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Aczel-Alsina Aggregation Operators on Spherical Fuzzy Rough Set and Their Application Section of Solar Panel

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Abstract: The Spherical fuzzy rough set (SFRS), which is based on approximations and is handled in this work, is a key idea for handling uncertainty when data is taken from real-world situations. The most adaptable operational laws based on the parameter for fuzzy frameworks are the Aczel-Alsina t-norm (AATN) and Aczel-Alsina t-conorm (AATCN), which are crucial for data interpolation. In this paper, operators based on AATN and AATCN are developed: spherical fuzzy rough Aczel-Alsina weighted geometric (SFRAAWG), spherical fuzzy rough Aczel-Alsina ordered weighted geometric (SFRAAOWG), and spherical fuzzy rough Aczel-Alsina hybrid weighted geometric (SFRAAHWG). A few fundamental properties of the generated SFRAAWG, SFRAAOWG, and SFRAAHWG operators are defined and given examples. The multi-criteria decision-making (MADM) problem is applied to the developed SFRAAWG operator. Additionally, the sensitivity of the SFRAAG operator is examined. The developed AOs are compared to a few pre-existing AOs and their significance is evaluated.

Keywords: Spherical fuzzy set (SFS); Rough set (RS); Spherical fuzzy rough set (SFRS); Aczel-Alsina t-norm (AATN); Aczel-Alsina t-conorm (AATCN); Multi-criteria decision-making (MADM)

1 Introduction

We resolve the MADM problem in this paper. Decision makers can use the defined decision alternatives when the decision environment is complex. The lack of complete information could affect the results of decisions. The fuzzy set (FS) theory was first presented by Zadeh [1]. In this theory, an element's belongingness to a set is described by its membership grade (MG). On occasion, it is demonstrated that the applicability of the FS concept is limited. For instance, all information pertaining to two or more aspects is independent of the other. With the aid of an MG and a non-membership grade (NMG), Atanassov [2] introduced the intuitionistic FS (IFS), a adjusted version of the FS that can holder complex and indefinite data. The IFS theory has been functional to many different problems and has gathered a lot of helpfulness [2]. The IFS data set is limited, though, and is predicated on the illogical requirement that the total of MG and NMG lie inside the unit [0,1]. Yager [3] introduced the idea of Pythagorean fuzzy sets (PyFS), a modified form of IFS, to deal with messy and erroneous data. Like IFS, PyFS has a controlled range and is based on the condition that the sum of the squares for MG and NMG fall confidential the assortment [0,1]. (0.5, 0.9) cannot be observed as a PyFV if the sum of the squares of the MG and NMG for a particular PyFV exceeds the unit interval, for instance, if MG is set at 0.5 and NMG is set at 0.9. To solve this issue, Yager [4] produced the q-rung orthopair fuzzy set (q-ROFS), which can handle complex and uncertain conditions, such as the effort mentioned above. A q-ROF value (q-ROFV) is a pair of MG and NMG such that the sum of their particular q powers must fall within the unit interval. We can choose any MG and NMG from [0,1] for each duplet (MG, NMG) by using the parameter q. The q is such that $0 \leq MG^q + NMG^q \leq 1$. There is also a certain amount of abstinence and refusal in human opinion. This suggests that these kinds of perplexing problems are beyond the capabilities of the earlier generalized form of FS structures. Cuong and Kreinovich [5] first defined a Picture Fuzzy Set (PFS) that includes abstinence great (AD), MG, and NMG in order to address these kinds of problems. PFS's range is likewise contained in the interval [0,1]. Subsequently, Mahmood et al. [6] proposed that the spherical FS (SFS) is the generalized form of PFS. After making some modifications, Ullah et al. [7] introduced a new concept known as the spherical FS (SFS) by taking the qth.

Helmers and Weiss [8] developed the idea of assessing the battery life cycle using MADM. Helmers et al. [9] introduced the concept of a life cycle evaluation based on real data from MADM for electric vehicles. Lundström and

Hellström [10] increased the app's scope for assessing electric automobiles. Stopka et al. [11] evaluated electric car charging with the use of a traction supply system based on MADM. Wieckowski et al. [12] conducted a careful and complex analysis to advance the theory on electric vehicles in MADM. Naeem and Ali [13] presented the concept of AASF AOs and its application in solar energy. Ali and Naeem [14] developed the concept of AA AOs in the context of MADM. Ali [15] advanced the theory of analysis on AA AOs in SF, demonstrating its application in MADM. Hussain et al. [16] applied MADM to the evaluation of solar panels. Akram et al. [17] provided a theoretical foundation on AA AOs with applications in energy resources. Shahabad et al. [18] formulated a theory of energy planning using SFAOs. Akram and Ashraf [19] introduced the concept of SFRs in MADM. Zheng et al. [20] developed the theory of SFRs employing the average aggregation operator and its applications in MADM. Zeng et al. [21] proposed a new concept of SFRs in a hybrid model with the TOPSIS method. Huang et al. [22] discussed the concept of SFRs with applications in MADM. Mohammad et al. [23] utilized SFRs in smart e-tourism data management. Hashmi et al. [24] applied SFRs in MADM. Hussain and Pamucar [25] used rough sets with Schweizer-Sklar TNM and TCM. Hussain et al. [26] presented the concept of PyFRS with the AA aggregation operator. Sarfraz [27] discussed the application of MADM in the context of recycled water.

An important role in information aggregation is played by the t-norm (TNM) and t-conorm (TCNM) [28]. Numerous TNM and TCNM varieties have been introduced, based on various variables. To name a few, Dombi [29] were instances of TNMs and TCNMs. Consequently, these TNMs and TCNMs have been used to create a number of AOs. The foundation for AOs [30, 31] is the Frank TNM and TCNM. The AOs [32] originate from the Einstein TNM and TCNM. The core elements of the AOs [33, 34] are the Einstein TNM and TCNM. Moreover, AOs begin with Einstein TNM and TCNM [35]. AOs in are predicated on Frank's TCNM and TNM [36, 37]. Moreover, AOs are based on Frank TNM and TCNM [38]. Important operational laws known as AATNM and AATCNM were introduced by Aczél and Alsina [39] and help with the fusion of information based on parameters. The flexibility of the AATNM and AATCNM makes them appropriate for use with any fuzzy framework. Senapati et al. [40] introduced the concepts of AATCNM and the AATNM to resolve the MADM problem. Its application is also made feasible by the significance of the AATNM and AATCNM [41].

SFRS is a tool for handling imprecise and imperfect data since it can remove unimportant characteristics early on and aggregate information based on approximations. As discussed above, in order to get around MADM issues, researchers have developed AOs for a SFRS framework based on different TNMs and TCNMs. Furthermore, Figueroa-García [42] assessed multiple TNM and TCNM types and found that the AATNM and AATCNM were the most advantageous and versatile in the combination of data. We found that the literature lacks geometric AOs for the SFRS based on AATNM and AATCNM. The objective of this work is to determine the geometric AOs (GAOs) for SFRS based on AATNM and AATCNM.

This article's main contribution is as follows.

- To begin with, in order to address the SFRVs, new operational laws have been introduced. As a result, AATNM and AATCNM are the foundation for a few basic SFRV operations.
- We have created some new methods for the fusion of data in the form of the SFRV collection, based on the operational laws for the SFRVs that were developed using the AATNM and AATCNM. In this study, the operators SFRAAWG, SFRAAOWG, and SFRAAHWG are developed.
- We look into a few basic and required requirements for the developed operators.
- An algorithm is given to explain how the developed AOs are applied in the section of Solar Panel, demonstrating the use of the suggested operators.
- The application of the suggested AOs in the section of Solar Panel is demonstrated by solving a numerical example.
- The obtained results are compared with some previous results to provide justification.

Section 2 presents the basic ideas for developing the GAOs for SFRS based on AATNM and AATCNM. Section 3 develops the procedures for SFRS values (SFRVs) based on the AATNM and AATCNM. Based on the operational laws for SFRVs, the SFRAAWG, SFRAAOWG, and SFRAAHWG operators are developed in Section 3. A number of essential traits are also mentioned and illustrated in Section 3. Section 4 suggests how to use the newly created SFRAAWG operator. Section 4 also contains the resolution to the SFRAAWG operator's MAGDM problem. In Section 4, the obtained results are analyzed at different parameter values and then compared with earlier studies. The results are graphically displayed and tabulated. The research is finally summarized in Section 5.

2 Methodology

We go over some fundamental ideas in this section, including SFS, RS, SFRS, and the score function for SFR values (SFRVs), AATN, and AATCN.

Definition 1 [2]: Let P represent the cosmos. Then, an expression defined as an SFS in P is

$$Z = \left\{ z, \left(\xi_z, \ddot{\Upsilon}_z, \mathfrak{b}_z \right) : z \in P \right\}$$

In this case, $\ddot{\Upsilon}_z : P \to [0,1], \xi_z : P \to [0,1], \mathfrak{b}_z : P \to [0,1]$ such that the requirement $0 \leq \mathrm{n}_z^2 + \ddot{\Upsilon}_z^2 + \mathfrak{b}_z^2 \leq 1$. The MD, AD, and NMD are represented by the numbers $\xi_z, \ddot{\Upsilon}_z$ and $\mathfrak{b}_z$. Let $Z = \left(\xi_z, \ddot{\Upsilon}_z, \mathfrak{b}_z\right)$, $Z_Q = \left(\xi_{zQ}, \ddot{\Upsilon}_{zQ}, \mathfrak{b}_{zQ}\right)$ for $Q = 1, 2$ are two SFVs and $\mathfrak{b} > 0$ be any real number. The SFVs' fundamental functions are then listed below.

1. $Z_1 \cup Z_2 = \left(\vee\left(\xi_{z1}, \xi_{z2}\right), \wedge\left(\ddot{\Upsilon}_{z1}, \ddot{\Upsilon}_{z2}\right), \wedge\left(\mathfrak{b}_{z1}, \mathfrak{b}_{z2}\right)\right)$;
2. $Z_1 \cap Z_2 = \left(\wedge\left(\xi_{z1}, \xi_{z2}\right), \vee\left(\ddot{\Upsilon}_{z1}, \ddot{\Upsilon}_{z2}\right), \vee\left(\mathfrak{b}_{z1}, \mathfrak{b}_{z2}\right)\right)$;
3. $Z_1 \oplus Z_2 = \left(\sqrt{\xi_{z1}^2 + \xi_{z2}^2 - \xi_{z1}^2 \xi_{z2}^2}, \ddot{\Upsilon}_{z1}, \ddot{\Upsilon}_{z2}, \mathfrak{b}_{z1}, \mathfrak{b}_{z2}\right)$;
4. $Z_1 \otimes Z_2 = \left(\xi_{z1}\xi_{z2}, \sqrt{\ddot{\Upsilon}_{z1}^2 + \ddot{\Upsilon}_{z1}^2 - \ddot{\Upsilon}_{z1}^2 \ddot{\Upsilon}_{z1}^2}, \sqrt{\mathfrak{b}_{z1}^2 + \mathfrak{b}_{z1}^2 - \mathfrak{b}_{z1}^2 \mathfrak{b}_{z1}^2}\right)$;
5. $Z^c = \left(\ddot{\Upsilon}_z, \xi_z, \mathfrak{b}_z\right)$, where Z^c is the complement of the SFV Z;
6. $\sigma Z = \left(\sqrt{1 - (1 - \xi_Z^2)^\sigma}, \ddot{\Upsilon}_Z^\sigma, \mathfrak{b}_Z^\sigma\right)$;
7. $Z^\sigma = \left(\xi_Z^\sigma, \sqrt{1 - \left(1 - \ddot{\Upsilon}_Z^2\right)^\sigma}, \sqrt{1 - (1 - \mathfrak{b}_Z^2)^\sigma}\right)$.

Definition 2 [43]: Let $\Omega \in P \times P$ be a relation, and let P be the universe. Next

1. Ω is reflexive iff $(\lambda, \lambda)\forall\lambda \in P$
2. Ω is symmetric if $(\lambda, \aleph) \in \Omega$ then $(\aleph, \lambda) \in \Omega \forall \lambda, \aleph \in P$
3. Ω is transitive if $\forall \lambda, \aleph, \omega \in P$ if $(\aleph, \omega) \in \Omega$ and $(\omega, \lambda) \in \Omega$ then $(\aleph, \lambda) \in \Omega$

Definition 3 [43]: Let P be the universal set and Ω be the relation. Compared to mapping $\Omega^* : P \to \mathcal{A}(P)$ is

$$\Omega^*(a) = \{\lambda \in P : (a, \lambda) \in \Omega\}, \text{ for } a \in P$$

where, (P, Ω) is referred to as the crisp space of the approximation and $\Omega^*(a)$ is designated as the successor neighborhood of an element a concerning Ω. The definitions of the LA and UA for any set $\mathfrak{W} \subseteq P$ are given below.

$$\Omega^{LA}(\mathfrak{W}) = \{\lambda \in \Omega \mid \Omega^*(\lambda) \subseteq \mathfrak{W}\}$$
$$\Omega^{UA}(\mathfrak{M}) = \{\lambda \in \Omega \mid \Omega^*(\lambda) \cap \mathfrak{W} \neq \phi\}$$

The set $\left\{\left(\Omega^{LA}(\mathfrak{W}), \Omega^{UA}(\mathfrak{W})\right)\right\} m$ is said to be an RS based on LA and UA.

Definition 4 [43]: Let Ω be the relation from SFS$(P \times P)$ and P be the universal set. Then

1. Ω is reflexive iff $\mathrm{n}_\Omega(\lambda, \lambda) = 1$ and $\ddot{\Upsilon}_\Omega(\lambda, \lambda) = 0 \forall \lambda \in P$
2. Ω is symmetric if $\forall (\lambda, \aleph) \in P \times P$ then $\xi_\Omega(\aleph, \lambda) = \xi_\Omega(\lambda, \aleph) \forall \lambda, \aleph \in P$ and $\ddot{\Upsilon}_\Omega(\aleph, \lambda) = \ddot{\Upsilon}_\Omega(\lambda, \aleph)$
3. Ω is transitive if $\forall \lambda, \aleph, \omega \in P$ if $(\aleph, \omega) \in \Omega$ and $(\omega, \lambda) \in \Omega$ then $\xi_\Omega(\aleph, \lambda) \geq \bigvee [\xi_\Omega(\aleph, \omega) \wedge \xi_\Omega(\omega, \lambda)]$ and $\ddot{\Upsilon}_\Omega(\aleph, \lambda) \geq \bigwedge \left[\ddot{\Upsilon}_\Omega(\aleph, \omega) \wedge \ddot{\Upsilon}_\Omega(\omega, \lambda)\right]$.

Definition 5 [44]: The SF space of the approximation is defined as (P, Ω) where P is the universal set and $\Omega \in P \times P$ is the SF relation. The definitions of the $SFLA$ and $SFUA$ are given below for any set $\mathfrak{W} \subseteq SFS(P)$

$$\Omega^{\mathrm{SFUA}}(\mathfrak{W}) = \left\{\lambda, \xi_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda), \ddot{\Upsilon}_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda), \mathfrak{b}_{\Omega^{\mathrm{SLUA}}(\mathfrak{W})}(\lambda) \mid \lambda \in P\right\}$$

$$\Omega^{\mathrm{SFLA}}(\mathfrak{W}) = \left\{\lambda, \xi_{\Omega^{\mathrm{SFLA}}(\mathfrak{W})}(\lambda), \ddot{\Upsilon}_{\Omega^{\mathrm{SFLA}}(\mathfrak{W})}(\lambda), \mathfrak{b}_{\Omega^{\mathrm{SLLA}}(\mathfrak{W})}(\lambda) \mid \lambda \in P\right\}$$

where,

$$\xi_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda) = \bigvee_{\aleph \in \mathrm{P}} \left[\xi_{\Omega(\lambda)}(\lambda, \aleph) \vee \xi_{\mathfrak{W}(\lambda)}\right]$$

$$\ddot{\Upsilon}_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda) = \bigwedge_{\aleph \in \mathrm{P}} \left[\ddot{\Upsilon}_{\Omega(\lambda)}(\lambda, \aleph) \wedge \ddot{\Upsilon}_{\mathfrak{W}(\lambda)}\right]$$

$$\mathfrak{b}_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda) = \bigwedge_{\aleph \in \mathrm{P}} \left[\mathfrak{b}_{\Omega(\lambda)}(\lambda, \aleph) \wedge \ddot{\Upsilon}_{\mathfrak{W}(\lambda)}\right]$$

$$\xi_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda) = \bigwedge_{\aleph \in \mathrm{P}} \left[\xi_{\Omega(\lambda)}(\lambda, \aleph) \wedge \xi_{\mathfrak{W}(\lambda)}\right]$$

$$\ddot{\Upsilon}_{\Omega^{\mathrm{SFLA}}(\mathfrak{W})}(\lambda) = \bigvee_{\aleph \in \mathrm{P}} \left[\ddot{\Upsilon}_{\Omega(\lambda)}(\lambda, \aleph) \vee \ddot{\Upsilon}_{\mathfrak{W}(\lambda)} \right]$$

$$\mathfrak{b}_{\Omega^{\mathrm{SFLA}}(\mathfrak{W})}(\lambda) = \bigvee_{\aleph \in \mathrm{P}} \left[\mathfrak{b}_{\Omega(\lambda)}(\lambda, \aleph) \vee \ddot{\Upsilon}_{\mathfrak{W}(\lambda)} \right]$$

With condition $0 \leq \xi_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda) + \ddot{\Upsilon}_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda) + \mathfrak{b}_{\Omega^{\mathrm{SFUA}}(\mathfrak{W})}(\lambda) \leq 1$ and $0 \leq \xi_{\Omega^{\mathrm{SFLA}}(\mathfrak{W})}(\lambda) + \ddot{\Upsilon}_{\Omega^{\mathrm{SFLA}}(\mathfrak{W})}(\lambda) + \mathfrak{b}_{\Omega^{\mathrm{SFLA}}(\mathfrak{W})}(\lambda) \leq 1$. The set $\left\{ \left(\Omega^{\mathrm{SFLA}}(\mathfrak{W}), \Omega^{\mathrm{SFUA}}(\mathfrak{W}), \Omega^{\mathrm{SFUA}}(\mathfrak{W}) \right) \right\}$ is said to be an SFRS based on SFLA and SFUA.
Definition 6 [39]: The definition of the AATN and

$$F_{\mathbb{X}}^{\Theta}(\mathcal{U}, \beta) = \begin{cases} F_{\mathrm{C}}(\mathcal{U}, \beta) & \text{if } \Theta = 0 \\ \min(\mathcal{U}, \beta) & \text{if } \Theta \to \infty \\ e^{-\left((-\ln \mathcal{U})^{\Theta} + (-\ln \mathcal{U})^{\Theta} \right)^{1/\Theta}} & \text{otherwise.} \end{cases}$$

And AATCN is defined as

$$S_{\mathbb{X}}^{\Theta}(\mathcal{U}, \beta) = \begin{cases} F_{\mathrm{C}}(\mathcal{U}, \beta) & \text{if } \Theta = 0 \\ \max(\mathcal{U}, \beta) & \text{if } \Theta \to \infty \\ 1 - \mathrm{e}^{-\left(\left(-\ln\left(1 - \mathcal{U}^2\right)\right)^{\Theta} + \left(-\ln\left(1 - \beta^2\right)\right)^{\Theta} \right)^{1/\Theta}} & \text{otherwise.} \end{cases}$$

where, $\Theta \in [0, \infty]$.

3 Aggregation Operators Based on AATN and AATCN

The operational laws of SFRVs based on the AATN and AATCN are contained in this section.
Definition 7: Let $\hat{\mathrm{R}}_Q = \left(\left(\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b} \right), \left(\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b} \right) \right)$, $Q = 1, 2$ be the assembly of SFRVs. Then

$$\begin{gathered} \hat{\mathrm{R}}_1 \oplus \hat{\mathrm{R}}_2 = \\ \left(\begin{pmatrix} \sqrt{1 - e^{-\left((-\aleph n(1 - (\xi_1^{\aleph b})^2))^{\Theta} + (-\aleph n(\xi_2^{\aleph b})^2)^{\Theta} \right)^{1/\Theta}}}, \\ e^{-\left(\left(-\aleph n \ddot{\Upsilon}_1^{\aleph b}\right)^{\Theta} + \left(-\aleph n \ddot{\Upsilon}_2^{\aleph b}\right)^{\Theta} \right)^{1/\Theta}}, \\ e^{-\left(\left(-\aleph n \mathfrak{b}_1^{\aleph b}\right)^{\Theta} + \left(-\aleph n \mathfrak{b}_2^{\aleph b}\right)^{\Theta} \right)^{1/\Theta}} \end{pmatrix} \begin{pmatrix} \sqrt{1 - e^{-\left((-\aleph n(1 - (\xi_1^{\partial b})^2))^{\Theta} + (-\aleph n(\xi_2^{\partial b})^2)^{\Theta} \right)^{1/\Theta}}}, \\ e^{-\left(\left(-\aleph n \ddot{\Upsilon}_1^{\partial b}\right)^{\Theta} + \left(-\aleph n \ddot{\Upsilon}_2^{\partial b}\right)^{\Theta} \right)^{1/\Theta}}, \\ e^{-\left(\left(-\aleph n \mathfrak{b}_1^{\partial b}\right)^{\Theta} + \left(-\aleph n \mathfrak{b}_2^{\partial b}\right)^{\Theta} \right)^{1/\Theta}} \end{pmatrix} \right) \\ \hat{\mathrm{R}}_1 \otimes \hat{\mathrm{R}}_2 = \\ \left(\begin{pmatrix} \sqrt{1 - e^{-\left((-\aleph n(1 - (\xi_1^{\aleph b})^2))^{\Theta} + (-\aleph n(\xi_1^{\aleph b})^2)^{\Theta} \right)^{1/\Theta}}}, \\ e^{-\left(\left(-\aleph n \ddot{\Upsilon}_1^{\aleph b}\right)^{\Theta} + \left(-\aleph n \ddot{\Upsilon}_2^{\aleph b}\right)^{\Theta} \right)^{1/\Theta}}, \\ e^{-\left(\left(-\aleph n \mathfrak{b}_1^{\aleph b}\right)^{\Theta} + \left(-\aleph n \mathfrak{b}_2^{\aleph b}\right)^{\Theta} \right)^{1/\Theta}} \end{pmatrix} \begin{pmatrix} \sqrt{1 - e^{-\left((-\aleph n(1 - (\xi_1^{\partial b})^2))^{\Theta} + (-\aleph n(\xi_1^{\partial b})^2)^{\Theta} \right)^{1/\Theta}}}, \\ e^{-\left(\left(-\aleph n \ddot{\Upsilon}_1^{\partial b}\right)^{\Theta} + \left(-\aleph n \ddot{\Upsilon}_2^{\partial b}\right)^{\Theta} \right)^{1/\Theta}}, \\ e^{-\left(\left(-\aleph n \mathfrak{b}_1^{\partial b}\right)^{\Theta} + \left(-\aleph n \mathfrak{b}_2^{\partial b}\right)^{\Theta} \right)^{1/\Theta}} \end{pmatrix} \right) \end{gathered} \tag{1}$$

Definition 8: Let $\hat{\mathrm{R}}_Q = \left(\left(\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b} \right), \left(\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b} \right) \right)$, $Q = 1, 2, \cdots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV such that $\sum_{Q=1}^{\Lambda} w_Q = 1$. Then

$$\begin{gathered} SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \cdots, \hat{\mathrm{R}}_\Lambda) = \bigotimes_{Q=1}^{\Lambda} \hat{\mathrm{R}}_Q^{w_Q} = \\ \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda} \left(-\aleph n(\xi_Q^{\aleph b})\right)^{\Theta} \right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda} \left(-\aleph n(1 - \ddot{\Upsilon}_Q^{\aleph b})\right)^{\Theta} \right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda} \left(-\aleph n(1 - \mathfrak{b}_Q^{\aleph b})\right)^{\Theta} \right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda} \left(-\aleph n(\xi_Q^{\partial b})\right)^{\Theta} \right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda} \left(-\aleph n(1 - \ddot{\Upsilon}_Q^{\partial b})\right)^{\Theta} \right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda} \left(-\aleph n(1 - \mathfrak{b}_Q^{\partial b})\right)^{\Theta} \right)^{1/\Theta}}} \end{pmatrix} \end{gathered} \tag{2}$$

The following investigation is made into some SFRAAWG operator attributes.
In Theorem 1, we explain that the SFRAAWG operator's aggregated value is also an SFRV.

Theorem 1: Let $\hat{\mathrm{R}}_Q = \left(\left(\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}\right), \left(\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}\right)\right)$, $Q = 1, 2, \cdots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Then the value obtained after the aggregation is SFRV and

$$SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \cdots, \hat{\mathrm{R}}_\Lambda) =$$
$$\begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_Q^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_Q^{\partial b})\right)^{\Theta}\right)^{1/\Theta}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix}$$

Proof: We prove Theorem 1 in the following way by applying the mathematical induction principle.

First, we check for $\Lambda = 2$

$$SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2) =$$
$$\begin{pmatrix} e^{-\left(\sum_{Q=1}^{2}\left(-\aleph n(\xi_Q^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{2}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{2}\left(-\aleph n(1-(\mathfrak{b}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{2}\left(-\aleph n(\xi_Q^{\partial b})\right)^{\Theta}\right)^{1/\Theta}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{2}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{2}\left(-\aleph n(1-(\mathfrak{b}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix}$$

Which is an SFRV.

Now, consider Eq. (2) true for $\Lambda = n$

$$SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_n) =$$
$$\begin{pmatrix} e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(\xi_Q^{\aleph b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{\frac{1}{\Theta}}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\mathfrak{b}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{\frac{1}{\Theta}}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(\xi_Q^{\partial b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\partial b})^2)\right)^{\Theta}\right)^{\frac{1}{\Theta}}}} \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\mathfrak{b}_Q^{\partial b})^2)\right)^{\Theta}\right)^{\frac{1}{\Theta}}}} \end{pmatrix}$$

We have to prove Eq. (2) true for $\Lambda = n + 1$, which is as follows:

$$SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_n, \hat{\mathrm{R}}_{n+1}) =$$
$$\begin{pmatrix} e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(\xi_Q^{\aleph b})\right)^{\Theta}+\left(-\aleph n(\xi_{n+1}^{\aleph b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\aleph b})^2)\right)^{\Theta}+\left(-\aleph n(1-\ddot{\Upsilon}_{n+1}^{\aleph b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\mathfrak{b}_Q^{\aleph b})^2)\right)^{\Theta}+\left(-\aleph n(1-\mathfrak{b}_{n+1}^{\aleph b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(\xi_Q^{\partial b})\right)^{\Theta}+\left(-\aleph n(\xi_{n+1}^{\partial b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\partial b})^2)\right)^{\Theta}+\left(-\aleph n(1-\ddot{\Upsilon}_{n+1}^{\partial b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n}\left(-\aleph n(1-(\mathfrak{b}_Q^{\partial b})^2)\right)^{\Theta}+\left(-\aleph n(1-\mathfrak{b}_{n+1}^{\partial b})\right)^{\Theta}\right)^{\frac{1}{\Theta}}}} \end{pmatrix}$$

Next, we have,

$$SFRAAWG(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_{n+1}) =$$
$$\begin{pmatrix} e^{-\left(\sum_{Q=1}^{n+1}\left(-\aleph n(\xi_Q^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n+1}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n+1}\left(-\aleph n(1-(\mathfrak{b}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{n+1}\left(-\aleph n(\xi_Q^{\partial b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n+1}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{n+1}\left(-\aleph n(1-(\mathfrak{b}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix}$$

Which is SFRV. Hence proof is completed.

In Theorem 1, we prove that the SFRAAWG operator is idempotent.

Theorem 2 (Idempotency): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \cdots, \Lambda$ be the collection of the SFRVs and w_Q is the weight of the Qth SFRV. Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b})) = ((\xi^{\aleph b}, \ddot{\Upsilon}^{\aleph b}, \mathfrak{b}^{\aleph b}), (\xi^{\partial b}, \ddot{\Upsilon}^{\partial b}, \mathfrak{b}^{\partial b})) = \hat{\mathrm{R}}$, $\forall Q = 1, 2, \cdots, \Lambda$. Then

$$SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) = ((\xi^{\aleph b}, \ddot{\Upsilon}^{\aleph b}, \mathfrak{b}^{\aleph b}), (\xi^{\partial b}, \ddot{\Upsilon}^{\partial b}, \mathfrak{b}^{\partial b})) = \hat{\mathrm{R}}$$

Proof: As $\hat{R}_Q = \left(\left(\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}\right), \left(\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}\right)\right) = \left(\left(\xi^{\aleph b}, \ddot{\Upsilon}^{\aleph b}, \mathfrak{b}^{\aleph b}\right), \left(\xi^{\partial b}, \ddot{\Upsilon}^{\partial b}, \mathfrak{b}^{\partial b}\right)\right)$, so we have

$$SFRAAWG(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_\Lambda) = (\hat{R}, \hat{R}, \ldots, \hat{R})$$

$$= \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_Q^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_Q^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_Q^{\partial b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix}$$

$$= \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi^{\partial b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_Q^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix}$$

$$= ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))\hat{R}$$

Hence proof is completed.

Theorem 3 (Boundedness): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Let $\hat{\mathrm{R}}_Q^s$ is the smallest and $\hat{\mathrm{R}}_Q^g$ is the greatest SFRV. Then

$$\hat{\mathrm{R}}_Q^s \leq SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) \leq \hat{\mathrm{R}}_Q^g$$

Theorem 4 (Monotonicity): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ and $\hat{\mathrm{R}}_Q^v = ((\xi_Q^{v\aleph b}, \ddot{\Upsilon}_Q^{v\aleph b}, \mathfrak{b}_Q^{v\aleph b}), (\xi_Q^{v\partial b}, \ddot{\Upsilon}_Q^{v\partial b}, \mathfrak{b}_Q^{v\partial b}))$ be the assemblages of the SFRVs such that $\hat{\mathrm{R}}_Q \leq \hat{\mathrm{R}}_Q^v$. Then

$$SFRAAWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) \leq SFRAAWG(\hat{\mathrm{R}}_1^v, \hat{\mathrm{R}}_2^v, \ldots, \hat{\mathrm{R}}_\Lambda^v)$$

The SFRAAWG operator gathers the data into SFRVs without the need for any separation. Sorting the SFRVs can also be used to pool them for better results. Thus, the following is the definition of the SFRAAOWG operator.

Definition 9: Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the collection of the SFRVs and w_Q is the weight of the Qth SFRV.

$$SFRAAOWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) = \bigotimes_{Q=1}^{\Lambda} \hat{\mathrm{R}}_{L(Q)}^{w_Q}$$

$$= \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_{L(Q)}^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_{L(Q)}^{\partial b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \tag{3}$$

In this case, $L(Q)$ is the SFRV permutation $(Q = 1, 2, \ldots, \Lambda)$ such that $L(Q-1) > L(Q)$. Some of the fundamental properties of the SFRAAWG operator are examined in the ensuing study.

SFRV is also the aggregate value that the SFRAAOWG operator obtains, according to Theorem 5.

Theorem 5: Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Then the value obtained after the aggregation is SFRV and

$$SFRAAOWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) =$$

$$\begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_{L(Q)}^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix} \begin{pmatrix} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\xi_{L(Q)}^{\partial b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{pmatrix}$$

Theorem 6 (Idempotency): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b})) = ((\xi^{\aleph b}, \ddot{\Upsilon}^{\aleph b}, \mathfrak{b}^{\aleph b}), (\xi^{\partial b}, \ddot{\Upsilon}^{\partial b}, \mathfrak{b}^{\partial b})) = \hat{\mathrm{R}}, \forall Q = 1, 2, \ldots, \Lambda$. Then

$$SFRAAOWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) = ((\xi^{\aleph b}, \ddot{\Upsilon}^{\aleph b}), (\xi^{\partial b}, \ddot{\Upsilon}^{\partial b})) = \hat{\mathrm{R}}$$

Theorem 7 (Boundedness): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial} b, \ddot{\Upsilon}_Q^{\partial} b, \mathfrak{b}_Q^{\partial} b))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Let $\hat{\mathrm{R}}_Q^s$ be the smallest and $\hat{\mathrm{R}}_Q^g$ the greatest SFRV. Then

$$\hat{\mathrm{R}}_Q^s \leq SFRAAOWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) \leq \hat{\mathrm{R}}_Q^g$$

Theorem 8 (Monotonicity): Let $\hat{R}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ and $\hat{R}_Q^v = ((\xi_Q^{v\aleph b}, \ddot{\Upsilon}_Q^{v\aleph b}, \mathfrak{b}_Q^{v\aleph b}), (\xi_Q^{v\partial b}, \ddot{\Upsilon}_Q^{v\partial b}, \mathfrak{b}_Q^{v\partial b}))$ be the collections of the SFRVs such that $\hat{R}_Q \leq \hat{R}_Q^v$.

$$SFRAAOWG(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_\Lambda) \leq SFRAAOWG(\hat{R}_1^v, \hat{R}_2^v, \ldots, \hat{R}_\Lambda^v)$$

The SFRAAWG and SFRAAOWG operators combine the data using the weights, either in an ordered or unordered manner. Nonetheless, weights can identify facades in both designated GAOs. Therefore, to solve the problem, we define the SFRAAHWG operator as follows.

Definition 10: Let $\hat{R}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Then

$$SFRAAHWG(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_\Lambda) = \bigotimes_{Q=1}^{\Lambda} \tilde{\hat{R}}_{L(Q)}^{w_Q} =$$

$$\left(\begin{array}{c} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\tilde{\xi}_{L(Q)}^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{array} \right) \left(\begin{array}{c} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\tilde{\xi}_{L(Q)}^{\partial b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{array} \right) \quad (4)$$

The SFRVs ($Q = 1, 2, \ldots, \Lambda$) are permuted to produce $L(Q-1) > L(Q)$ and $\tilde{\hat{R}} = zw\hat{R}$ which exhibits a strong balancing coefficient z. Here are some studies of the fundamental properties of the SFRAAHWG operator.

SFRV also applies to the aggregate value generated by the SFRAAHWG operator, according to Theorem 9.

Theorem 9: Let $\hat{R}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. The value that results from the aggregation is then SFRV and

$$SFRAAHWG(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_\Lambda) =$$

$$\left(\begin{array}{c} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\tilde{\xi}_{L(Q)}^{\aleph b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\aleph b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{array} \right) \left(\begin{array}{c} e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(\tilde{\xi}_{L(Q)}^{\partial b})\right)^{\Theta}\right)^{1/\Theta}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\ddot{\Upsilon}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}}, \\ \sqrt{1 - e^{-\left(\sum_{Q=1}^{\Lambda}\left(-\aleph n(1-(\mathfrak{b}_{L(Q)}^{\partial b})^2)\right)^{\Theta}\right)^{1/\Theta}}} \end{array} \right)$$

Theorem 10 (Idempotency): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b})) = ((\xi^{\aleph b}, \ddot{\Upsilon}^{\aleph b}, \mathfrak{b}^{\aleph b}), (\xi^{\partial b}, \ddot{\Upsilon}^{\partial b}, \mathfrak{b}^{\partial b})) = \hat{\mathrm{R}}, \forall Q = 1, 2, \ldots, \Lambda$. Then

$$SFRAAHWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) = ((\xi^{\aleph b}, \ddot{\Upsilon}^{\aleph b}, \mathfrak{b}^{\aleph b}), (\xi^{\partial b}, \ddot{\Upsilon}^{\partial b}, \mathfrak{b}^{\partial b})) = \hat{\mathrm{R}}$$

Theorem 11 (Boundedness): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ be the assembly of the SFRVs and w_Q is the weight of the Qth SFRV. Let $\hat{\mathrm{R}}_Q^s$ be the smallest and $\hat{\mathrm{R}}_Q^g$ the greatest SFRV. Then

$$\hat{R}_Q^s \leq SFRAAHWG(\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_\Lambda) \leq \hat{R}_Q^g$$

Theorem 12 (Monotonicity): Let $\hat{\mathrm{R}}_Q = ((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$, $Q = 1, 2, \ldots, \Lambda$ and $\hat{\mathrm{R}}_Q^v = ((\xi_Q^{v\aleph b}, \ddot{\Upsilon}_Q^{v\aleph b}, \mathfrak{b}_Q^{v\aleph b}), (\xi_Q^{v\partial b}, \ddot{\Upsilon}_Q^{v\partial b}, \mathfrak{b}_Q^{v\partial b}))$ be the assemblies of the SFRVs such that $\hat{\mathrm{R}}_Q \leq \hat{\mathrm{R}}_Q^v$. Then

$$SFRAAHWG(\hat{\mathrm{R}}_1, \hat{\mathrm{R}}_2, \ldots, \hat{\mathrm{R}}_\Lambda) \leq SFRAAOWG(\hat{\mathrm{R}}_1^v, \hat{\mathrm{R}}_2^v, \ldots, \hat{\mathrm{R}}_\Lambda^v)$$

4 A Multi-Attribute Decision Making Method Based on Suggested Research and Uses

Finding the best option out of all the viable options that are assessed based on various attributes is a MADM problem. Suppose that the set of q is $\{\mathcal{Q}_1, \mathcal{Q}_2, \ldots, \mathcal{Q}_q\}$. A group of options from which one is to be chosen based on the characteristics $\{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_r\}$, where r is the number of characteristics. The set of h experts with weights $W_n \in [0, 1], n = 1, 2, \ldots, h$ such that $\sum_{n=1}^{h} W_n = 1$ is denoted by $\{X_1, X_2, \ldots, X_h\}$. The procedures for selecting a substitute are outlined below.

Step 1: Expert data is collected in the form of SFRVs, with the result that $((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$. If a "cost" attribute already exists, we have to update it by getting the complement and swapping out the MD and NMD so that $((\xi_Q^{\aleph b}, \ddot{\Upsilon}_Q^{\aleph b}, \mathfrak{b}_Q^{\aleph b}), (\xi_Q^{\partial b}, \ddot{\Upsilon}_Q^{\partial b}, \mathfrak{b}_Q^{\partial b}))$.

Step 2: After normalization, the data in the form of SFRVs is combined with the known GAOs operator to determine the unique aggregated value of each alternative's characteristics.

Step 3: Next, the total individual SFRVs to calculate the overall value of the aggregation against each alternative, characteristics are aggregated with the help of the GAO's operator.

Step 4: Each option is assessed using the combined SFRVs and score values.

Step 5: With the score values from step 4, we use them to order the options.

Positive impact of Solar Panel: Counselors who participate in the solar panel segment learn about the life-changing possibilities of using solar energy. Students gain knowledge of the financial advantages, which include long-term energy bill savings and possible government incentives, in addition to the environmental benefits, which include lowering carbon emissions and protecting natural resources. Through a thorough examination of the installation, maintenance, and integration of solar panels into their lifestyle or business operations, clients gain the necessary knowledge and self-assurance to commence their journey towards sustainable energy.

Importance of Solar Panel: The potential of solar panels to transform energy production and provide a sustainable substitute for fossil fuels while reducing the environmental effects of conventional energy sources is what makes them so important. Solar panels help reduce greenhouse gas emissions, fight climate change, and protect natural resources by utilizing the power of sunlight. Furthermore, solar energy improves energy security and independence for people, communities, and countries by offering a dependable and renewable source of electricity. The widespread use of solar panels promotes technological innovation, job creation, and energy access for marginalized populations in addition to its positive effects on the environment and economy. This leads to the development of a cleaner, more resilient, and equitable energy future.

Effects of Solar Panel: Solar panels have a significant and wide-ranging impact on our lives, changing how we generate and use energy. Solar panels provide many advantages by utilizing sunlight, such as lower electricity costs, increased energy independence, and less environmental impact. By reducing greenhouse gas emissions and dependency on finite fossil fuels, solar energy helps to preserve natural resources and fight climate change. Furthermore, the use of solar panels increases access to clean, reasonably priced electricity for people and communities worldwide while also promoting technological innovation, job creation, and community resilience. Solar panels have the potential to have a positive impact on almost every aspect of our lives, from powering homes and businesses to providing electricity to remote areas and disaster relief efforts. This could help shape a more sustainable and optimistic future for future generations.

4.1 Case Study Experimentation

In this case study, the decision-maker assesses various solar panel kinds while taking the MADM problem's discussed algorithm into account. An efficient and workable aggregation model that is an expanded form of PFRVs is the SFRVs environment. Examine the following five alternative of solar panels, now we choose the best alternative of solar panel.

ϕ_1 **Wind turbines**: Produce electricity by utilizing wind energy.

ϕ_2 **Hydroelectric power**: Generate sustainable energy by using the flow of water.

ϕ_3 **Biomass energy**: Produce useful energy sources from organic materials.

ϕ_4 **Geothermal energy**: Use the heat from the Earth to create power.

ϕ_5 **Tidal power**: Use the tides' natural movement to produce electricity.

The various solar panel types mentioned above are assessed based on a few characteristics, which are described as follows:

ϱ_1 **Renewable**: Relies on inexhaustible sunlight for energy generation.

ϱ_2 **Sustainable**: Minimizes environmental impact and lessens dependency on fossil fuels.

ϱ_3 **Low maintenance**: After installed, requires little maintenance.

ϱ_4 **Versatile**: Adaptable to a range of configurations and sizes, from rooftop solar panels for homes to massive solar farms.

The decision-maker assesses solar panel information while taking the aforementioned characteristics into account. The decision-maker theoretically assigns the criteria $(0.37, 0.32, 0.31)^T$ a specific amount of weight. The decision-maker applies the suggested methodologies to assess a suitable building material according to the suggested MADM process algorithm. The following section includes the weights for the characteristics used to allocate SFRVs $(0.24, 0.18, 0.21, 0.19, 0.18)^T$.

The decision-making process's evaluation process: We used the derived methodologies of the SFRAAWG and SFRAAOWG operators, based on the above algorithm of the MADM problem, to evaluate a suitable optimal option taking certain reliable characteristics into consideration.

Step 1: The decision matrices in Table 1, are first normalized. Normalization is not required in this example because there is no cost type characteristics.

Table 1. Values assigned decision maker

			$\boldsymbol{\varrho_1}$						$\boldsymbol{\varrho_2}$			
	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial b}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
ϕ_1	0.32	0.32	0.45	0.45	0.21	0.22	0.35	0.54	0.54	0.45	0.23	0.18
ϕ_2	0.22	0.22	0.23	0.34	0.21	0.44	0.32	0.39	0.54	0.55	0.12	0.32
ϕ_3	0.44	0.11	0.16	0.22	0.11	0.39	0.22	0.22	0.54	0.34	0.18	0.19
ϕ_4	0.23	0.15	0.22	0.16	0.44	0.36	0.45	0.19	0.54	0.47	0.26	0.45
ϕ_5	0.44	0.18	0.14	0.15	1	0.43	0.35	0.33	0.54	0.38	0.31	0.38
			$\boldsymbol{\varrho_3}$						$\boldsymbol{\varrho_4}$			
	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
ϕ_1	0.32	0.29	0.17	0.35	0.21	0.35	0.23	0.22	0.13	0.34	0.44	0.35
ϕ_2	0.42	0.45	0.18	0.45	0.22	0.55	0.32	0.19	0.86	0.43	0.32	0.21
ϕ_3	0.44	0.22	0.19	0.45	0.27	0.25	0.33	0.22	0.53	0.53	0.22	0.25
ϕ_4	0.22	0.45	0.11	0.39	0.21	0.53	0.44	0.26	0.21	0.45	0.27	0.22
ϕ_5	0.43	0.19	0.32	0.52	0.15	0.45	0.43	0.39	0.22	0.54	0.31	0.44

Step 2: We aggregate the SFRVs to determine the characteristics that were independently collected from the all-decision using the SFRAAWG and SFRAAOWG operator and the expert weight. The outcomes are displayed as follows in Table 2.

Table 2. Aggregated values acquired by the SFRAAWG operator

			$\boldsymbol{\varrho_1}$						$\boldsymbol{\varrho_2}$			
	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial b}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
ϕ_1	0.500	0.243	0.344	0.615	0.766	0.165	0.528	0.421	0.418	0.615	0.751	0.135
ϕ_2	0.398	0.166	0.173	0.519	0.766	0.336	0.500	0.299	0.418	0.695	0.836	0.242
ϕ_3	0.607	0.083	0.120	0.398	0.844	0.296	0.398	0.166	0.418	0.519	0.788	0.142
ϕ_4	0.409	0.113	0.165	0.328	0.610	0.273	0.615	0.143	0.418	0.632	0.730	0.344
ϕ_5	0.607	0.136	0.105	0.315	0.853	0.328	0.528	0.251	0.418	0.555	0.696	0.288
			$\boldsymbol{\varrho_3}$						$\boldsymbol{\varrho_4}$			
	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
ϕ_1	0.500	0.220	0.127	0.528	0.766	0.265	0.409	0.166	0.097	0.519	0.610	0.265
ϕ_2	0.590	0.347	0.135	0.615	0.759	0.426	0.500	0.143	0.726	0.598	0.689	0.157
ϕ_3	0.607	0.166	0.142	0.615	0.723	0.188	0.509	0.166	0.410	0.680	0.759	0.188
ϕ_4	0.398	0.347	0.082	0.564	0.766	0.410	0.607	0.197	0.157	0.615	0.723	0.165
ϕ_5	0.598	0.143	0.242	0.672	0.811	0.344	0.598	0.299	0.165	0.687	0.696	0.336

Now we aggregate the SFRVs to determine the characteristics that were independently collected from the all-decision using the SFRAAOWG operator and the expert weight. The outcomes are displayed as follows in Table 3.

Step 3: Once the aggregating characteristics have been aggregated in Table 4, they can be individually aggregated using the SFRAAWG and SFRAAOWG operator, which is provided below.

Now the aggregating characteristics have been aggregated in Table 5, they can be individually aggregated using the SFRAAOWG operator, which is provided below.

Table 3. Aggregated values acquired by the SFRAAOWG operator

	ϱ_1						ϱ_2					
	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
ϕ_1	0.707	0.941	0.882	0.620	0.414	0.973	0.687	0.823	0.825	0.620	0.435	0.982
ϕ_2	0.776	0.972	0.970	0.694	0.414	0.887	0.707	0.911	0.825	0.552	0.301	0.942
ϕ_3	0.627	0.993	0.986	0.776	0.287	0.912	0.776	0.972	0.825	0.694	0.379	0.980
ϕ_4	0.769	0.987	0.973	0.820	0.629	0.926	0.620	0.979	0.825	0.607	0.467	0.882
ϕ_5	0.627	0.982	0.989	0.827	0.272	0.892	0.687	0.937	0.825	0.667	0.516	0.917
	ϱ_3						ϱ_4					
	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
ϕ_1	0.707	0.952	0.984	0.687	0.414	0.930	0.769	0.972	0.991	0.694	0.629	0.930
ϕ_2	0.640	0.880	0.982	0.620	0.425	0.818	0.707	0.979	0.473	0.634	0.525	0.975
ϕ_3	0.627	0.972	0.980	0.620	0.477	0.965	0.700	0.972	0.832	0.566	0.425	0.965
ϕ_4	0.776	0.880	0.993	0.660	0.414	0.832	0.627	0.961	0.975	0.620	0.477	0.973
ϕ_5	0.634	0.979	0.942	0.573	0.342	0.882	0.634	0.911	0.973	0.559	0.516	0.887

Table 4. SFRAAWG operator-obtained aggregated individual values

ϱ_1						ϱ_2					
$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
0.397	0.204	0.279	0.322	0.572	0.330	0.417	0.357	0.470	0.517	0.556	0.298
ϱ_3						ϱ_4					
$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
0.436	0.320	0.195	0.511	0.545	0.401	0.421	0.245	0.595	0.529	0.698	0.286

Table 5. SFRAAOWG operator-obtained aggregated individual values

ϱ_1						ϱ_2					
$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
0.625	0.993	0.990	0.662	0.850	0.963	0.622	0.973	0.880	0.546	0.830	0.980
ϱ_3						ϱ_4					
$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\aleph\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$	$\xi^{\aleph\mathfrak{b}}$	$\xi^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\ddot{\Upsilon}^{\partial\mathfrak{b}}$	$\mathfrak{b}^{\aleph\mathfrak{b}}$	$\mathfrak{b}^{\partial\mathfrak{b}}$
0.600	0.976	0.994	0.556	0.829	0.948	0.616	0.984	0.985	0.535	0.784	0.979

Step 4: To determine the ranking of the alternatives, we now obtain the score values for each, which are given as follows in Table 6.

Table 6. Score values obtained by SFRAAWG and SFRAAOWG operators

Operator	Score Values
SFRAAW	$\ddot{\Upsilon}(\varrho_1) = -0.0735, \ddot{\Upsilon}(\varrho_2) = -0.0764, \ddot{\Upsilon}(\varrho_3) = -0.0364, \ddot{\Upsilon}(\varrho_4) = -0.1314$
SFRAAOWG	$\ddot{\Upsilon}(\varrho_1) = -0.6967, \ddot{\Upsilon}(\varrho_2) = -0.6715, \ddot{\Upsilon}(\varrho_3) = -0.7140, \ddot{\Upsilon}(\varrho_4) = -0.7118$

Step 5: Given that Table 7 provides the highest score value among all other alternatives, ϱ_1 is the best option.

Table 7. Score values obtained

Operator	Score Values
SFRAAWG	$\varrho_1 \succ \varrho_4 \succ \varrho_3 \succ \varrho_2$
SFRAAOWG	$\varrho_1 \succ \varrho_2 \succ \varrho_4 \succ \varrho_3$

Form Figure 1 the results of the SFRAAWG and SFRAAOWG operator same when increase the value of ϱ now we easily find that ϱ_1 have the best alternative because the ϱ_1 given the same result in all parameters.

4.2 Sensitivity Analysis

In the sections that follow, we look at how the ranking results change based on the value of the parameter Θ. Tables 8 and 9 display the sensitivity of the SFRAAWG and SFRAAOWG operator, respectively.

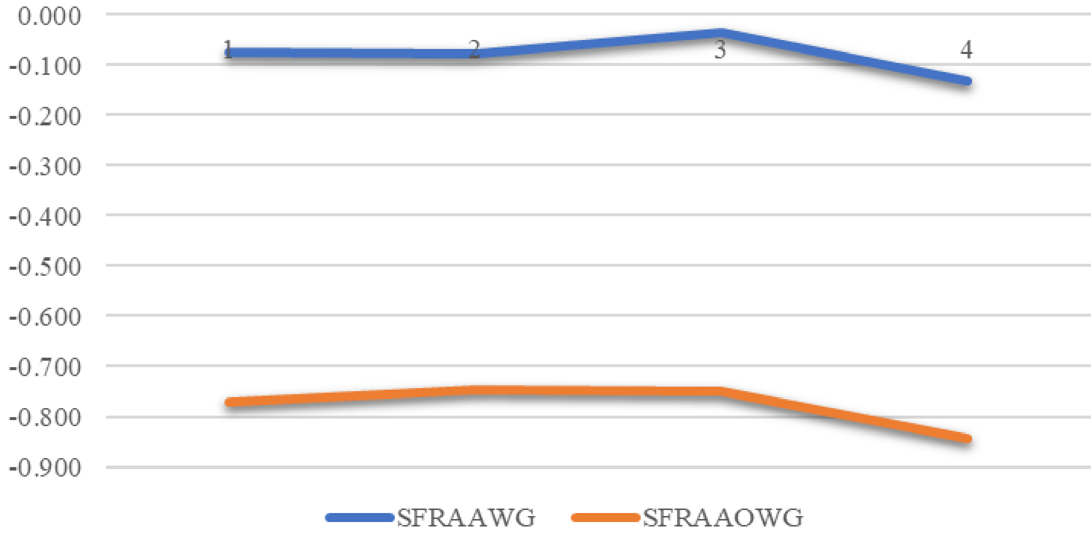

Figure 1. Present the score value of SRAAWG and SRFAAOWG operator

Table 8. Sensitivity analysis SFRAAWG operators

Θ	SFRAAWG
2	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
3	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
4	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
5	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
10	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
15	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
30	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
40	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$
50	$\varrho_1 > \varrho_4 > \varrho_3 > \varrho_2$

The alternatives to the SFRAAWG operator are ranked in Table 8 for parameter values between $\Theta = 2$ to $\Theta = 50$; The result is the same for all parameter values, with ϱ_3 being the best. To improve understanding, take a look at Figure 2's depiction of the sensitivity of the SFRAAWG and SFRAAOWG operators.

Table 9. Sensitivity analysis SFRAAOWG operators

Θ	SFRAAOWG
2	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
3	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
4	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
5	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
10	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
15	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
30	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
40	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$
50	$\varrho_1 > \varrho_2 > \varrho_4 > \varrho_3$

Depending on the Θ value, the score has different values. However, the alternatives maintain their ranks, and the SFRAAWG operator's ranking indicates that ϱ_3 is the most efficient manager overall Figure 3's.

4.3 Comparative Analysis

Hussain et al. [26] introduced the AA aggregation operator within the framework of the PyFRS and applied it to MADM problems. Akram and Ashraf [19] presented the concept of SFRS in the context of MADM. Ali [15] developed the theory of analysis on AA AOs in SF and provided applications for MADM. Hussain et al. [16] applied the MADM to the evaluation of solar panels. In the following, we compared the proposed AOs with these AOs in Table 10 as follows.

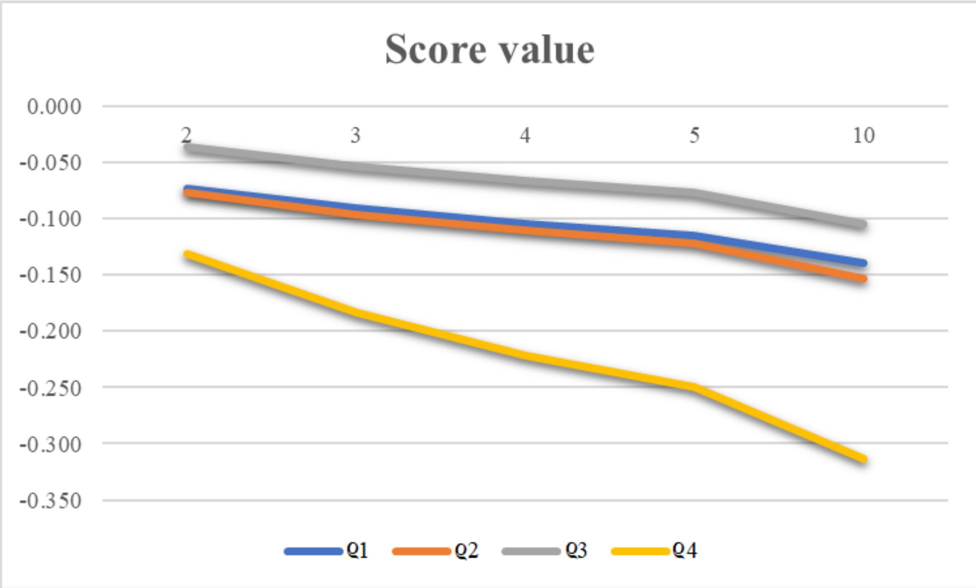

Figure 2. Sensitivity analysis of SRAAWG operator

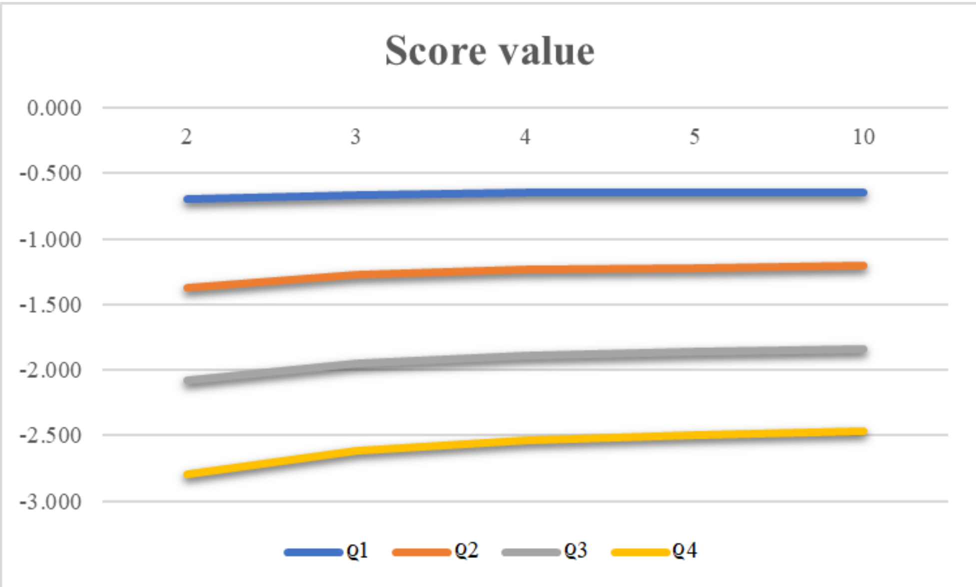

Figure 3. Sensitivity analysis of SRAAWG operator

Table 10. Comparative study SFRAAWG and SFRAAOWG operators

Operator	Score Values
Huassain et al. [26]	$\varrho_3 > \varrho_2 > \varrho_4 > \varrho_1$
Huassain et al. [26]	$\varrho_3 > \varrho_4 > \varrho_2 > \varrho_1$
Akram et al. [19]	$\varrho_3 > \varrho_4 > \varrho_2 > \varrho_1$
Akram et al. [19]	$\varrho_3 > \varrho_4 > \varrho_2 > \varrho_1$
Ali et al. [15]	$\varrho_3 > \varrho_4 > \varrho_2 > \varrho_1$
Ali et al. [15]	$\varrho_3 > \varrho_4 > \varrho_2 > \varrho_1$
Hussain et al. [16]	Not applicable
Hussain et al. [16]	Not applicable

The comparison of various AOs with the suggested AOs in this article is shown in Table 10. Observingly, the most efficient ϱ_3 obtained by SFRAAWG, but SFRAAOWG given the best option is ϱ_1. When we compare the result in different parameter then show that ϱ_3 has the best option. After that we compare the result some old operator then we see that ϱ_3 has the best option.

5 Conclusions

Some basic SFRV operations are developed in this paper, based on the AATN and AATCN. An analysis of the basic properties of the AOs for the information aggregation in the form of PyFRS is carried out after they are developed using the methods described. The developed technique is applied to assess the sector managers' performance in response to an actual MADM scenario. The outcomes are compared to specific existing AOs. The results for the AATN and AATCN at different parameter values are investigated. The extremely flexible and practical AATNM and AATCNM serve as the foundation for the developed techniques, SFRAAWG, OFRAAOWG, and SFRAAHWG operators. The most suitable manager obtained from SFRAAWG operators is ϱ_3. However, because of the way it aggregates, the SFRAAWG operator is more dependable than the SFRAAWA operator. However, because they are based on SFRS, which can cover the most information in the form of SFRVs from real-life scenarios, the SFRAAWA and SFRAAWG operators are more reliable AOs than the current AOs. The first reduction of the characteristics is also aided by SFRS. Since SFRS relies on further fuzzy approximations rather than precise ones, it is a more reliable framework than SFS. We wish to use the recently developed approach with the frameworks mentioned conducted by [45].

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Data Availability

Not Available.

Conflicts of Interest

The authors declare that none of the work reported in this paper could have been influenced by any known competing financial interests or personal relationships.

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Nomenclature

SFRS	Spherical Fuzzy Rough Set
AATN	Aczel-Alsina t-norm
AATCN	Aczel-Alsina t-conorm
SFRAAWG	Spherical Fuzzy Rough Aczel-Alsina Weighted Geometric
SFRAAOWG	Spherical Fuzzy Rough Aczel-Alsina Ordered Weighted Geometric
SFRAAHWG	Spherical Fuzzy Rough Aczel-Alsina Hybrid Weighted Geometric
MADM	Multi-criteria Decision-Making
AOs	Aggregation Operators
RS	Rough Set
SFS	Spherical Fuzzy Set