and proposed a new scale [0, 1] to overcome these shortcomings. However, upon closer examination, it has been observed that Abdel-Basset et al. made several mathematically incorrect assumptions in their neutrosophic environment approach:

For instance, consider the trapezoidal neutrosophic pairwise comparison matrix.

$$\tilde{P}=\begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.2,0.3,0.4,0.5) & p & p \\ p & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.75,0.9) & p \\ p & p & (0.5,0.5,0.5,0.5) & (0.3,0.4,0.5,0.8) \\ p & p & p & (0.5,0.5,0.5,0.5) \end{bmatrix}$$

with $(n-1)$ judgments.

Using Step 2 and Step 3 of Abdel-Basset et al.'s method, as discussed in the Section 3, the matrix $\tilde{P}$ can be obtained as follows.

$$\begin{aligned}
&P_{13}=p_{12}+p_{23}-(0.5,0.5,0.5,0.5)=(0.3,0.5,0.65,0.9),\\
&P_{31}=1-P_{13}=1-(0.3,0.5,0.65,0.9)=(0.1,0.35,0.5,0.7),\\
&P_{32}=p_{31}+p_{12}-(0.5,0.5,0.5,0.5)=(-0.2,0.15,0.4,0.7),\\
&P_{21}=1-P_{12}=1-(0.2,0.3,0.4,0.5)=(0.5,0.6,0.7,0.8),\\
&P_{14}=p_{13}+p_{34}-(0.5,0.5,0.5,0.5)=(0.1,0.4,0.65,1.2),\\
&P_{24}=p_{21}+p_{14}-(0.5,0.5,0.5,0.5)=(0.1,0.5,0.85,1.5),\\
&P_{41}=1-P_{14}=1-(0.1,0.4,0.65,1.2)=(-0.2,0.35,0.6,0.9),\\
&P_{42}=1-P_{24}=1-(0.1,0.5,0.85,1.5)=(-0.5,0.15,0.5,0.9),\\
&P_{43}=1-P_{34}=1-(0.3,0.4,0.5,0.8)=(0.2,0.5,0.6,0.7),
\end{aligned}$$

Using the above values, the incomplete matrix $\tilde{P}$ is transformed into the matrix $\tilde{P}_1$

$$\tilde{P}_1=\begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.2,0.3,0.4,0.5) & (0.3,0.5,0.65,0.9) & (0.1,0.4,0.65,1.2) \\ (0.5,0.6,0.7,0.8) & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.75,0.9) & (0.1,0.5,0.85,1.5) \\ (0.1,0.35,0.5,0.7) & (-0.2,0.15,0.4,0.7) & (0.5,0.5,0.5,0.5) & (0.3,0.4,0.5,0.8) \\ (-0.2,0.35,0.6,0.9) & (-0.5,0.15,0.5,0.9) & (0.2,0.5,0.6,0.7) & (0.5,0.5,0.5,0.5) \end{bmatrix}.$$

Using the using the expression (1) and (2) transform the matrix $\tilde{P}_1$ into the matrix $\tilde{P}_2$

$$\tilde{P}_2=\begin{bmatrix} (0.5,0.5,0.5,0.5) & (0.2,0.3,0.4,0.5) & (0.3,0.5,0.65,0.9) & (0.1,0.4,0.65,1) \\ (0.5,0.6,0.7,0.8) & (0.5,0.5,0.5,0.5) & (0.6,0.7,0.75,0.9) & (0.1,0.5,0.85,1) \\ (0.1,0.35,0.5,0.7) & (0.2,0.15,0.4,0.7) & (0.5,0.5,0.5,0.5) & (0.3,0.4,0.5,0.8) \\ (0.2,0.35,0.6,0.9) & (0.5,0.15,0.5,0.9) & (0.2,0.5,0.6,0.7) & (0.5,0.5,0.5,0.5) \end{bmatrix}.$$

To check the consistency of matrix $\tilde{P}_2$, expression (2) and (3) from Step 3 of Abdel-Basset et al.'s method, as discussed in previous section, are employed. The matrix $\tilde{P}_2$ is transformed into four different matrices as follows:

$$L=(L_{ij})_{n\times n}=\begin{bmatrix} 0.5 & 0.2 & 0.3 & 0.1 \\ 0.8 & 0.5 & 0.6 & 0.1 \\ 0.7 & 0.7 & 0.5 & 0.2 \\ 0.9 & 0.9 & 0.9 & 0.5 \end{bmatrix}, M=(M_{ij})_{n\times n}=\begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.4 \\ 0.6 & 0.5 & 0.7 & 0.5 \\ 0.35 & 0.2 & 0.5 & 0.4 \\ 0.35 & 0.5 & 0.5 & 0.5 \end{bmatrix}$$

$$M'=(M'_{ij})_{n\times n}=\begin{bmatrix} 0.5 & 0.4 & 0.65 & 0.65 \\ 0.7 & 0.5 & 0.75 & 0.85 \\ 0.5 & 0.4 & 0.5 & 0.5 \\ 0.6 & 0.5 & 0.6 & 0.5 \end{bmatrix} \text{ and } U=(U_{ij})_{n\times n}=\begin{bmatrix} 0.5 & 0.5 & 0.9 & 1 \\ 0.5 & 0.5 & 0.9 & 1 \\ 0.1 & 0.2 & 0.5 & 0.8 \\ 0.2 & 0.5 & 0.2 & 0.5 \end{bmatrix}.$$

Therefore, it can be easily verified that none of the matrices $L=(L_{ij})_{n\times n}, M=(M_{ij})_{n\times n}, M'=(M'_{ij})_{n\times n}$ and $U=(U_{ij})_{n\times n}$ are satisfying the additive reciprocal property of pairwise comparison matrices i.e., $p_{ij}+p_{ji}=1; i,j=1,2,\dots,n$. For example, for the elements of $L=(L_{ij})_{n\times n}, l_{23}+l_{32}=0.6+0.7=1.3\neq 1; l_{34}+l_{43}=0.2+0.9=1.1\neq 1$. Similarly, for the matrices $M=(M_{ij})_{n\times n}, M'=\left(M'_{ij}\right)_{n\times n}$ and $U=(U_{ij})_{n\times n}$ the elements $m_{12}+m_{21}=0.3+0.6=0.9\neq 1,\quad m'_{12}+m'_{21}=0.4+0.7=1.1\neq 1$ and $u_{23}+u_{32}=0.9+0.2=1.1\neq 1$ respectively are not satisfying the property $p_{ij}+p_{ji}=1; i,j=1,2,\dots,n$.

Consequently, matrices $L=(L_{ij})_{n\times n}, M=(M_{ij})_{n\times n}, M'=\left(M'_{ij}\right)_{n\times n}$ and $U=(U_{ij})_{n\times n}$ are found to be inconsistent, implying that matrix $\tilde{P}_2$ is not consistent and will never be consistent when applying Abdel-Basset et al.'s method [9]. However, Basset et al. claimed that matrix $\tilde{P}_2$ is consistent, which is mathematically incorrect.

To determine the propriety weights of the alternatives from the trapezoidal neutrosophic pairwise comparison matrix $\tilde{P}=\{\tilde{p}_{ij}\}_{n\times n}$, Basset et al. used expressions (3) and (4) to transform matrix $\tilde{P}=\{\tilde{p}_{ij}\}_{n\times n}$ into the corresponding crisp deterministic pairwise comparison matrix $P = \{p_{ij}\}_{n\times n}$ [9]. Nonetheless, it was observed that the transformed crisp deterministic pairwise comparison matrix $P = \{p_{ij}\}_{n\times n}$, obtained by applying Basset et al.'s method as discussed in previous section, does not satisfy the property $p_{ij} + p_{ji} = 1; i, j = 1, 2, \ldots, n$, thus not constituting a crisp pairwise comparison matrix. The following examples clearly indicate that the conditions $p_{ij} + p_{ji} = 1; \forall i, j$ are not necessarily satisfied for the elements p_{ij} of the transformed crisp matrix when applying Abdel-Basset et al.'s method [9].

Therefore, it is scientifically incorrect to use Abdel-Basset et al.'s method [9], as discussed in previous section, to determine the priority weights of a trapezoidal neutrosophic pairwise comparison matrix.

Example 4.1 Suppose

$$\tilde{p} = \begin{pmatrix} \langle(0.5, 0.5, 0.5, 0.5)\rangle & \langle(0.2, 0.3, 0.4, 0.5; 0.7, 0.2, 0.5)\rangle \\ \langle(0.5, 0.6, 0.7, 0.8; 0.7, 0.2, 0.5)\rangle & \langle(0.5, 0.5, 0.5, 0.5)\rangle \end{pmatrix}$$

represents a consistent trapezoidal neutrosophic pairwise comparison matrix. Utilizing Abdel-Basset et al.'s method [9], as described in previous section, the examined trapezoidal neutrosophic pairwise comparison matrix will be converted into the corresponding crisp matrix.

$$P = \begin{pmatrix} 0.50 & S(\langle(0.2, 0.3, 0.4, 0.5; 0.7, 0.2, 0.5)\rangle) \\ S(\langle(0.5, 0.6, 0.7, 0.8; 0.7, 0.2, 0.5)\rangle) & 0.50 \end{pmatrix}$$
$$= \begin{pmatrix} 0.5 & \frac{(0.2+0.3+0.4+0.5)}{16} \times 2 + (0.7 - 0.2 - 0.5) \\ \frac{(0.5+0.6+0.7+0.8)}{16} \times 2 + (0.7 - 0.2 - 0.5) & 0.5 \end{pmatrix}$$
$$= \begin{pmatrix} 0.50 & 0.17 \\ 0.32 & 0.50 \end{pmatrix}.$$

Using Step 6 of Abdel-Basset et al.'s method, the normalized priority weights are found to be $P = \begin{bmatrix} 0.43, & 0.57 \end{bmatrix}^T$ [9]. However, the condition $p_{ij} + p_{ji} = 0.17 + 0.32 = 0.49 \neq 1 \forall i, j$ is not satisfied for the transformed crisp matrix P. Consequently, determining the priority weights of a crisp non-pairwise comparison matrix is a meaningless task and may mislead the decision-maker. Thus, Abdel-Basset et al.'s method [9], as discussed in the previous section, cannot be applied to determine the priority weights of the considered trapezoidal neutrosophic pairwise comparison matrix.

4.1 Modified Method

To address the limitations and reduce the computational complexity inherent in Abdel-Basset et al [9], as discussed in Section 4, a modified approach is proposed. Using the definition of the likelihood-based comparison relations [11] to preserve the reciprocal property of crisp pairwise comparison matrices. The steps of the modified approach for constructing crisp pairwise comparison matrices are as follows:

Step 1: A single-valued trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$, is constructed, where $\tilde{p}_{ij} = \left\langle \left(L_{a_{ij}}, M_{a_{ij}}, M'_{a_{ij}} U_{a_{ij}}\right); T_{a_{ij}}, I_{a_{ij}}, F_{a_{ij}} \right\rangle; i \neq j$ and $\tilde{p}_{ij} = 0.50; i = j; i, j = 1, 2, \ldots, n$ representing $(n-1)$ judgments between the criterion with respect to the goal of the problem. The process is similar for alternatives with respect to the corresponding criterion.

Step 2: The incomplete trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$, with $(n-1)$ judgments, is transformed into the corresponding incomplete crisp pairwise comparison matrix $P = \{p_{ij}\}_{n\times n}$ using the specified ranking functions.

$$R_1(\langle(L, M, M', U); T, I, F\rangle) = \frac{1}{6}[(L + M + M' + U + T + I + F)]$$
$$R_2(\langle(L, M, M', U); T, I, F\rangle) = \frac{1}{6}[(L - M + M' + U + T + I + F)]$$
$$R_3(\langle(L, M, M', U); T, I, F\rangle) = \frac{1}{6}[(L + M - M' + U + T + I + F)]$$
$$R_4(\langle(L, M, M', U); T, I, F\rangle) = \frac{1}{6}[(L + M + M' - U + T + I + F)]$$
$$R_5(\langle(L, M, M', U); T, I, F\rangle) = \frac{1}{6}[(L + M + M' + U - T + I + F)]$$
$$R_6(\langle(L, M, M', U); T, I, F\rangle) = \frac{1}{6}[(L + M + M' + U + T - I + F)]$$
$$R_7(\langle(L, M, M', U); T, I, F\rangle) = \frac{1}{6}[(L + M + M' + U + T + I - F)]$$

Step 3: The incomplete crisp pairwise comparison matrix $P = \{p_{ij}\}_{n\times n}$ is transformed into a complete crisp pairwise comparison matrix by employing Theorem 1 from the study [9] and the relation $p_{ij} = 1 - p_{ji}; \forall i, j$.

Step 4: The consistency of the transformed crisp pairwise comparison matrix $P = \{p_{ij}\}_{n\times n}$, obtained in Step 3, is examined.

Case (i): If the transformed crisp matrix $P = \{p_{ij}\}_{n\times n}$ is consistent, the trapezoidal neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ is also considered consistent, and the process proceeds to the next step.

Case (ii): If the transformed crisp matrix $P = \{p_{ij}\}_{n\times n}$ is not consistent, the neutrosophic pairwise comparison matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$ is also considered inconsistent, and the process returns to Step 2 to repair the matrix $\tilde{P} = \{\tilde{p}_{ij}\}_{n\times n}$.

Step 5: The matrix $P = [p_{ij}]_{n\times n}$ is transformed into the matrix $P' = \left[p'_{ij}\right]_{n\times n}$, where $p'_{ij} = \frac{p_{ij}}{\sum_{k=1}^{n} p_{kj}}$, for $i, j = 1, 2, \ldots, n$.

Step 6: A column matrix, $= [w_{i1}]_{n\times 1}$ is determined, where, $w_{i1} = \frac{\sum_{k=1}^{n} p'_{ik}}{n}$ for $i = 1, 2, \ldots, n$.

Step 7: It is verified whether $W(A_i) > W(A_j)$ or $W(A_i) < W(A_j)$ or $W(A_i) = W(A_j)$.

Case (i): If $W(A_i) = W(A_j)$ then $A_i = A_j$,

Case (ii): If $W(A)_i > W(A_j)$ then $A_i > A_j$,

Case (iii): If $W(A_i) < W(A_j)$ then $A_i < A_j$.

4.2 Exact Transformation of Trapezoidal Neutrosophic Pairwise Comparison Matrix into the Crisp Pairwise Comparison Matrix

To accurately determine the weights of criteria and alternatives, it is essential to transform the trapezoidal neutrosophic pairwise comparison matrix into the corresponding crisp pairwise comparison matrix without losing information provided by the decision maker. By applying the steps of the modified method discussed in Section 4.1, it can be confirmed, through the example presented in Section 4, that the transformed crisp matrix consistently preserves the additive reciprocal property of the crisp pairwise comparison matrix, i.e., $a_{ij} = 0.5, i = j$ and $p_{ij} + p_{ji} = 1, i \neq j, \forall i, j = 1, 2, \ldots, n$. Consider the trapezoidal neutrosophic pairwise comparison matrix $\tilde{P}$.

$$\tilde{P} = \begin{bmatrix} 0.5 & \left\langle \begin{matrix} (0.2, 0.3, 0.4, 0.5; \\ 0.7, 0.2, 0.5) \end{matrix} \right\rangle & p_{13} & p_{14} \\ p_{21} & 0.5 & \left\langle \begin{matrix} (0.6, 0.7, 0.75, 0.9; \\ 0.5, 0.2, 0.1) \end{matrix} \right\rangle & p_{24} \\ p_{31} & p_{32} & 0.5 & \left\langle \begin{matrix} (0.3, 0.4, 0.5, 0.8; \\ 0.7, 0.2, 0.5) \end{matrix} \right\rangle \\ p_{41} & p_{42} & p_{43} & 0.5 \end{bmatrix}$$

Employing Step 2 of the modified method, as delineated in Section 5, the trapezoidal neutrosophic pairwise comparison matrix $\tilde{P}$ is transformed into the corresponding crisp pairwise comparison matrix P_1.

$$P_1 = \begin{bmatrix} 0.5 & 0.4667 & P_{13} & P_{14} \\ P_{21} & 0.5 & 0.6250 & P_{24} \\ P_{31} & P_{32} & 0.5 & 0.5667 \\ P_{41} & P_{42} & P_{43} & 0.5 \end{bmatrix}.$$

Subsequently, Step 3 of the modified method (Section 4.1) is utilized to convert the incomplete crisp pairwise comparison matrix P_1 into the corresponding complete crisp pairwise comparison matrix P_2, with the following values:

$$\begin{aligned} &P_{13} = p_{12} + p_{23} - 0.5 = 0.5917, P_{31} = 1 - P_{13} = 1 - 0.5917 = 0.4083, \\ &P_{32} = p_{31} + p_{12} - 0.5 = 0.3750, P_{21} = 1 - P_{12} = 1 - 0.4667 = 0.5333, \\ &P_{14} = p_{13} + p_{34} - 0.5 = 0.6584, P_{24} = p_{21} + p_{14} - 0.5 = 0.6917, \\ &P_{41} = 1 - P_{14} = 1 - 0.6584 = 0.3416, P_{42} = 1 - P_{24} = 1 - 0.6917 = 0.3083, \\ &P_{43} = 1 - P_{34} = 1 - 0.5667 = 0.5333. \end{aligned}$$

$$P_2 = \begin{bmatrix} 0.5 & 0.4667 & 0.5917 & 0.6584 \\ 0.5333 & 0.5 & 0.6250 & 0.6917 \\ 0.4083 & 0.3750 & 0.5 & 0.5667 \\ 0.3416 & 0.3083 & 0.4333 & 0.5 \end{bmatrix}.$$

Consequently, it can be readily confirmed that the elements of the transformed crisp matrix P_2 satisfy $a_{ij} = 0.5, i = j$ and $p_{ij} + p_{ji} = 1, i \neq j, \forall i, j = 1, 2, \ldots, n$, thus preserving the additive reciprocal property of the additive pairwise comparison matrix. As a result, the crisp pairwise comparison matrix P_2 is consistent, which implies that the corresponding trapezoidal neutrosophic pairwise comparison matrix is also consistent. Furthermore, by implementing Steps 5, 6, and 7 of the modified method, as outlined in previous section, the corresponding normalized priority weights of P_2 are determined to be 0.2777, 0.2948, 0.2308, and 0.1967, respectively.

5 Illustrative Example

Abdel-Basset et al. [9] addressed a real-life problem of identifying the most popular search engine among four available options: (i) Google, (ii) Yahoo Search, (iii) Ask, and (iv) Bing. The evaluation was based on four criteria: (i) Core technology, (ii) Query functionality, (iii) Security, and (iv) User interface. The authors employed their proposed method to illustrate the solution. However, as discussed in Section 4, Abdel-Basset et al.'s method exhibited some shortcomings, which led to imprecise results for the real-life problem [9]. In this section, the exact result of the same problem is derived using the modified method. The steps of the modified method are applied to obtain the precise ranking of the MCDM problem [9] as follows:

Step 1: The decision maker's information regarding the criteria relative to the problem's goal is represented by the incomplete trapezoidal neutrosophic pairwise comparison matrix $\tilde{P}_C$. Similarly, the alternatives with respect to criteria C_1, C_2, C_3 and C_4 are presented in matrices $\tilde{P}_{AC_1}, \tilde{P}_{AC_2}, \tilde{P}_{AC_3}$ and $\tilde{P}_{AC_4}$, respectively.

$$\tilde{P}_C = \begin{bmatrix} 0.5 & \left\langle \begin{matrix}(0.2,0.3,0.4,0.5;\\ 0.7,0.2,0.5)\end{matrix} \right\rangle & p_{13} & p_{14} \\ p_{21} & 0.5 & \left\langle \begin{matrix}(0.6,0.7,0.75,0.9;\\ 0.5,0.2,0.1)\end{matrix} \right\rangle & p_{24} \\ p_{31} & p_{32} & 0.5 & \left\langle \begin{matrix}(0.3,0.4,0.5,0.8;\\ 0.7,0.2,0.5)\end{matrix} \right\rangle \\ p_{41} & p_{42} & p_{43} & 0.5 \end{bmatrix}$$

$$\tilde{P}_{AC_1} = \begin{bmatrix} 0.5 & \left\langle \begin{matrix}(0.5,0.6,0.7,0.8;\\ 0.7,0.2,0.5)\end{matrix} \right\rangle & p_{13} & p_{14} \\ p_{21} & 0.5 & \left\langle \begin{matrix}(0.4,0.5,0.6,0.7;\\ 0.5,0.2,0.1)\end{matrix} \right\rangle & p_{24} \\ p_{31} & p_{32} & 0.5 & \left\langle \begin{matrix}(0.2,0.3,0.4,0.5;\\ 0.6,0.4,0.2)\end{matrix} \right\rangle \\ p_{41} & p_{42} & p_{43} & 0.5 \end{bmatrix}$$

$$\tilde{P}_{AC_2} = \begin{bmatrix} 0.5 & p_{12} & \left\langle \begin{matrix}(0.4,0.5,0.7,0.9;\\ 0.5,0.2,0.1)\end{matrix} \right\rangle & p_{14} \\ p_{21} & 0.5 & p_{23} & p_{24} \\ p_{31} & p_{32} & 0.5 & \left\langle \begin{matrix}(0.2,0.5,0.6,0.8;\\ 0.6,0.4,0.2)\end{matrix} \right\rangle \\ p_{41} & \left\langle \begin{matrix}(0.2,0.4,0.5,0.8;\\ 0.3,0.1,0.5)\end{matrix} \right\rangle & p_{43} & 0.5 \end{bmatrix}$$

$$\tilde{P}_{AC_3} = \begin{bmatrix} 0.5 & \left\langle \begin{matrix}(0.6,0.7,0.9,1;\\ 0.7,0.2,0.5)\end{matrix} \right\rangle & p_{13} & p_{14} \\ p_{21} & 0.5 & \left\langle \begin{matrix}(0.6,0.7,0.8,0.9;\\ 0.5,0.2,0.1)\end{matrix} \right\rangle & p_{24} \\ p_{31} & p_{32} & 0.5 & \left\langle \begin{matrix}(0.2,0.5,0.6,0.8;\\ 0.6,0.4,0.2)\end{matrix} \right\rangle \\ p_{41} & p_{42} & p_{43} & 0.5 \end{bmatrix}$$

$$\tilde{P}_{AC_4} = \begin{bmatrix} 0.5 & \left\langle \begin{matrix}(0.5,0.6,0.7,0.8;\\ 0.7,0.2,0.5)\end{matrix} \right\rangle & p_{13} & p_{14} \\ p_{21} & 0.5 & \left\langle \begin{matrix}(0.4,0.5,0.6,0.7;\\ 0.5.0.2,0.1)\end{matrix} \right\rangle & p_{24} \\ p_{31} & p_{32} & 0.5 & \left\langle \begin{matrix}(0.2,0.3,0.4,0.5;\\ 0.6,0.4,0.2)\end{matrix} \right\rangle \\ p_{41} & p_{42} & p_{43} & 0.5 \end{bmatrix}.$$

Step 2: Steps 2 and 3 of the modified method, as discussed in Section 4.1, are applied to transform the incomplete trapezoidal neutrosophic pairwise comparison matrix $\tilde{P}_C$ and the incomplete trapezoidal neutrosophic pairwise comparison matrices of alternatives $\tilde{P}_{AC_1}, \tilde{P}_{AC_2}, \tilde{P}_{AC_3}$ and $\tilde{P}_{AC_4}$ into the following crisp pairwise comparison matrices:

$$P_{\mathrm{C}} = \begin{bmatrix} 0.5 & 0.4667 & P_{13} & P_{14} \\ P_{21} & 0.5 & 0.6250 & P_{24} \\ P_{31} & P_{32} & 0.5 & 0.5667 \\ P_{41} & P_{42} & P_{43} & 0.5 \end{bmatrix}.$$

Now, using the Step 3 of the modified method, discussed in Section 5, to transform the incomplete crisp pairwise comparison matrix P_C into the corresponding complete crisp pairwise comparison matrix P_C, by using the following values:

$$P_{13} = p_{12} + p_{23} - 0.5 = 0.5917, P_{31} = 1 - P_{13} = 1 - 0.5917 = 0.4083,$$
$$P_{32} = p_{31} + p_{12} - 0.5 = 0.3750, P_{21} = 1 - P_{12} = 1 - 0.4667 = 0.5333,$$
$$P_{14} = p_{13} + p_{34} - 0.5 = 0.6584, P_{24} = p_{21} + p_{14} - 0.5 = 0.6917,$$
$$P_{41} = 1 - P_{14} = 1 - 0.6584 = 0.3416, P_{42} = 1 - P_{24} = 1 - 0.6917 = 0.3083,$$
$$P_{43} = 1 - P_{34} = 1 - 0.5667 = 0.5333.$$

$$P_{\mathrm{C}} = \begin{bmatrix} 0.5 & 0.4667 & 0.5917 & 0.6584 \\ 0.5333 & 0.5 & 0.6250 & 0.6917 \\ 0.4083 & 0.3750 & 0.5 & 0.5667 \\ 0.3416 & 0.3083 & 0.4333 & 0.5 \end{bmatrix}.$$

Similarly, for the alternatives as shown in matrices $P_{AC_1}, P_{AC_1}, P_{AC_2}, P_{AC_3}$ and P_{AC_4} respectively.

$$P_{AC_1} = \begin{bmatrix} 0.5000 & 0.6667 & 0.5000 & 0.3000 \\ 0.3333 & 0.5000 & 0.4333 & 0.1333 \\ 0.5000 & 0.5667 & 0.5000 & 0.3667 \\ 0.7000 & 0.8667 & 0.6333 & 0.5000 \end{bmatrix} P_{AC_2} = \begin{bmatrix} 0.5000 & 0.1000 & 0.3833 & 0.2333 \\ 0.9000 & 0.5000 & 0.7833 & 0.5667 \\ 0.6167 & 0.2167 & 0.5000 & 0.3500 \\ 0.7667 & 0.4333 & 0.6500 & 0.5000 \end{bmatrix}$$

$$P_{AC_3} = \begin{bmatrix} 0.5000 & 0.7667 & 0.9000 & 0.9500 \\ 0.2333 & 0.5000 & 0.6333 & 0.6833 \\ 0.1000 & 0.3667 & 0.5000 & 0.5500 \\ 0.0500 & 0.3167 & 0.4500 & 0.5000 \end{bmatrix} P_{AC_4} = \begin{bmatrix} 0.5000 & 0.6667 & 0.5000 & 0.3000 \\ 0.3333 & 0.5000 & 0.4333 & 0.1333 \\ 0.5000 & 0.5667 & 0.5000 & 0.3667 \\ 0.7000 & 0.8667 & 0.6333 & 0.5000 \end{bmatrix}$$

Step 3: Utilizing Step 3 of the modified method, as outlined in Section 4.1, the incomplete crisp pairwise comparison matrix P_C is transformed into the corresponding complete crisp pairwise comparison matrix, employing the given values.

Step 4: Step 4 of the modified method, detailed in Section 4.1, is used to examine the consistency of all the transformed crisp pairwise comparison matrices. It can be easily verified that the matrices $P_{AC_1}, P_{AC_1}, P_{AC_2}, P_{AC_3}$ and P_{AC_4} satisfy the property $a_{ij} = 0.5, i = j$ and $p_{ij} + p_{ji} = 1, i \neq j, \forall i, j = 1, 2, \ldots, n$ i.e., thus preserving the additive reciprocal property of the additive pairwise comparison matrix.

Step 5: Steps 5 and 6 of the modified method, presented in Section 4.1, are employed to obtain normalized priority weights of criteria C_1, C_2, C_3 and C_4, which are 0.2777, 0.2948, 0.2308, and 0.1967, respectively. The normalized priority weights of the alternatives corresponding to criteria C_1, C_2, C_3 and C_4 are shown in Table 1.

Table 1. Normalized priority weights of the alternatives corresponding to the criteria C_1, C_2, C_3 and C_4

Alternatives	Priority weights corresponding C_1	Priority weights corresponding C_2	Priority weights corresponding C_3	Priority weights corresponding C_4
A_1	0.2799	0.1541	0.4189	0.2799
A_2	0.1944	0.3326	0.2575	0.1944
A_3	0.2286	0.2165	0.1769	0.2286
A_4	0.2970	0.2968	0.1466	0.2970

Table 2. Overall ranking order of the alternatives

Alternatives	Abdel-Basset et al.'s existing Method [9]		Proposed modified method	
	C_i	Rank	C_i	Rank
A_1	0.2681	1	0.2749	1
A_2	0.2305	3	0.2497	3
A_3	0.2073	4	0.2131	4
A_4	0.2562	2	0.2622	2

Step 6: Finally, using Step 6, the ranking of alternatives based on global priority weights, i.e., the product of criteria and alternatives, is displayed in Table 2. The ranking order of the alternatives, as determined by both Abdel-Basset et al.'s existing method [9] and the proposed modified method, is also presented in Table 2.

6 Conclusions

In this study, a modified neutrosophic AHP technique based on trapezoidal neutrosophic numbers has been developed to address the limitations of the existing method proposed by Abdel-Basset et al [9]. A thorough investigation of an essential property of the pairwise comparison matrix has been conducted, revealing that the existing method fails to maintain the reciprocal property of the pairwise comparison matrix. Consequently, the enhanced method has been introduced to preserve this property and improve the accuracy of the decision-making process.

By applying the proposed modified method to the decision-making problem presented in Abdel-Basset et al.'s work [9], a comparison has been made between the results obtained using the existing method and those derived from the modified method. This comparison demonstrates the effectiveness and superiority of the modified method in overcoming the shortcomings of the existing technique.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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