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Optimizing Earthquake Response with Fermatean Probabilistic Hesitant Fuzzy Sets: A Decision Support Framework

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Abstract: Reducing the devastating effects of earthquakes is the main objective of planning for earthquake response. The decision-making process is essential to this attempt. However, it is particularly difficult because of the inherent uncertainties. A sophisticated methodological approach was proposed to handle these uncertainties in this study. The approach makes use of Fermatean probabilistic hesitant fuzzy sets (FePHFSs), and emphasizes the resilience of algebraic operations and their crucial role in improving the effectiveness of decision-making. In particular, a noteworthy development in the field of multiple attribute decision making (MADM) is the introduction of novel probabilistic hesitant fuzzy sets (PHFSs) aggregation operators, which are created by carefully synthesizing algebraic operations with the Combined Compromise Solution (CoCoSo) method. A key component of this technique is the application of the CoCoSo strategy, which is well known for its resilience in optimal goal selection and uses various aggregation strategies to effectively navigate the complex, multicriteria decision-making environment. A thorough numerical case study illustrates the adaptability and efficacy of this method and highlights its potential in practical settings. Decision-makers now have a new and effective tool that helps them make better informed and trustworthy decisions even in the face of uncertainty by combining PHFS with the CoCoSo technique. This method offers real-world implications for improving disaster response plans in addition to advancing the theory of decision support systems.

Keywords: Fermatean probabilistic hesitant fuzzy sets; Probabilistic hesitant fuzzy sets; Algebraic t-norm & t-conorm; CoCoSo method; Decision making

1 Introduction

Leading the way in disaster management, earthquake response planning requires carefully thought out plans to minimize possible damage and save lives and property. Earthquakes are inherently unpredictable and have a wide-ranging impact. Therefore, it is critical that response planning processes incorporate sound decision-making processes. Effective decision-making is crucial in this situation since it forms the basis of successful reaction tactics. With so many moving parts, unknowns, and trade-offs, the decision-making process in earthquake response planning is inherently complicated. To attain the best results, it is essential to have a thorough grasp of seismic risks in addition to a smart decision-making methodology that can handle scenarios with multiple criteria with ease. The goal of this research is to improve the field of earthquake response planning by presenting an integrated methodology that combines algebraic operations and Fermatean PHFSs. The complex relationships between these factors are investigated. It aims to improve decision-making procedures and raise the effectiveness of earthquake response plans. This study centers on the possible revolutionary influence of these breakthroughs in the field of catastrophe management.

As the intricacy of decision-making in earthquake response planning is addressed, it becomes clear that a smart, flexible instrument is required. In this case, MADM proves to be useful and effective. Well-known for its effectiveness in decision analysis, MADM works especially well in situations where a variety of factors and characteristics affect the choice of the best response plan. The methodical manner in which MADM evaluates and ranks alternatives, taking into consideration a wide range of frequently contradictory factors, is what makes it so strong.

Within the domain of earthquake response planning, where crucial choices impact the distribution of resources, staffing levels, and execution of mitigating actions, MADM provides a methodical framework to skillfully handle innate uncertainties and intricacies. Fermatean PHFSs are integrated into MADM. This improves its capacity to capture the complex interactions between decision factors. By addressing the hesitation and imprecision frequently linked to seismic risk assessments, this integration improves the process of making decisions.

The application of MADM in earthquake response planning goes beyond its ability to perform systematic analysis. It appears as a driving force behind decision outcomes optimization, efficiency promotion, and general response strategy effectiveness augmenting. It is hoped that investigating Fermatean PHFSs and algebraic operations in the MADM framework can reveal a revolutionary instrument. This tool gives decision-makers useful insights while still acknowledging the complexity of earthquake response preparation. This study aims to shed light on the unmatched value of MADM in this crucial area and emphasize how crucial it is for making wise decisions in the face of seismic uncertainty.

Zadeh proposed the idea of fuzzy sets (FSs) in 1965. This laid the groundwork for a fundamental method of handling uncertainty in MADM problems. Rather than specifying the value of an alternative ξ_i exactly at a given criterion ς_j, the intrinsic imprecision is acknowledged by proposing that the value be roughly δ_{ij}. The FS is defined as $F = \{\delta_j, \mu_\Delta(\delta_j)|\delta_j \in \Delta\}$ on $\Delta = \delta_1, \delta_2, ..., \delta_n$, where $\mu_\Delta(\delta_j)$ denotes the positive grade of membership for $\delta_j \in \Delta$. Notably, the typical FS framework does not take into account negative grades of $\delta_j \in \Delta$. But there have been doubts about these sets' dependability. This has prompted the creation of a number of extensions in disciplines including engineering, medical diagnostics, and decision-making. Under certain conditions, extensions such as intuitionistic FSs (IFSs) [1], Pythagorean FSs [2], and Fermatean FSs (FeFSs) [3] have been proposed to address uncertainty more effectively by incorporating both positive and negative grades of $\delta_j \in \Delta$. Unlike intuitionistic FSs and Pythagorean FSs, FeFSs excel in representing imprecise human judgments during decision-making and handling a broader range of uncertainty [4–7].

Recognizing the superior capabilities of FeFSs, numerous studies have incorporated FeFS methodologies to address complex MADM problems. Notably, Verma [8] extended the WASPAS method within an FeFS framework for the selection of healthcare waste disposal sites [9]. In response to the challenges posed by COVID-19, Garg et al. [10] utilized FFS aggregation functions in assessing COVID-19 facilities. Furthermore, Sergi and Sari [11] applied FeFSs to handle the uncertainty associated with parameters in capital budgeting. Fuzzy transportation challenges were successfully addressed by Sahoo [12] through the utilization of FeFS parameters and a score function.

The introduction of hesitant fuzzy sets (HFS) [13] has significantly enhanced traditional FS in the realm of MCDM, particularly in efficiently managing uncertainties posed by expert judgments. A comprehensive overview and future perspective on HFSs was provided in a study by Rodrguez et al. [14], which elucidated that HFSs (i) enhance the elicitation of expert preferences and (ii) offer a more versatile and flexible preference structure, thereby reducing uncertainties.

The amalgamation of HFSs with IFSs led to the creation of a novel FS category termed intuitionistic hesitant fuzzy sets, as investigated by Peng et al. [15]. Additionally, the integration of HFSs with Pythagorean FSs resulted in the development of Pythagorean hesitant fuzzy sets, introduced by Khan et al. [16]. In a subsequent advancement, Kirisci [17] presented Fermatean hesitant fuzzy sets, a fusion of HFS and FFS, and demonstrated their application in a medical case study. Nonetheless, certain limitations in HFS, such as information loss and the oversight of occurrence probabilities, were identified [14]. Addressing these issues, Qian et al. [18] introduced PHFSs, integrating probability elements into HFS. Constructed on this basis, Batool et al. [19] improved the idea even more by introducing Pythagorean PHFSs, which are limited by the requirement that the square sum of the positive and negative hesitant adhesions' degrees be less than or equal to 1.

Comparing FePHFSs to Pythagorean PHFSs, a substantial improvement has been made. Motivated by the limitations of Pythagorean PHFSs, where the square sum of positive and negative hesitant grades is constrained to be less than or equal to one. A more comprehensive constraint is introduced by FePHFSs, which limits the cube total of positive and negative hesitant grades to be less than or equal to one. Consequently, FePHFSs is utilized to effectively address expert uncertainties while efficiently considering the occurrence probability of expanding the application domains. For example, Sahoo [20] extended the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method with FeFSs for the purpose of bridge selection. To find the best sanitizer, Akram et al. [21] used TOPSIS with FeFSs and Einstein averaging operators to tackle the problems brought on by COVID-19. Additionally, the optimization of laboratory selection for COVID-19 testing was achieved by Gül [22] through the amalgamation of SAW, ARAS, and VIKOR methods within an FeFS framework. These studies collectively underscore the adaptability and efficacy of FeFSs across diverse decision-making contexts. Further related work is detailed in literature [23–28].

The structure of this article is as follows: The introduction section provides a comprehensive overview of the study. Section 2 is dedicated to elucidating fundamental concepts crucial for understanding the subsequent material.

Section 3 introduces a novel methodology employing FePHFS. Thereafter, Section 4 presents a case study focusing on earthquake response, accompanied by a detailed discussion. The article culminates with Section 5, which summarizes the key findings and conclusions.

2 Preliminaries

This section elucidates key concepts fundamental to the methodology employed in this research, focusing on their implications and contributions to the field.

Definition 2.1: Let $\Im$ denote a universal set. A FePHFS on $\Im$ is defined as:

$$\Theta = \{\theta, \chi_{\hbar_{\cdot}}(\theta)/\wp, \varrho_{\hbar_{\cdot}}(\theta)/\wp^{\bullet} \mid \theta \in \Theta\} \tag{1}$$

where, $\theta \in \Theta, \chi_{\hbar_{\bullet}}(\theta)$ and $\varrho_{\hbar_{\bullet}}(\theta)$ represent sets comprising certain values within the range [0,1]. These sets $\chi_{\hbar_{\bullet}}(\theta)\wp$ and $\varrho_{\hbar_{\bullet}}(\theta)/\wp^{\bullet}$, known as probable positive and negative grades respectively, and $\wp, \wp^{\bullet}$ signifies the probabilities of the grades.

Additionally, $0 \leq \hbar_{\bullet_i}, \eth_{\bullet_j} \leq 1$ and $0 \leq \wp_i, \wp_j^{\bullet} \leq 1, \sum_{i=1}^{\curlywedge} \wp_i \leq 1 \& \sum_{j=1}^{\curlywedge} \wp_i^{\bullet} \leq 1$, with $\curlywedge$ being a positive integer, indicate the cardinality of the FePHFS. It is imperative that the following criteria are satisfied:

$$\left(min\left(\chi_{\hbar_{\bullet}}(\theta)\right)\right)^3 + \left(max\left(\varrho_{\hbar_{\bullet}}(\theta)\right)\right)^3 \leq 1, \left(max\left(\chi_{\hbar_{\bullet}}(\theta)\right)\right)^3 + \left(min\left(\varrho_{\hbar_{\bullet}}(\theta)\right)\right)^3 \leq 1 \tag{2}$$

In this context, a Fermatean probabilistic hesitant fuzzy number (FPHFN) is represented by the pair $(\chi_{\hbar_{\bullet_*}}(\theta)/\wp_*, \varrho_{\hbar_{\bullet_*}}(\theta)/\wp_*^{\bullet})$.

Definition 2.2: Given two FePHFNs, $\Theta_1 = \left(\chi_{\hbar_{\bullet_1}}(\theta)/\wp_1, \varrho_{\hbar_{\bullet_1}}(\theta)/\wp_1^{\bullet}\right)$ and $\Theta_2 = \left(\chi_{\hbar_{\bullet_2}}(\theta)/\wp_2, \varrho_{\hbar_{\bullet_2}}(\theta)/\wp_2^{\bullet}\right)$, the fundamental operational laws are established as follows:

1. $\Theta_1 \cup \Theta_2 = \left\{ \bigcup_{\substack{\hbar_{\bullet_1} \in \chi_{\hbar_{\bullet_1}}, \wp_1 \in \wp \\ \hbar_{\bullet_2} \in \chi_{\hbar_{\bullet_2}}, \wp_2 \in \wp}} \left(max\left(\hbar_{\bullet_1}/\wp_1, \hbar_{\bullet_2}/\wp_2\right)\right), \bigcup_{\substack{\eth_{\bullet_1} \in \varrho_{\hbar_{\bullet_1}}, \wp_1^{\bullet} \in \wp^{\bullet} \\ \eth_{\bullet_2} \in \varrho_{\eth_{\bullet_2}}, \wp_2^{\bullet} \in \wp^{\bullet}}} \left(min\left(\eth_{\bullet_1}/\wp_1^{\bullet}, \eth_{\bullet_2}/\wp_2^{\bullet}\right)\right)\right\}$.
2. $\Theta_1 \cap \Theta_2 = \left\{ \bigcap_{\substack{\hbar_{\bullet_1} \in \chi_{\hbar_{\bullet_1}}, \wp_1 \in \wp \\ \hbar_{\bullet_2} \in \chi_{\hbar_{\bullet_2}}, \wp_2 \in \wp}} \left(min\left(\hbar_{\bullet_1}/\wp_1, \hbar_{\bullet_2}/\wp_2\right)\right), \bigcup_{\substack{\eth_{\bullet_1} \in \varrho_{\hbar_{\bullet_1}}, \wp_1^{\bullet} \in \wp^{\bullet} \\ \eth_{\bullet_2} \in \varrho_{\eth_{\bullet_2}}, \wp_2^{\bullet} \in \wp^{\bullet}}} \left(max\left(\eth_{\bullet_1}/\wp_1^{\bullet}, \eth_{\bullet_2}/\wp_2^{\bullet}\right)\right)\right\}$.
3. If $\Theta_2 = \left(\chi_{\hbar_{\bullet_2}}(\theta)/\wp_2, \varrho_{\hbar_{\bullet_2}}(\theta)/\wp_2^{\bullet}\right)$, then $\Theta_2^c = \left(\varrho_{\hbar_{\bullet_2}}(\theta)/\wp_2^{\bullet}, \chi_{\hbar_{\bullet_2}}(\theta)/\wp_2\right)$.

Definition 2.3: For two FePHFNs, $\Theta_1 = \left(\chi_{\hbar_{\cdot_1}}(\theta)/\wp_1, \varrho_{\hbar_{\cdot_1}}(\theta)/\wp_1^{\bullet}\right)$ and $\Theta_2 = \left(\chi_{\hbar_{\cdot_2}}(\theta)/\wp_2, \varrho_{\hbar_{\cdot_2}}(\theta)/\wp_2^{\bullet}\right)$, and a given $\lambda > 0$, their operations are articulated as:

1. $\Theta_1 \oplus \Theta_2 = \left\{ \bigcup_{\substack{\hbar_{\bullet_1} \in \chi_{\hbar_{\bullet_1}}, \wp_1 \in \wp \\ \hbar_{\bullet_2} \in \chi_{\hbar_{\bullet_2}}, \wp_2 \in \wp}} \left(\sqrt[3]{\hbar_{\bullet_1}^3 + \hbar_{\bullet_2}^3 - \hbar_{\bullet_1}^3 \hbar_{\bullet_2}^3}/\wp_1\wp_2\right), \bigcup_{\substack{\eth_{\bullet_1} \in \varrho_{\hbar_{\bullet_1}}, \wp_1^{\bullet} \in \wp^{\bullet} \\ \eth_{\bullet_2} \in \varrho_{\eth_{\bullet_2}}, \wp_2^{\bullet} \in \wp^{\bullet}}} \left(\eth_{\bullet_1}\eth_{\bullet_2}/\wp_1^{\bullet}\wp_2^{\bullet}\right)\right\}$.
2. $\Theta_1 \otimes \Theta_2 = \left\{ \bigcup_{\substack{\hbar_{\bullet_1} \in \chi_{\hbar_{\bullet_1}}, \wp_1 \in \wp \\ \hbar_{\bullet_2} \in \chi_{\hbar_{\bullet_2}}, \wp_2 \in \wp}} \left(\hbar_{\bullet_1}\hbar_{\bullet_2}/\wp_1\wp_2\right), \bigcup_{\substack{\eth_{\bullet_1} \in \varrho_{\hbar_{\bullet_1}}, \wp_1^{\bullet} \in \wp^{\bullet} \\ \eth_{\bullet_2} \in \varrho_{\eth_{\bullet_2}}, \wp_2^{\bullet} \in \wp^{\bullet}}} \left(\sqrt[3]{\eth_{\bullet_1}^3 + \eth_{\bullet_2}^3 - \eth_{\bullet_1}^3 \eth_{\bullet_2}^3}/\wp_1^{\bullet}\wp_2^{\bullet})\right)\right\}$.
3. $\curlywedge\Theta_2 = \left\{ \bigcup_{\hbar_{\bullet_2} \in \chi_{\hbar_{\bullet_2}}, \wp_2 \in \wp} \left(\sqrt[3]{1 - \left(1 - \hbar_{\bullet_2}^3\right)^{\curlywedge}}/\wp_2\right) \bigcup_{\eth_{\bullet_2} \in \varrho_{\hbar_{\bullet_2}}, \wp_2^{\bullet} \in \wp^{\bullet}} \left(\eth_{\bullet_2}^{\curlywedge}/\wp_2^{\bullet}\right)\right\}$.
4. $\Theta_2^{\curlywedge} = \left\{ \bigcup_{\hbar_{\bullet_2} \in \chi_{\hbar_{\bullet_2}}, \wp_2 \in \wp} \left(\hbar_{\bullet_2}^{\curlywedge}/\wp_2\right) \bigcup_{\eth_{\bullet_2} \in \varrho_{\hbar_{\bullet_2}}, \wp_2^{\bullet} \in \wp^{\bullet}} \left(\sqrt[3]{1 - \left(1 - \eth_{\bullet_2}^3\right)^{\curlywedge}}/\wp_2^{\bullet}\right)\right\}$.

Definition 2.4: The score function for a FePHFN, represented as $\Theta = \left(\chi_{\hbar_{\bullet_*}}(\theta)/\wp_*, \varrho_{\hbar_{\bullet_*}}(\theta)/\wp_*^{\bullet}\right)$, is defined as follows:

$$\sqsupset(\Theta) = \left(\frac{1}{\blacksquare_{\Theta}} \sum_{\hbar_{\bullet_i} \in \chi_{\hbar_{\bullet_i}}, \wp_i \in \wp} \left(\hbar_{\bullet_i}\wp_i\right)\right)^3 - \left(\frac{1}{\blacklozenge_{\Theta}} \sum_{\eth_{\bullet_i} \in \varrho_{\hbar_{\bullet_i}}, \wp_i^{\bullet} \in \wp^{\bullet}} \left(\eth_{\bullet_i}\wp_i^{\bullet}\right)\right)^3 \tag{3}$$

where, $\blacksquare_{\Theta}$ and $\blacklozenge_{\Theta}$ denote the number of elements in respective sets $\chi_{\hbar_{\bullet_i}}$ and $\varrho_{\hbar_{\bullet_i}}$.

Definition 2.5: An accuracy function for any FePHFN, denoted as $\Theta = \left(\chi_{\hbar_{\bullet_*}}(\theta)/\wp_*, \varrho_{\hbar_{\bullet_*}}(\theta)/\wp_*^{\bullet}\right)$, is formulated as:

$$\urcorner(\Theta) = \left(\frac{1}{\blacksquare_{\Theta}} \sum_{\hbar_{\bullet_i} \in \chi_{\hbar_{\bullet_i}}, \wp_i \in \wp} \left(\hbar_{\bullet_i}\wp_i\right)\right)^3 + \left(\frac{1}{\blacklozenge_{\Theta}} \sum_{\eth_{\bullet_i} \in \varrho_{\hbar_{\bullet_i}}, \wp_i^{\bullet} \in \wp^{\bullet}} \left(\eth_{\bullet_i}\wp_i^{\bullet}\right)\right)^3 \tag{4}$$

where, $\blacksquare_{\Theta}$ and $\blacklozenge_{\Theta}$ represent the number of elements in $\chi_{\hbar_{\bullet i}}$ and $\varrho_{\hbar_{\bullet i}}$, respectively.

Definition 2.6: Given FePHFNs $\Theta_1 = \big(\chi_{\hbar_{\bullet 1}}(\theta)/\wp_1, \varrho_{\hbar_{\bullet 1}}(\theta)/\wp_1^{\bullet}\big)$ and $\Theta_2 = \big(\chi_{\hbar_{\bullet 2}}(\theta)/\wp_2, \varrho_{\hbar_{\bullet 2}}(\theta)/\wp_2^{\bullet}\big)$, the following relationships are established:

1. $\beth(\Theta_1) > \beth(\Theta_2) \Rightarrow \Theta_1 > \Theta_2$.
2. $\beth(\Theta_1) < \beth(\Theta_2) \Rightarrow \Theta_1 < \Theta_2$.
3. If $\beth(\Theta_1) = \beth(\Theta_2)$, then go for accuracy:
   - $\daleth(\Theta_1) > \daleth(\Theta_2) \Rightarrow \Theta_1 > \Theta_2$.
   - $\daleth(\Theta_1) < \daleth(\Theta_2) \Rightarrow \Theta_1 < \Theta_2$.
   - $\daleth(\Theta_1) = \daleth(\Theta_2) \Rightarrow \Theta_1 \approx \Theta_2$.

Definition 2.7: For a collection of FePHFNs, termed $\Delta_i = \big(\chi_{\hbar_{\bullet i}}/\wp_i, \varrho_{\hbar_{\bullet i}}/\wp_i^{\bullet}\big)(i = 1, 2, ..., \ell)$, and a Fermatean probabilistic hesitant fuzzy weighted average (FePHFWA) $\ell \rightarrow$, the FePHFWA operator is defined as:

$$\begin{aligned} FePHWA(\Theta_1, \Theta_2, \ldots, \Theta_\ell) &= \Xi_1\Theta_1 \oplus \Xi_2\Theta_2 \oplus \cdots \oplus \Xi_\ell\Theta_\ell \\ &= \sum_{i=1}^{\ell} \Xi_i\Theta_i \end{aligned} \tag{5}$$

where, $\Xi = (\Xi_1, \Xi_2, ..., \Xi_\ell)$ are identified as the weights, and $\Xi_i \geq 0, \sum_{i=1}^{\ell} \Xi_i = 1$.

Definition 2.8: For any collection of FePHFNs, represented as $\Delta_i = \big(\chi_{\hbar_{\bullet i}}/\wp_i, \varrho_{\hbar_{\bullet i}}/\wp_i^{\bullet}\big)(i = 1, 2, ..., \ell)$, the aggregation outcome utilizing the FePHFWA is defined by:

$$FePHWA(\Theta_1, \Theta_2, \ldots, \Theta_\ell) = \left\{ \bigcup_{\substack{\hbar_{\bullet i} \in \chi_{\hbar_{\bullet i}} \\ \wp_i \in \wp}} \sqrt[3]{1 - \prod_{i=1}^{\ell}\left(1 - (\hbar_{\bullet i})^3\right)^{\Xi}} \Big/ \prod_{i=1}^{\ell} \wp_i, \bigcup_{\substack{\eth_{\bullet i} \in \varrho_{\hbar_{\bullet i}} \\ \wp_i^{\bullet} \in \wp^{\bullet}}} \prod_{i=1}^{\ell} (\eth_{\bullet 2})^{\Xi} \Big/ \prod_{i=1}^{\ell} \wp_i^{\bullet} \right\}. \tag{6}$$

Definition 2.9: Considering a set of FePHFNs, denoted as $\Delta_i = \big(\chi_{\hbar_{\bullet i}}/\wp_i, \varrho_{\hbar_{\bullet i}}/\wp_i^{\bullet}\big)(i = 1, 2, ..., \ell)$, and the Fermatean probabilistic hesitant fuzzy weighted geometric (FePHFWG) operator: FePHFN $\ell \rightarrow$ FePHFN, the FePHFWG operator is articulated as:

$$\begin{aligned} FePHWG(\Theta_1, \Theta_2, \ldots, \Theta_\ell) &= \Xi_1\Theta_1 \otimes \Xi_2\Theta_2 \otimes \cdots \otimes \Xi_\ell\Theta_\ell \\ &= \prod_{i=1}^{\ell} \Xi_i\Theta_i. \end{aligned} \tag{7}$$

where, $\Xi = (\Xi_1, \Xi_2, ..., \Xi_\ell)$ are designated as the weights, and $\Xi_i \geq 0, \sum_{i=1}^{\ell} \Xi_i = 1$.

Definition 2.10: For any collection of FePHFNs, indicated as $\Delta_i = \big(\chi_{\hbar_{\bullet i}}/\wp_i, \varrho_{\hbar_{\bullet i}}/\wp_i^{\bullet}\big)(i = 1, 2, ..., \ell)$, the aggregation result derived from the application of the FePHFWG operator is:

$$FePHWG(\Theta_1, \Theta_2, \ldots, \Theta_\ell) = \left\{ \bigcup_{\substack{\hbar_{\bullet i} \in \chi_{\hbar_{\bullet i}} \\ \wp_i \in \wp}} \prod_{i=1}^{\ell} (\hbar_{\bullet i})^{\Xi} \Big/ \prod_{i=1}^{\ell} \wp_i, \bigcup_{\substack{\eth_{\bullet i} \in \varrho_{\hbar_{\bullet i}} \\ \wp_i^{\bullet} \in \wp^{\bullet}}} \sqrt[3]{1 - \prod_{i=1}^{\ell}\left(1 - (\eth_{\bullet i})^3\right)^{\Xi}} \Big/ \prod_{i=1}^{\ell} \wp_i^{\bullet} \right\}. \tag{8}$$

3 FePHFS-CoCoSo Methodology

The objective of this section is to develop an innovative decision-making method termed the Fermatean probabilistic hesitant fuzzy combined compromise solution (FePHFS-CoCoSo), thereby handling MADM situations, particularly those in which FePHFNs are used to express the decision information. The FePHFS-CoCoSo approach focuses on using the CoCoSo methodology. The objective is to determine the priority order of different schemes by using an aggregation operator and a score function that is provided.

This approach combines the previously described models with FePHFS. The purpose is to achieve increased accuracy and consistency in decision-making when faced with uncertainty. A systematic illustration of the FePHFS-CoCoSo method's technique highlights the strategy used to achieve more reasonable and accurate decision analysis in uncertain circumstances.

3.1 Construction of a Comprehensive Decision Matrix Incorporating FePHFS for Decision-Making

This subsection focuses on the integration of FePHFS with MADM approaches to manage inherent ambiguity in scenarios involving decision-making. The best decisions are made when the best possibilities are carefully chosen, and algorithms are essential to this process. According to this paradigm, an algorithm is a methodical series of processes that are purposefully created in order to determine the optimal solution for a certain problem. A novel algorithm integrating FePHFS information is proposed. A collection of α options ($\xi = \xi_1, \xi_2, ..., \xi_\alpha$) and λ criteria ($\varsigma = \varsigma_1, \varsigma_2, ..., \varsigma_\beta$) can be accommodated by this algorithm. The choice matrix $\Gamma = [\gimel_{\pounds\lambda}]_{\alpha\times\beta}$ includes all these components. The algorithm provides decision-makers with a methodical tool to navigate intricate decision scenarios and provides an organized way to handle uncertainty inside the FePHFS framework. Weight vectors, denoted as $\Xi_h = \{\Xi_1, \Xi_2, ..., \Xi_\lambda\}$, signify the relative importance of various properties.

Algorithm

Step 1: A decision matrix $\Gamma = [\gimel_{\pounds\lambda}]_{\alpha\times\beta}$ was first created. It includes criteria (ς_λ) and options $\xi_\pounds$, each of which has a particular weighting (Ξ_λ).

$$\Gamma = [\gimel_{\pounds\lambda}]_{\alpha\times\beta} \begin{pmatrix} \left(\chi_{\hbar_{\bullet 11}}/\wp_{11}, \varrho_{\hbar_{\bullet 11}}/\wp^{\bullet}_{11}\right) & \left(\chi_{\hbar_{\bullet 12}}/\wp_{12}, \varrho_{\hbar_{\bullet 12}}/\wp^{\bullet}_{12}\right) & \cdots & \cdots & \left(\chi_{\hbar_{\bullet 1\lambda}}/\wp_{1\lambda}, \varrho_{\hbar_{\bullet 1\lambda}}/\wp^{\bullet}_{1\lambda}\right) \\ \left(\chi_{\hbar_{\bullet 21}}/\wp_{21}, \varrho_{\hbar_{\bullet 21}}/\wp^{\bullet}_{21}\right) & \left(\chi_{\hbar_{\bullet 22}}/\wp_{22}, \varrho_{\hbar_{\bullet 22}}/\wp^{\bullet}_{22}\right) & \cdots & \cdots & \left(\chi_{\hbar_{\bullet 2\lambda}}/\wp_{2\lambda}, \varrho_{\hbar_{\bullet 2\lambda}}/\wp^{\bullet}_{2\lambda}\right) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \left(\chi_{\hbar_{\bullet \pounds 1}}/\wp_{\pounds 1}, \varrho_{\hbar_{\bullet \pounds 1}}/\wp^{\bullet}_{\pounds 1}\right) & \left(\chi_{\hbar_{\bullet \pounds 2}}/\wp_{\pounds 2}, \varrho_{\hbar_{\bullet \pounds 2}}/\wp^{\bullet}_{\pounds 2}\right) & \cdots & \cdots & \left(\chi_{\hbar_{\bullet \pounds\lambda}}/\wp_{\pounds\lambda}, \varrho_{\hbar_{\bullet \pounds\lambda}}/\wp^{\bullet}_{\pounds\lambda}\right) \end{pmatrix}$$

Step 2: The data is normalized. Then particular attention is paid to the attributes related to costs.

$$\Gamma^{\clubsuit} = \begin{cases} \gimel_{\pounds\lambda}, & \text{if Benefit Attribute,} \\ \gimel^c_{\pounds\lambda}, & \text{if Cost Attribute,} \end{cases}$$

where, $\gimel_{\pounds\lambda} = \left(\chi_{\hbar_{\bullet\pounds\lambda}}/\wp_{\pounds\lambda}, \varrho_{\hbar_{\bullet\pounds\lambda}}/\wp^{\bullet}_{\pounds\lambda}\right)$ and $\gimel^c_{\pounds\lambda} = \left(\varrho_{\hbar_{\bullet\pounds\lambda}}/\wp^{\bullet}_{\pounds\lambda}, \chi_{\hbar_{\bullet\pounds\lambda}}/\wp_{\pounds\lambda}\right)$.

Step 3: The normalized group decision matrix $\Gamma^{\clubsuit} = [\gimel_{\pounds\lambda}]_{\alpha\times\beta}$ and the FePHFWA operator are utilized to determine the weighted sum measure.

$$FePHWA(\Theta_1, \Theta_2, \ldots, \Theta_\ell) = \left\{ \bigcup_{\hbar_{\bullet i}\in\chi_{\hbar_{\bullet i}}, \wp_i\in\wp} \sqrt[3]{1 - \prod_{i=1}^{\ell}\left(1 - \hbar^3_{\bullet i}\right)^{\Xi}} \Big/ \prod_{i=1}^{\ell}\wp_i, \bigcup_{\eth_{\bullet i}\in\varrho_{\hbar_{\bullet i}}, \wp^{\bullet}_i\in\wp^{\bullet}} \prod_{i=1}^{\ell}\left(\eth_{\bullet 2}\right)^{\Xi} \Big/ \prod_{i=1}^{\ell}\wp^{\bullet}_i \right\}.$$

Step 4: The weighted sum measure is determined utilizing the normalized group decision matrix $\Gamma^{\clubsuit} = [\gimel_{\pounds\lambda}]_{\alpha\times\beta}$ and the FePHFWG operator.

$$FePHWG(\Theta_1, \Theta_2, \ldots, \Theta_\ell) = \left\{ \bigcup_{\hbar_{\bullet i}\in\chi_{\hbar_{\bullet i}}, \wp_i\in\wp} \prod_{i=1}^{\ell}\left(\hbar_{\bullet 2}\right)^{\Xi} \Big/ \prod_{i=1}^{\ell}\wp_i, \bigcup_{\eth_{\bullet i}\in\varrho_{\hbar_{\bullet i}}, \wp^{\bullet}_i\in\wp^{\bullet}} \sqrt[3]{1 - \prod_{i=1}^{\ell}\left(1 - \left(\eth_{\bullet i}\right)^3\right)^{\Xi}} \Big/ \prod_{i=1}^{\ell}\wp^{\bullet}_i \right\}.$$

Step 5: The weighted product measure, denoted as, $\nabla(\xi_i)$, and the weighted sum measure, denoted as, $\mho(\xi_i)$, scores are calculated.

Step 6: The relative importance measure of the scheme is calculated using three appraisal score strategies.

$$\yen^1_i = \frac{\nabla_i + \mho_i}{\sum_{i=1}^{\pounds}(\nabla_i + \mho_i)}, \tag{9}$$

$$\yen^2_i = \frac{\nabla_i}{\min_i \nabla_i} + \frac{\mho_i}{\min_i \mho_i}, \tag{10}$$

$$\yen^3_i = \frac{\sigma\nabla_i + (1-\sigma)\mho_i}{\sigma\max_i \nabla_i + (1-\sigma)\max_i \mho_i}, \sigma \in [0,1]. \tag{11}$$

Step 7: The ultimate appraisal index $\yen_i$ was computed by amalgamating the aforementioned three scoring strategies.

$$\yen_i = \sqrt[3]{\yen^1_i\yen^2_i\yen^3_i} + \frac{\yen^1_i + \yen^2_i + \yen^3_i}{3}. \tag{12}$$

Step 8: The schemes are arranged in decreasing order based on their $\yen_i$ values.

4 Case Study

Earthquake disaster response planning is of paramount significance in safeguarding communities and minimizing the devastating impacts of seismic events, which involves the meticulous organization of resources, personnel, and infrastructure to effectively respond to the immediate aftermath of an earthquake. The significance lies in its capacity to save lives through swift evacuation, search and rescue operations, and the provision of critical medical aid. Furthermore, the planning encompasses long-term recovery efforts, emphasizing the restoration of essential infrastructure for sustained community resilience. The criticality of every instant is emphasized in catastrophe scenarios. The efficacy of a well-executed response plan is recognized as crucial in determining the difference between life and death. Prioritizing effective resource allocation, promptly dispatching emergency services, and restoring critical systems are essential to building communities that can withstand earthquakes.

Making decisions in the framework of earthquake catastrophe response planning is urgent and vital, because post-earthquake surroundings are dynamic and frequently unpredictable, emergency responders and decision-makers must act quickly and with knowledge. These choices have an immediate effect on the safety of the impacted people and set the stage for long-term healing.

The effectiveness of disaster response is acknowledged to be greatly impacted by decisions made regarding the order of importance for evacuation routes, the deployment of search and rescue teams, the distribution of medical resources, and the restoration of vital infrastructure. The pressing nature of these choices emphasizes the need for thorough planning, efficient channels of communication, and the flexibility to adjust plans as conditions change.

Decision-makers in earthquake catastrophe response must weigh immediate life-saving measures against longer-term rehabilitation initiatives. The distribution of resources, which includes medical care, evacuation planning, and infrastructure repair, requires a strategic understanding of the particular difficulties posed by seismic disasters as well as a careful assessment of priorities. The foundation of earthquake disaster response planning is determined to be effective decision-making. It affects the effectiveness and success of response operations as well as the resilience of impacted communities in the end.

The preparation for earthquake response includes shelter setting and evacuation (ξ_1), search and rescue operations (ξ_2), medical aid and emergency healthcare (ξ_3), and infrastructure assessment and restoration (ξ_4). A thorough evacuation strategy is first created. It leads residents toward approved safe areas and makes emergency shelters with the bare requirements accessible is essential. Search and rescue operations are prioritized to locate and assist individuals trapped in collapsed buildings or other dangerous situations. In addition, medical triage centers and emergency healthcare facilities must be established. The purposed is to treat injured patients and provide the required medical assistance. Finally, to support the entire recovery process, it is essential to quickly analyze and prioritize the restoration of vital infrastructure, including as roads, bridges, and utility services.

The following factors must be taken into account while analyzing certain criteria for earthquake reaction over chosen alternatives:

- **Impact on saving lives** (ς_1): Every option's capacity to preserve lives and safeguard the welfare of the impacted populace is assessed.
- **Resource efficiency** (ς_2): The efficient utilization of resources, encompassing manpower, equipment, and supplies, for each alternative is assessed.
- **Long-term recovery** (ς_3): The impact of each alternative on the long-term recovery and resilience of the community post-earthquake is considered.

Step 1: An analysis is initiated with a FePHF information matrix, denoted as Matrix 3. This matrix encompasses four alternatives $\xi = \{\xi_1, \xi_2, \xi_3, \xi_4\}$ and three criteria $\varsigma = \{\varsigma_1, \varsigma_2, \varsigma_3\}$, as indicated in Table 1. Each criterion is assigned specific weightings $\Xi_1 = 0.314, \Xi_2 = 0.355, \Xi_3 = 0.331$.

Table 1. FePHF information matrix analysis

	ς_1	ς_2	ς_3
ξ_1	$(0.2/0.6, 0.3/0.4)(0.3/1)$	$(0.45/1)(0.2/0.6, 0.8/0.4)$	$(0.7/0.9, 0.6/0.1)(0.6/0.7, 0.7/0.3)$
ξ_2	$(0.8/0.3, 0.1/0.7)(0.1/1)$	$(0.5/1)(0.3/0.7, 0.4/0.3)$	$(0.9/0.1)(0.3/0.6, 0.2/0.4)$
ξ_3	$(0.05/0.5, 0.2/0.5)(0.1/1)$	$(0.1/1)(0.3/0.4, 0.4/0.6)$	$(0.5/0.5, 0.6/0.5)(0.3/0.9, 0.1/0.1)$
ξ_4	$(0.4/0.4, 0.6/0.6)(0.5/1)$	$(0.7/1)(0.1/0.5, 0.1/0.5)$	$(0.2/0.1)(0.3/0.2, 0.6/0.8)$

Step 2: In this instance, normalization of the data is deemed unnecessary due to the absence of cost requirements.
Steps 3 & 4: The FePHF information matrix (Table 1) is utilized, applying the FePHFWA and the FePHFWG operators to determine the weighted sum and product measures.
Step 5: The scores for both the weighted sum measure $\nabla(\xi_i)$ and the weighted product measure $\mho(\xi_i)$ are determined, as presented in Table 2.

Table 2. Score values

$\nabla(\xi_i)$	$\mho(\xi_i)$
0.0013	-0.0014
0.0519	0.0125
0.0008	-0.0003
0.0227	0.0077

Steps 6 & 7: The relative importance measure of each scheme is calculated using three distinct appraisal score strategies, followed by the computation of the ultimate appraisal index $¥_i$, as illustrated in Table 3.
Step 8: The schemes are ranked in descending order based on their $¥_i$ values, as indicated in Table 3.

Table 3. Computation outcomes of the extended FePHF-CoCoSo method

Alternatives	$¥_i^1$	$¥_i^2$	$¥_i^3$	$¥_i$	**Ranking**
ξ_1	-0.0007	2.7145	-0.0010	0.9163	3
ξ_2	0.6762	57.8461	1	23.2352	1
ξ_3	0.0049	1.2194	0.0073	0.4459	4
ξ_4	0.3195	23.7801	0.4726	9.7221	2

5 Conclusion

In summary, this study introduces an innovative approach to earthquake response planning, integrating FePHFSs and algebraic operations within the framework of decision-making. The critical role of decision-making in response planning is emphasized, highlighting the significant impact of aggregation operators in formulating effective strategies. The novel aggregation operators, derived from the synthesis of algebraic operations and the CoCoSo method, offer a promising development in enhancing MADM processes. The comprehensive numerical case study presented herein not only demonstrates the adaptability and efficacy of this methodology but also establishes a robust framework for addressing the uncertainties inherent in earthquake response planning and decision-making.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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