Journal of Operational and Strategic Analytics
https://www.acadlore.com/journals/JOSA

Advancing Cybersecurity Strategies for Multinational Corporations: Novel Distance Measures in q-Rung Orthopair Multi-Fuzzy Systems

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Received: 01-26-2024 **Revised:** 03-03-2024 **Accepted:** 03-12-2024

Citation: P. Mahalakshmi, J. Vimala, K. Jeevitha, and S. Nithya Sri, "Advancing cybersecurity strategies for multinational corporations: Novel distance measures in q-rung orthopair multi-fuzzy systems," *J. Oper. Strateg Anal.*, vol. 2, no. 1, pp. 49–55, 2024. https://doi.org/10.56578/josa020105.

Abstract: In the realm of cybersecurity, the formulation of comprehensive strategies is imperative for multinational corporations to protect against pervasive cyber threats. Recent developments in the field of intuitionistic multi-fuzzy sets (IMFSs) have heralded q-rung orthopair multi-fuzzy sets (MFSs) as a pivotal tool for encapsulating ambiguity and uncertainty within complex scenarios. The essence of this study lies in the introduction of two innovative distance measures tailored for q-rung orthopair MFSs (q-ROM^{k}FSs) of dimension k, enhancing the capacity to delineate distinctions between such sets effectively. Employing score functions pertinent to q-ROM^{k}FSs, this research extends its application to the sphere of Multi-Attribute Decision Making (MADM), presenting a methodological advancement in decision-making processes. The efficacy of the proposed measures is elucidated through a comparative analysis with existing methodologies in MADM, thereby underscoring the superiority of the introduced approach. This investigation not only contributes to the enrichment of the theoretical underpinnings of q-ROMFSs but also propels their practical application in cybersecurity strategy formulation for multinational entities. The study employs the Euclidean and Hamming distance measures as benchmarks, supplemented by the development of a score and accuracy function, to furnish a comprehensive tool for addressing cybersecurity challenges.

Keywords: Multi-attribute decision making; q-rung orthopaor multi-fuzzy set; Euclidean distance measure, Hamming distance measure; Score and accuracy function

1 Introduction

The cybersecurity strategy offers benefits by establishing a framework that is inherently conducive to early detection. A set of established rules and procedures, coupled with ongoing monitoring, aids in identifying anything unusual promptly. The theory of intuitionistic fuzzy sets (IFSs) [1], developed by Atanassov in 1986, adheres to the restriction that the sum of the membership function (MemF) and the non-membership function (NMemF) cannot exceed one and is defined by its MemF and NMemF. Yager [2] also developed the idea of Pythagorean fuzzy sets (PFSs), which is an extension of IFS and is defined by MemF and NMemF, with the restriction that the square sum of MemF and NMemF cannot exceed one.

In 2017, Yager [3] presented a broad class of sets called q-rung orthopair fuzzy sets (q-ROFSs). The q-th power of the support against and for, added together, is restricted under this framework to equal one. As q increases, users have more freedom to express their opinions about membership grades since a wider variety of acceptable orthopairs are available. Over the years, with the increasing complexity of human life and thought processes, it has become evident that the traditional fuzzy set structure may not adequately address the growing intricacies and uncertainties.

In response to this limitation, Sebastian and Ramakrishnan [4] introduced the concept of multi-fuzzy sets (MFSs) and and subsequently expanded upon this concept [5]. Subsequently, in an effort to enhance the adaptability and versatility of MFSs, Das and Kar [6] introduced the concept of IMFSs. Szmidt and Kacprzyk [7] conducted the initial exploration by extending established distance measures like the Hamming distance and Euclidean distance to the interval valued type-2 fuzzy sets (IT-2FSs) domain and comparing them with methods employed for traditional fuzzy sets. Nevertheless, Wang and Xin [8] proposed that the distance measure introduced by Szmidt and Kacprzyk [7] proved ineffective in specific scenarios. As a result, several novel distance measures for pattern recognition were formulated and put into practice. Later on, Das et al. [9] also extended Hamming, Euclidean, and their normalized versions to the IMFSs framework.

In practical scenarios, distance measures play a crucial role in quantifying the degrees of disparity between entities. Numerous distance measures have been explored in the literature concerning various extensions of fuzzy set theory, soft sets, IFS, PFS, and q-ROFS [10–13]. However, there is a scarcity of distance measures tailored specifically for MFSs. This paper addresses this gap by introducing several extended distance measures for MFSs. The major goal of this research is to offer some innovative distance measures using q-rung orthopair multi-fuzzy information, drawing inspiration from the fact that q-ROFSs have a great deal of capacity to simulate imprecise and ambiguous information in real-world applications. In order to demonstrate the effectiveness of these unique distance measures, they have finally been tested on the multi-attribute decision-making (MADM) issue.

Inspired by these considerations, Pethaperumal et al. [14] formulated the algebraic concepts of q-ROMFSs. Similarly, akin to a q-rung extension of multi-fuzzy set theory, Jeevitha et al. [15] introduced the notion of LDMFSs. In recent times, this extension has found application in q-rung orthopair multi-fuzzy soft sets (q-ROMFS$_t$S) [16] and linear Diophantine multi-fuzzy soft sets (LDMFS$_t$S) [17]. As a result, q-ROM^{k}FSs are better than IFMSs at handling the ambiguity and ambiguous information that arise in real-world problem-solving circumstances. In future, we have the potential to expand the concept of q-ROM^{k}FSs to include lattice ordering [18].

The flexibility of q-ROM^{k}FSs offers a way to deal with problems where there are different membership values for every element, particularly when there is a lot of ambiguity. The application of q-rung orthopair multi-fuzzy-based information may improve the accuracy of alternative evaluations in the MADM process. Therefore, the focus of this work is on the distance measures and score function of q-ROMFSs. The distance measure is important for a number of real-world applications, including texture analysis, image segmentation, disease diagnosis, and more. However, only a limited number of researchers have delved into the development of multi-fuzzy distance measures and their significance. As a result, this work presents new distance measures and score functions tailored for q-ROM^{k}FSs.

2 Preliminaries

Definition 2.1 Let $\hat{\mathcal{O}}$ be the initial universal set. A fuzzy set $\hat{\mathcal{Z}}$ is defined as

$$\hat{\mathcal{Z}} = \{\langle o_i, r_{\hat{\mathcal{Z}}}(o_i)\rangle | o_i \in \hat{\mathcal{O}}\}$$

where, the function $r_{\hat{\mathcal{Z}}}(o_i) : \hat{\mathcal{O}} \longrightarrow [0,1]$ characterizes the MemF for every $o_i \in \hat{\mathcal{O}}$ with the condition that $0 \leq r_{\hat{\mathcal{Z}}}(o_i) \leq 1$ [19].

Definition 2.2 Let k be a positive integer. A Mk F-set (M^{k}F-set) $\hat{\mathcal{F}}$ of dimension k over $\hat{\mathcal{O}}$ is defined as

$$\hat{\mathcal{F}} = \left\{\langle o_i, r^m_{\hat{\mathcal{F}}}(o_i)\rangle | o_i \in \hat{\mathcal{O}}\right\}$$

where, $r^m_{\hat{\mathcal{F}}}(o_i) = \left(r^1_{\hat{\mathcal{F}}}(o_i), r^2_{\hat{\mathcal{F}}}(o_i), \cdots, r^k_{\hat{\mathcal{F}}}(o_i)\right)$ is the multi-MemF of $\hat{\mathcal{F}}$, and the set of all MFS of dimension k is denoted as M^{k}FS($\hat{\mathcal{O}}$) [4].

Definition 2.3 In 2017, Yager [3] defined the mathematical structure of q-ROFSs as $\hat{\mathcal{P}}$, given as

$$\hat{\mathcal{P}} = \left\{\left\langle o_i, \left(r_{\hat{\mathcal{P}}}(o_i), s_{\hat{\mathcal{P}}}(o_i)\right)\right\rangle | o_i \in \hat{\mathcal{O}}\right\}$$

where, $\hat{\mathcal{O}}$ represents the initial universal set, and $r_{\hat{\mathcal{P}}} : \hat{\mathcal{O}} \longrightarrow [0,1]$ and $s_{\hat{\mathcal{P}}} : \hat{\mathcal{O}} \longrightarrow [0,1]$ characterize the MemF and NMemF of an element $o_i \in \hat{\mathcal{P}}$ to the set $\hat{\mathcal{P}}$, respectively, under a restriction $0 \leq \left(r_{\hat{\mathcal{P}}}(o_i)\right)^q + \left(s_{\hat{\mathcal{P}}}(o_i)\right)^q \leq 1$, with q ≥ 1. $o_i \in \hat{\mathcal{O}}$, the degree of indeterminacy is given by $\Pi_{\hat{\mathcal{P}}}(o_i) = \sqrt[q]{1 - \left(r_{\hat{\mathcal{P}}}(o_i)\right)^q - \left(s_{\hat{\mathcal{P}}}(o_i)\right)^q}$ and $(r_{\hat{\mathcal{P}}}, s_{\hat{\mathcal{P}}})$ represents a q-rung orthopair fuzzy number (q-ROFN).

Definition 2.4 In 2013, Das and Kar [6] defined the mathematical structure of as an IMFS $\hat{\mathcal{H}}$ of dimension k over $\hat{\mathcal{O}}$ given as

$$\hat{\mathcal{H}} = \left\{\left\langle o_i, \left((r^1_{\hat{\mathcal{H}}}(o_i), s^1_{\hat{\mathcal{H}}}(o_i)), (r^2_{\hat{\mathcal{H}}}(o_i), s^2_{\hat{\mathcal{H}}}(o_i)), \cdots, (r^k_{\hat{\mathcal{H}}}(o_i), s^k_{\hat{\mathcal{H}}}(o_i))\right)\right\rangle | o_i \in \hat{\mathcal{O}}\right\}$$

where, $\hat{\mathcal{O}}$ represents the initial universal set, and $r^m_{\hat{\mathcal{H}}} : \hat{\mathcal{O}} \longrightarrow [0,1]$ and $s^m_{\hat{\mathcal{H}}} : \hat{\mathcal{O}} \longrightarrow [0,1]$ characterize the multi-MemF and multi-NMemF of an element $o_i \in \hat{\mathcal{H}}$ to the set $\hat{\mathcal{H}}$ respectively, under a restriction $0 \leq r^m_{\hat{\mathcal{H}}}(o_i) + s^m_{\hat{\mathcal{H}}}(o_i) \leq 1$, m = 1,2,$\cdots$,k. For each $o_i \in \hat{\mathcal{O}}$, the degree of indeterminacy is given by $\Pi^m_{\hat{\mathcal{H}}}(o_i) = 1 - r^m_{\hat{\mathcal{H}}}(o_i) - s^m_{\hat{\mathcal{H}}}(o_i)$.

Further, the intuitionistic multi-fuzzy number (IM^{k}FN) is given by $\hat{\mathcal{H}}$ = ($r^m_{\hat{\mathcal{H}}}$, $s^m_{\hat{\mathcal{H}}}$) in which $r^m_{\hat{\mathcal{H}}}$, $s^m_{\hat{\mathcal{H}}} \in$ [0,1] and 0 $\leq r^m_{\hat{\mathcal{H}}} + s^m_{\hat{\mathcal{H}}} \leq 1$, m = 1, 2, $\cdots$, k.

Definition 2.5 In 2023, Vimala et al. [16] defined the mathematical structure of a q-ROM^{k}FS $\hat{\mathcal{T}}$ of dimension k over $\hat{\mathcal{O}}$ given as

$$\hat{\mathcal{T}} = \left\{ \left\langle o_i, \Big((r^1_{\hat{\mathcal{T}}}(o_i), s^1_{\hat{\mathcal{T}}}(o_i)), (r^2_{\hat{\mathcal{T}}}(o_i), s^2_{\hat{\mathcal{T}}}(o_i)), \cdots, (r^k_{\hat{\mathcal{T}}}(o_i), s^k_{\hat{\mathcal{T}}}(o_i)) \Big) \right\rangle | o_i \in \hat{\mathcal{O}} \right\}$$

where, $\hat{\mathcal{O}}$ represents the initial universal set, and $r^m_{\hat{\mathcal{T}}} : \hat{\mathcal{O}} \longrightarrow [0,1]$ and $s^m_{\hat{\mathcal{T}}} : \hat{\mathcal{O}} \longrightarrow [0,1]$ characterize the multi-MemF and multi-NMemF of an element $o_i \in \hat{\mathcal{T}}$ to the set $\hat{\mathcal{T}}$ respectively, under a restriction $0 \leq \Big(r^m_{\hat{\mathcal{T}}}(o_i)\Big)^q + \Big(s^m_{\hat{\mathcal{T}}}(o_i)\Big)^q \leq 1$, with $q \geq 1$ and $m = 1, 2, \cdots, k$. For each $o_i \in \hat{\mathcal{O}}$, the degree of indeterminacy is given by $\Pi^m_{\hat{\mathcal{T}}}(o_i) = \sqrt[q]{1 - \Big(r^m_{\hat{\mathcal{T}}}(o_i)\Big)^q - \Big(s^m_{\hat{\mathcal{T}}}(o_i)\Big)^q}$.

Further, the q-rung orthopair multi-fuzzy number (q-ROM^{k}FN) is given by $\hat{\mathcal{T}}$ = ($r^m_{\hat{\mathcal{T}}}$, $s^m_{\hat{\mathcal{T}}}$) in which $r^m_{\hat{\mathcal{T}}}$, $s^m_{\hat{\mathcal{T}}} \in$ [0,1] and $0 \leq (r^m_{\hat{\mathcal{T}}})^q + (s^m_{\hat{\mathcal{T}}})^q \leq 1$, $q \geq 1$ and $m = 1, 2, \cdots, k$.

Example 2.6 Suppose $\hat{\mathcal{O}} = \{o_1, o_2, o_3, o_4\}$ represents a set of four universities. Mr. Y wants to choose a university by considering factors, such as tuition cost, location, program quality, campus facilities, student-faculty ratio, and career opportunities. Mr. Y wants to evaluate these universities based on 6-dimensional q-rung orthopair multi-fuzzy information.

$$\begin{aligned}\hat{\mathcal{T}} = \Big\{ &\langle o_1, (0.9, 0.6), (0.4, 0.7), (0.3, 0.5), (0.4, 0.5), (0.3, 0.6), (0.2, 0.6) \rangle, \\ &\langle o_2, (0.4, 0.6), (0.7, 0.9), (0.8, 0.4), (0.6, 0.3), (0.5, 0.9), (0.2, 0.4) \rangle, \\ &\langle o_3, (0.4, 0.6), (0.3, 0.5), (0.6, 0.2), (0.8, 0.5), (0.9, 0.2), (0.3, 0.5) \rangle, \\ &\langle o_4, (0.3, 0.4), (0.4, 0.5), (0.6, 0.4), (0.9, 0.6), (0.4, 0.8), (0.2, 0.6) \rangle \Big\}.\end{aligned}$$

3 Novel Score Functions and Distance Measures of q-ROMFSs

Motivated by the novel score function proposed by Peng et al. [20] for q-ROFN, this section presents an enhanced score function designed to address q-rung orthopair multi-fuzzy information. This improved function takes into account the degrees of multi-memF, non-memF, and hesitation. Consider a q-ROM^{k}FN of dimension m is denoted as $\mathcal{T} = \langle r^m, s^m \rangle$, where r^m and s^m represent the multi-memFs and non-memFs, respectively.

Definition 3.1 The score function $\mathfrak{S}(\mathcal{T})$ of the q-ROM^{k}FN $\hat{\mathcal{T}}$ = ($r^m_{\hat{\mathcal{T}}}$, $s^m_{\hat{\mathcal{T}}}$) of dimension m is defined as

$$\mathfrak{S}(\mathcal{T}) = \sum_{m=1}^{k} \left\{ (r^m_{\hat{\mathcal{T}}})^q - (s^m_{\hat{\mathcal{T}}})^q + \left(\frac{e^{(r^m_{\hat{\mathcal{T}}})^q - (s^m_{\hat{\mathcal{T}}})^q}}{e^{(r^m_{\hat{\mathcal{T}}})^q - (s^m_{\hat{\mathcal{T}}})^q} + 1} - \frac{1}{2} \right) (\Pi^m_T)^q \right\} \tag{1}$$

Example 3.2 An example can be employed to demonstrate the application of the score function. $\hat{\mathcal{T}}_1 = \langle (0.8, 0.2), (0.3, 0.5) \rangle$ and $\hat{\mathcal{T}}_2 = \langle (0.6, 0.4), (0.7, 0.3) \rangle$ are two q-ROM^{k}FSs of dimension 2. The degree of indeterminacy $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ are $\pi^1_{\hat{\mathcal{T}}_1} = \sqrt[4]{1 - (0.8)^4 - (0.2)^4} = 0.876$. Similarly, $\pi^2_{\hat{\mathcal{T}}_1} = \sqrt[4]{1 - (0.3)^4 - (0.5)^4} = 0.9294$, $\pi^2_{\hat{\mathcal{T}}_2} = \sqrt[4]{1 - (0.6)^4 - (0.2)^4} = 0.9654$ and $\pi^2_{\hat{\mathcal{T}}_2} = \sqrt[4]{1 - (0.7)^4 - (0.3)^4} = 0.9312$, respectively, and the score functions of $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ are $\mathfrak{S}(\hat{\mathcal{T}}_1)$= 0.4027 and $\mathfrak{S}(\hat{\mathcal{T}}_2)$ = 0.4021, respectively.

Definition 3.3 For any two q-ROMFSs $\mathcal{T}_1$=($r^m_{\hat{\mathcal{T}}_1}$, $s^m_{\hat{\mathcal{T}}_1}$) and $\mathcal{T}_2$=($r^m_{\hat{\mathcal{T}}_2}$, $s^m_{\hat{\mathcal{T}}_2}$) of dimension m, then

(1) If $\mathfrak{S}(\mathcal{T}_1) > \mathfrak{S}(\mathcal{T}_2)$, then $\mathcal{T}_1 > \mathcal{T}_2$;
(2) If $\mathfrak{S}(\mathcal{T}_1) < \mathfrak{S}(\mathcal{T}_2)$, then $\mathcal{T}_1 < \mathcal{T}_2$;
(3) If $\mathfrak{S}(\mathcal{T}_1) = \mathfrak{S}(\mathcal{T}_2)$, then
  (a) If $\Pi^m_1 > \Pi^m_2$, then $\mathcal{T}_1 < \mathcal{T}_2$;
  (b) If $\Pi^m_1 = \Pi^m_2$, then $\mathcal{T}_1 = \mathcal{T}_2$.

Definition 3.4 Let $\hat{\mathcal{T}}_1 = (r^m_{\hat{\mathcal{T}}_1}, s^m_{\hat{\mathcal{T}}_1})$ and $\hat{\mathcal{T}}_2 = (r^m_{\hat{\mathcal{T}}_2}, s^m_{\hat{\mathcal{T}}_2})$ be two q-ROM^{k}FSs. Then the Hamming distance between $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ in $\hat{\mathcal{O}} = \{o_1, o_2, \cdots, o_n\}$ of dimension k is defined as

$$\mathfrak{D}^q_H = \frac{1}{2k}\left(\sum_{i=1}^{n}\sum_{m=1}^{k}\left(\left|\left(r^m_{\hat{\mathcal{T}}_1}(o_i)\right)^q - \left(r^m_{\hat{\mathcal{T}}_2}(o_i)\right)^q\right| + \left|\left(s^m_{\hat{\mathcal{T}}_1}(o_i)\right)^q - \left(s^m_{\hat{\mathcal{T}}_2}(o_i)\right)^q\right| + \left|\left(\pi^m_{\hat{\mathcal{T}}_1}(o_i)\right)^q - \left(\pi^m_{\hat{\mathcal{T}}_2}(o_i)\right)^q\right|\right)\right), \quad o \in \hat{\mathcal{O}} \tag{2}$$

The normalized Hamming distance between $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ in $\hat{\mathcal{O}} = \{o_1, o_2, \cdots, o_n\}$ of dimension k is defined as

$$\mathfrak{N}^q_H = \frac{1}{2nk}\left(\sum_{i=1}^{n}\sum_{m=1}^{k}\left(\left|\left(r^m_{\hat{\mathcal{T}}_1}(o_i)\right)^q - \left(r^m_{\hat{\mathcal{T}}_2}(o_i)\right)^q\right| + \left|\left(s^m_{\hat{\mathcal{T}}_1}(o_i)\right)^q - \left(s^m_{\hat{\mathcal{T}}_2}(o_i)\right)^q\right| + \left|\left(\pi^m_{\hat{\mathcal{T}}_1}(o_i)\right)^q - \left(\pi^m_{\hat{\mathcal{T}}_2}(o_i)\right)^q\right|\right)\right), \quad o \in \hat{\mathcal{O}} \tag{3}$$

Definition 3.5 Let $\hat{\mathcal{T}}_1 = (r^m_{\hat{\mathcal{T}}_1}, s^m_{\hat{\mathcal{T}}_1})$ and $\hat{\mathcal{T}}_2 = (r^m_{\hat{\mathcal{T}}_2}, s^m_{\hat{\mathcal{T}}_2})$ be two q-ROM^{k}FSs. Then the Euclidean distance between $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ in $\hat{\mathcal{O}} = \{o_1, o_2, \cdots, o_n\}$ of dimension k is defined as

$$\mathfrak{D}^q_E = \sqrt{\frac{1}{2k}\left(\sum_{i=1}^{n}\sum_{m=1}^{k}\left(\left((r^m_{\hat{\mathcal{T}}_1}(o_i))^q - (r^m_{\hat{\mathcal{T}}_2}(o_i))^q\right)^2 + \left((s^m_{\hat{\mathcal{T}}_1}(o_i))^q - (s^m_{\hat{\mathcal{T}}_2}(o_i))^q\right)^2 + \left((\pi^m_{\hat{\mathcal{T}}_1}(o_i))^q - (\pi^m_{\hat{\mathcal{T}}_2}(o_i))^q\right)^2\right)\right)}, \quad o \in \hat{\mathcal{O}} \tag{4}$$

The normalized Euclidean distance between $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ in $\hat{\mathcal{O}} = \{o_1, o_2, \cdots, o_n\}$ of dimension k is defined as

$$\mathfrak{N}^q_E = \sqrt{\frac{1}{2nk}\left(\sum_{i=1}^{n}\sum_{m=1}^{k}\left(\left((r^m_{\hat{\mathcal{T}}_1}(o_i))^q - (r^m_{\hat{\mathcal{T}}_2}(o_i))^q\right)^2 + \left((s^m_{\hat{\mathcal{T}}_1}(o_i))^q - (s^m_{\hat{\mathcal{T}}_2}(o_i))^q\right)^2 + \left((\pi^m_{\hat{\mathcal{T}}_1}(o_i))^q - (\pi^m_{\hat{\mathcal{T}}_2}(o_i))^q\right)^2\right)\right)}, \quad o \in \hat{\mathcal{O}} \tag{5}$$

The application of the Hamming and Euclidean distance measures can be exemplified using an example below. If $\hat{\mathcal{T}}_1 = \langle(0.8, 0.2), (0.3, 0.5)\rangle$ and $\hat{\mathcal{T}}_2 = \langle(0.6, 0.4), (0.7, 0.3)\rangle$ are two q-ROM^{k}FSs of dimension 2. The degrees of indeterminacy $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ are $\pi^1_{\hat{\mathcal{T}}_1} = \sqrt[4]{1-(0.8)^4-(0.2)^4} = 0.876$. Similarly, $\pi^2_{\hat{\mathcal{T}}_1} = \sqrt[4]{1-(0.3)^4-(0.5)^4} = 0.9294$, $\pi^2_{\hat{\mathcal{T}}_2} = \sqrt[4]{1-(0.6)^4-(0.2)^4} = 0.9654$ and $\pi^2_{\hat{\mathcal{T}}_2} = \sqrt[4]{1-(0.7)^4-(0.3)^4} = 0.9312$, respectively, and the Hamming and Euclidean distance measures between $\hat{\mathcal{T}}_1$ and $\hat{\mathcal{T}}_2$ are $\mathfrak{D}^q_H(\hat{\mathcal{T}}_1, \hat{\mathcal{T}}_2) = 0.2186$ and $\mathfrak{D}^q_E(\hat{\mathcal{T}}_1, \hat{\mathcal{T}}_2) = 0.1634$, respectively.

4 Case Study: Development of a Cybersecurity Strategy for a Multi-National Corporation

In today's digitally interconnected world, cybersecurity is a top priority for organizations, particularly multinational corporations that handle vast amounts of sensitive data. This case study delves into the process of developing an effective cybersecurity strategy for a multinational corporation using MADM techniques. The MADM approach is chosen because it offers a systematic way to evaluate multiple criteria and alternatives, making it well-suited for complex decision-making scenarios like cybersecurity strategy development. MADM helps decision-makers consider various aspects simultaneously and prioritize them effectively.

Example: MADM problem focuses on selecting an optimal cybersecurity strategy for safeguarding sensitive healthcare data. Four distinct strategies, Strategy A (S_A), Strategy B (S_B), Strategy C (S_C), and an alternative approach, Strategy M (S_M), are evaluated based on five critical attributes to ensure effective data protection and compliance with regulatory standards. The universal set $\mathfrak{Z} = \{\mathfrak{z}_1, \mathfrak{z}_2, \mathfrak{z}_3, \mathfrak{z}_4\}$ functions as a complete collection of various cybersecurity situations in this cybersecurity scenario. These scenarios include a wide range of situations that cover the intricacies and challenges encountered in the field of cybersecurity, and they encapsulate the techniques that are currently being considered. Every component of the set symbolises a unique cybersecurity scenario or environment, adding to the vast array of possible outcomes that decision-makers need to consider and take into account. The evaluation of the strategies based on their attributes is depicted using the q-rung orthopair multi-fuzzy (q-ROM^{2}F) information of dimension 2, illustrated in Tables 1, 2, 3 and 4. Four attributes, detailed in Table 5, guide the evaluation process, addressing crucial aspects of cybersecurity, such as security effectiveness, cost, adaptability and scalability, user experience and productivity.

Table 1. $q - ROM^2FS$ decision matrix for strategy decision-making S_A (assuming q=4)

	$\mathfrak{z}_1$	$\mathfrak{z}_2$	$\mathfrak{z}_3$	$\mathfrak{z}_4$
A_1	[(0.5,0.9),(0.8,0.1)]	[(0.3,0.8),(0.3,0.4)]	[(0.2,0.7),(0.8,0.8)]	[(0.2,0.3),(0.4,0.7)]
A_2	[(0.3,0.2),(0.4,0.7)]	[(0.5,0.2),(0.4,0.7)]	[(0.2,0.3),(0.3,0.7)]	[(0.9,0.3),(0.4,0.6)]
A_3	[(0.5,0.7),(0.5,0.1)]	[(0.2,0.6),(0.8,0.7)]	[(0.8,0.6),(0.4,0.6)]	[(0.4,0.5),(0.5,0.1)]
A_4	[(0.4,0.3),(0.7,0.6)]	[(0.6,0.7),(0.2,0.5)]	[(0.6,0.2),(0.7,0.2)]	[(0.1,0.2),(0.8,0.5)]

Table 2. $q - ROM^2FS$ decision matrix for strategy decision-making S_B (assuming q=4)

	$\mathfrak{z}_1$	$\mathfrak{z}_2$	$\mathfrak{z}_3$	$\mathfrak{z}_4$
A_1	[(0.3,0.8),(0.7,0.4)]	[(0.4,0.8),(0.5,0.7)]	[(0.2,0.3),(0.3,0.8)]	[(0.4,0.7),(0.3,0.8)]
A_2	[(0.8,0.7),(0.4,0.3)]	[(0.6,0.5),(0.7,0.3)]	[(0.9,0.8),(0.5,0.3)]	[(0.7,0.5),(0.8,0.4)]
A_3	[(0.6,0.4),(0.4,0.5)]	[(0.6,0.1),(0.5,0.6)]	[(0.8,0.4),(0.6,0.1)]	[(0.7,0.5),(0.4,0.7)]
A_4	[(0.8,0.5),(0.6,0.5)]	[(0.7,0.6),(0.8,0.6)]	[(0.8,0.4),(0.9,0.4)]	[(0.6,0.5),(0.7,0.3)]

Table 3. $q - ROM^2FS$ decision matrix for strategy decision-making S_C (assuming q=4)

	$\mathfrak{z}_1$	$\mathfrak{z}_2$	$\mathfrak{z}_3$	$\mathfrak{z}_4$
A_1	[(0.2,0.9),(0.6,0.4)]	[(0.4,0.7),(0.5,0.8)]	[(0.3,0.9),(0.3,0.9)]	[(0.1,0.7),(0.2,0.9)]
A_2	[(0.3,0.7),(0.4,0.6)]	[(0.3,0.8),(0.2,0.8)]	[(0.2,0.9),,(0.4,0.2)]	[(0.8,0.4),(0.3,0.7)]
A_3	[(0.6,0.2),(0.7,0.1)]	[(0.5,0.7),(0.8,0.4)]	[(0.5,0.7),(0.5,0.6)]	[(0.3,0.7),(0.5,0.1)]
A_4	[(0.6,0.4),(0.3,0.5)]	[(0.5,0.8),(0.2,0.5)]	[(0.2,0.8),,(0.7,0.2)]	[(0.7,0.4),(0.3,0.7)]

Table 4. $q - ROM^2FS$ decision matrix for strategy decision-making S_M (assuming q=4)

	$\mathfrak{z}_1$	$\mathfrak{z}_2$	$\mathfrak{z}_3$	$\mathfrak{z}_4$
A_1	[(0.3,0.9),(0.2,0.9)]	[(0.2,0.9),(0.3,0.4)]	[(0.4,0.9),(0.6,0.8)]	[(0.1,0.9),(0.2,0.9)]
A_2	[(0.7,0.5),(0.1,0.3)]	[(0.8,0.5),(0.9,0.2)]	[(0.7,0.6),(0.8,0.5)]	[(0.8,0.4),(0.3,0.2)]
A_3	[(0.4,0.8),(0.3,0.5)]	[(0.5,0.7),(0.6,0.5)]	[(0.6,0.3),(0.8,0.4)]	[(0.3,0.4),(0.7,0.7)]
A_4	[(0.4,0.8),(0.4,0.8)]	[(0.6,0.8),(0.5,0.8)]	[(0.6,0.3),(0.2,0.5)]	[(0.6,0.7),(0.8,0.7)]

Table 5. Security evaluation criteria

Symbol	**Criteria**	**2-Dimensional Evaluation**	**Description**
o_1	Security Effectiveness	Threat Type	Evaluate the approach against diverse risks, including internal (insider threats), external (hackers), and emergent (zero-day vulnerabilities) threats.
		Attack Surface	Evaluate the plan's effectiveness in securing diverse attack surfaces, including network, endpoint, and application security.
o_2	Cost	Cost Category	Aggregate expenses into several groups, such as those associated with initial implementation, ongoing operations, and incident response.
		Cost Efficiency	Analyse the cost allocation among the strategy's many components, such as technology investments, personnel development costs, and compliance-related outlays.
o_3	Adaptability and scalability	Technological Adaptability	Examine the strategy's ability to adapt to technological advancements like the adoption of cloud services and the incorporation of cutting-edge security solutions.
		Growth Scalability	Analyse whether the plan can scale as the business grows, taking activities into new markets or geographic areas.
o_4	User Experience and Productivity	Usability	Examine the impact of security measures on user experience and how user-friendly they are.
		Productivity Metrics	To determine the impact of security measures on employee effectiveness, compare productivity indicators before and after deployment.

The results are shown in Table 6 (assuming q=4). When the data in Table 6 are compared to other distance measures, it is clear that the distance measure between S_C and S_M is the minimum. According to the idea of distance measure within q-ROM^{n}FSs, option S_C is closer to the optimal choice than S_M. As a result (Figure 1), S_C emerges as the most advantageous option. Among numerous considerations, S_C emerges as the best cybersecurity method for protecting sensitive healthcare data.

Table 6. q-ROM^{2}FS Distance measure for strategy decision-making (assuming q=4)

Distance Measure	(S_A, S_M)	(S_B, S_M)	(S_C, S_M)
Normalized Hamming Distance Measure $\mathfrak{N}^q_H$	0.711	0.672	**0.671**
Normalized Euclidean Distance Measure $\mathfrak{N}^q_E$	0.91	0.80	**0.68**

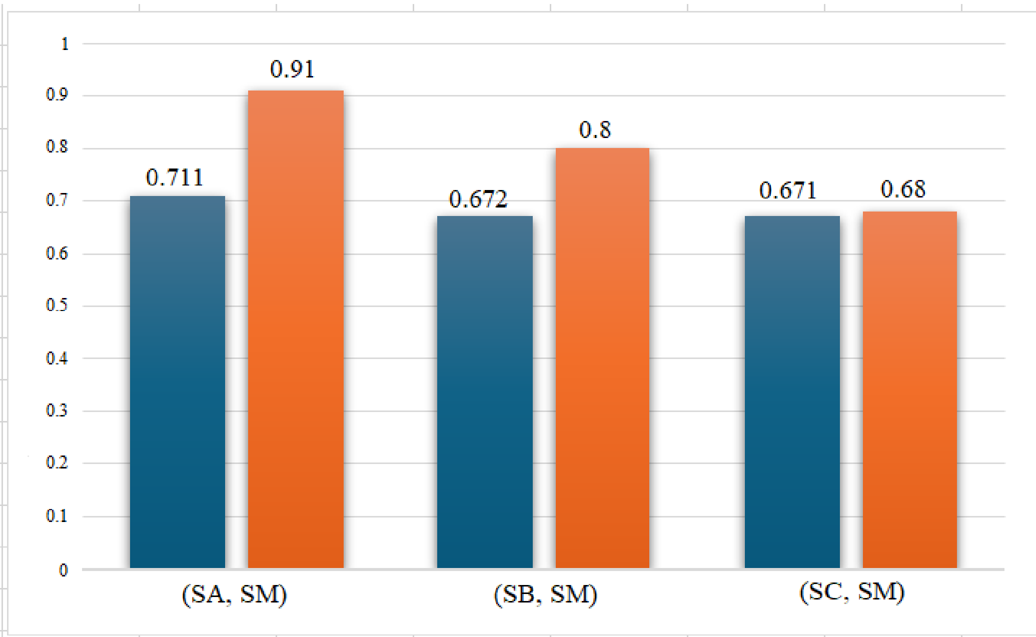

Figure 1. Ranking of strategies

5 Conclusions

This study introduces enhanced score functions for q-ROMFSs, building upon the existing score function of IMFSs and the improved score function of q-ROFSs. Additionally, two novel distance measures are introduced for q-ROMFSs, namely the Hamming and Euclidean distance measures. Subsequently, a MADM method is devised to assess the optimal selection of cybersecurity strategies using q-rung orthopair multi-fuzzy information. Finally, an illustrative example is presented to demonstrate the applicability and rationality of the proposed approach. The future study aims to broaden the developed method to include more MFSs and apply them to various fields, such as medical diagnosis and image processing.

Funding

The article has been written with the joint financial support of RUSA-Phase 2.0 grant sanctioned vide letter No.F 24-51/2014-U, Policy (TN Multi-Gen), Dept. of Edn. Govt. of India, Dt. 09.10.2018, DST-PURSE 2nd Phase programme vide letter No. SR/PURSE Phase 2/38 (G) Dt. 21.02.2017 and DST (FIST - level I) 657876570 vide letter No.SR/FIST/MS-I/2018/17 Dt.20.12.2018.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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