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Strategic Placement of Nuclear Power Plants in Pakistan: A Complex Polytopic Fuzzy Model Approach with Confidence Level Assessment

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Abstract: Confidence sets provide a robust method for addressing the uncertainty inherent in the membership degrees of elements within fuzzy sets (FSs). These sets enhance the capability of FSs to manage imprecise or uncertain data systematically. Analogous to repeated experimentation, the interpretation of confidence sets remains valid before sample observation. However, once the sample is examined, all confidence sets exclusively encompass parameter values of either 1 or 0. This study introduces novel techniques in the domain of confidence levels, specifically the Confidence Complex Polytopic Fuzzy Weighted Averaging (CCPoFWA) operator, confidence complex polytopic fuzzy ordered weighted averaging (CCPoFOWA) operator, and Confidence Complex Polytopic Fuzzy Hybrid Averaging (CCPoFHA) operator. These aggregation operators are indispensable tools in data analysis and decision-making, aiding in the understanding of complex systems across diverse fields. They facilitate the extraction of valuable insights from extensive datasets and streamline the presentation of information to enhance decision support. The efficacy and utility of the proposed methods are demonstrated through a detailed illustrative example, underscoring their potential in strategic decision-making for the placement of nuclear power plants in Pakistan.

Keywords: CCPoFWA operator; CCPoFOWA operator; CCPoFHA operator; Decision-making process; Nuclear power plant placement; Fuzzy sets; Confidence levels

1 Introduction

The FS theory of Zadeh [1] is fundamental to fuzzy logic and mathematics, extending classical set concepts to address uncertainty and vagueness by allowing elements to have varying degrees of membership. This flexibility makes FSs invaluable across fields like computer science, finance, management, healthcare, engineering, and environmental management. FS theory effectively models complex, real-world situations where binary logic is inadequate. However, it falls short when dealing with data that includes both unsatisfactory and satisfactory information. To address these limitations, Atanassov [2] introduced intuitionistic FSs (IFSs). IFSs offer a more comprehensive framework for managing complexities, enabling a better representation of ambiguity and hesitation. In IFSs, each element is represented as a pair (μ, v) with $\mu + v \leq 1$. Later on, Yager [3] introduced Pythagorean FSs (PyFSs) to overcome the limitations of IFSs such as $\mu + v \leq 1$ to $\mu^2 + v^2 \leq 1$, aiming to better manage uncertainty and hesitation. These extensions enhance the capabilities of IFS theory.

The existing models often fail to deliver the expected results, especially in real-world scenarios where calculating the neutral membership degree is critical. This limitation highlights the necessity for more advanced models capable of accurately managing these complexities. Consequently, there is a growing demand for improved solutions that can address these challenges effectively. Cuong and Kreinovich [4] introduced picture FSs (PcFSs) to address existing challenges innovatively. In PcFSs, each element is represented as (μ, η, v), where the sum $\mu + \eta + v \leq 1$. This framework provides a unique approach to representing uncertainty and ambiguity. Ashraf et al. [5] introduced spherical FSs (SpFSs) to address limitations in PcFSs, changing the constraint from $\mu+\eta+v \leq 1$ to $\mu^2+\eta^2+v^2 \leq 1$. This advancement offers a more robust framework for managing uncertainty and imprecision in complex decision-making scenarios. Their recent work demonstrates the improved applicability of SpFSs in such contexts. Beg et al. [6] introduced polytopic FSs (PoFSs) to address the limitations of earlier models like PcFSs and SpFSs, which were constrained by conditions such as $\mu + \eta + v \leq 1$ to $\mu^2 + \eta^2 + v^2 \leq 1$. By extending these conditions to $\mu^q + \eta^q + v^q \leq 1$, PoFSs provide an improved framework for managing uncertainty and imprecision in complex decision-making scenarios. This development enhances the ability to handle more intricate and uncertain data effectively.

Traditional FS theories and their extensions struggle with partial ignorance and temporal changes in data, making them unsuitable for periodic information. They fall short in handling the ambiguity, vagueness, and periodic variations in complex datasets, such as those found in facial recognition, image analysis, health analysis, biometric database analysis, and audio. Thus, a more advanced approach is required to manage and interpret these complex and incomplete datasets effectively. Ramot et al. [7] proposed complex FSs (CFSs), which enhance traditional FSs by incorporating complex-valued membership grades. This extension allows for more effective handling of multidimensional data. As a result, CFSs provide a richer representation of uncertainty and ambiguity in complex systems and data. This advancement aimed to enrich the modeling of complex systems where conventional FSs may fall short, offering a more nuanced approach to handling imprecision in mathematical contexts. CFSs introduce a higher level of flexibility, accommodating diverse degrees of uncertainty within mathematical frameworks. While FSs enable the modeling of imprecision through membership degrees, CFSs take this a step further by allowing membership values to be complex numbers. In a CFS, instead of simple real numbers, the membership function assigns a complex number to each element in the universal set. This complex number comprises a real part and an imaginary part, reflecting both the degree of membership and the phase or angle of membership for that specific element. This extension enables the modeling of scenarios where uncertainty not only involves the degree of membership but also the orientation or phase of that membership. They offer a richer representation of fuzzy information, particularly in situations where both magnitude and phase information are crucial. Consequently, CFSs serve as a valuable tool for addressing the limitations of traditional FSs in scenarios where standard fuzzy modeling may be inadequate. Alkouri and Salleh [8] introduced a novel concept called complex intuitionistic FSs (CIFSs). These sets combine the principles of complex numbers with IFSs [9]. This combination allows for a more sophisticated representation of uncertainty, imprecision, and hesitation encountered in decision-making processes. Ma et al. [10], Kumer and Bajej [11], Dick et al. [12], Rani and Garg [13], Liu and Zhang [14], and Garg and Reni [15] contributed additional research in the field, specifically focusing on CIFSs. Complex Pythagorean FSs (CPyFSs), introduced by Ullah et al. [16], expand upon traditional FS theory by incorporating additional parameters to handle more intricate forms of uncertainty in decision-making. In CPyFSs, elements are not only associated with membership degrees but also with degrees of non-membership and hesitancy. This comprehensive representation allows for simultaneous consideration of inclusion, exclusion, and uncertainty aspects. The degree of hesitancy captures the ambiguity or lack of confidence in the decision-making process. CPyFSs excel in scenarios with conflicting information or high uncertainty. They offer a robust framework with mathematical properties conducive to the development of efficient computational methods. Overall, CPyFSs provide a valuable tool for modeling and navigating complex decision-making problems (DMPs) in uncertain environments. Hezam et al. [17], Rahman and Iqbal [18], Rahman et al. [19], Rahman et al. [20] and Liu et al. [21] collectively introduced a variety of aggregation operators aimed at enhancing decision-making processes. These operators were designed to aggregate various inputs or criteria in decision-making scenarios, providing flexibility and adaptability to different contexts. These operators were applied across different decision-making processes, likely aiming to optimize outcomes or improve the efficiency of decision-making systems. Rahman [22] introduced complex polytopic FSs (CPoFSs) as an advancement of PoFSs, presenting a flexible framework for handling complex data and uncertainty in multiple domains. CPoFSs represent a pivotal concept in fuzzy logic and associated fields, offering adept management of intricate information and uncertainty. Their versatility makes them invaluable for analysis and decision-making across diverse contexts. With CPoFSs, practitioners can effectively navigate complex datasets and make informed decisions even in uncertain environments. This extension enriches the capabilities of FSs, enabling their application in various domains where traditional FSs may fall short.

Building on the strengths of the aforementioned models and their associated methods, this paper introduces several novel operators, including the CCPoFWA, CCPoFOWA, and CCPoFHA. These operators are designed to enhance decision-making processes by incorporating complex preference structures and aggregation techniques. To demonstrate their practical applicability and effectiveness, we conclude the paper with a detailed descriptive example. This example illustrates the superiority of these new operators in handling real-world scenarios, validating their efficacy and potential for broader application.

The subsequent sections of this study are organized as follows: Section 2 provides a comprehensive overview of current models relevant to this study, mapping out the existing framework and highlighting opportunities for enhancement. It aims to present the current state of the field and pinpoint potential areas for development. Section 3 introduces three innovative operators: CCPoFWA, CCPoFOWA, and CCPoFHA. These operators are new methodologies aimed at enhancing decision-making processes. Each operator likely brings unique advantages to address specific aspects of decision analysis. In Section 4, the study presents a decision-making algorithm that organizes the methodologies from Section 3 into a structured approach. This algorithm provides clear, step-by-step instructions for practitioners to effectively apply the discussed operators. It aims to facilitate the practical implementation of these methodologies in real-world scenarios. Section 5 probably builds on the ideas introduced earlier, providing additional depth through a new illustrative example. This section aims to clarify and expand upon the previous concepts. The example likely offers practical insights, making the theoretical aspects more relatable and understandable. Section 6 encapsulates the study's pivotal findings, highlighting their relevance and impact within the broader research context. The conclusion underscores the implications of these insights and suggests avenues for future exploration, encouraging further investigation to build on the foundation established by this research.

2 Preliminaries

CPoFSs were explored in this section, focusing on their score function and fundamental operational laws. CPoFSs offer a versatile framework for managing complex, multivariate, and uncertain information, valuable in various fields where these traits overlap. Score functions are essential for quantitatively evaluating the performance and effectiveness of fuzzy systems, helping to measure how well they handle uncertain or vague data. Operational laws provide a robust framework for navigating uncertainty and modeling intricate real-world scenarios, enhancing the understanding and management of complex systems.

Definition 1 [22]: Let C be a CPoFS and U be a universal set, then C can be defined mathematically on U as follows:

$$C = \left\{ \left\langle u, \mu(u)e^{ix(u)}, \eta(u)e^{iy(v)}, \nu(u)e^{iz(u)} \right\rangle \middle| u \in U \right\} \tag{1}$$

where, $\mu(u) : U \to [0,1], \eta(u) : U \to [0,1]$, and $v(u) : U \to [0,1]$ are the amplitude terms, with $(\mu(u))^q + (\eta(u))^q + (v(u))^q \leq 1; x(u) \in [0, 2\pi], y(u) \in [0, 2\pi]$, and $z(u) \in [0, 2\pi]$ are the phase terms, with conditions of $0 \prec \left(\frac{x(u)}{2\pi}\right)^q + \left(\frac{y(u)}{2\pi}\right)^q + \left(\frac{z(u)}{2\pi}\right)^q \leq 1$, and $i = \sqrt{-1}$ in a unit circle. Furthermore, its hesitancy or indeterminacy can be presented as $\pi_C = (1 - (\mu^q + \eta^q + v^q))^{\frac{1}{q}} e^{(1-(x^q+y^q+z^q))^{\frac{1}{q}}}$ of the element ℓ for all $u \in U$.

Definition 2 [22]: Let $\alpha_j = \left(\mu_j e^{ix_j}, \eta_j e^{iy_j}, v_j e^{iz_j}\right) (1 \leq j \leq 2)$ be a family of complex polytopic fuzzy numbers (CPoFNs) and a real number $\hbar$, with $\hbar \succ 0$, then the following conditions hold:

i) $\alpha_1 \oplus \alpha_2 = \left(\begin{array}{c} (\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q)^{\frac{1}{q}} \operatorname{expi} 2\pi \left(\left(\left(\frac{x_1}{2\pi}\right)^q + \left(\frac{x_2}{2\pi}\right)^q - \left(\frac{x_1}{2\pi}\right)^q \left(\frac{x_2}{2\pi}\right)^q \right)^{\frac{1}{q}} \right), \\ (\eta_1 \eta_2) \operatorname{expi} 2\pi \left(\left(\frac{y_1}{2\pi}\right) \left(\frac{y_2}{2\pi}\right) \right), (v_1 v_2) \operatorname{expi} 2\pi \left(\left(\frac{z_1}{2\pi}\right) \left(\frac{z_2}{2\pi}\right) \right) \end{array} \right)$

ii) $\hbar(\alpha) = \left(\left(1 - (1 - \mu^q)^{\hbar}\right)^{\frac{1}{q}} \exp i2\pi \left(1 - \left(1 - \left(\frac{x}{2\pi}\right)^q\right)^{\hbar}\right)^{\frac{1}{q}}, (\eta)^{\hbar} \exp i2\pi \left(\frac{y}{2\pi}\right)^{\hbar}, (v)^{\hbar} \exp i2\pi \left(\frac{z}{2\pi}\right)^{\hbar} \right)$

Definition 3 [22]: Let $\alpha = \left(\mu e^{ix}, \eta e^{iy}, v e^{iz}\right)$ be a CPoFN, then score and accuracy functions can be defined as follows:

Score function: $\operatorname{scor}(\alpha) = \frac{1}{3} [(1 + \mu^q + \mu^q - v^q) + (1 + x^q + y^q - z^q)]$, with $s(\alpha) \in [-2, 2]$

Accuracy function: $A(\alpha) = \frac{1}{2} [(1 + \max(\mu^q, \eta^q) - v^q) + (1 + \max(x^q, y^q) - z^q)]$, with $A(\alpha) \in [0, 2]$

Definition 4 [22]: Let $\alpha_j = \left(\mu_j e^{ix_j}, \eta_j e^{iy_j}, v_j e^{iz_j}\right) (1 \leq j \leq 2)$ be a family of CPoFNs, then

i) If $scor(\alpha_1) \succ scor(\alpha_2)$, this implies that $\alpha_1 \succ \alpha_2$.

ii) If $scor(\alpha_1) \prec scor(\alpha_2)$, this implies that $\alpha_1 \prec \alpha_2$.

iii) If $scor(\alpha_1) = scor(\alpha_2)$, then there are three conditions as follows:

a) If $A(\alpha_1) \succ A(\alpha_2)$, this implies that $\alpha_1 \succ \alpha_2$.

b) If $A(\alpha_1) \prec A(\alpha_2)$, this implies that $\alpha_1 \prec \alpha_2$.

c) If $A(\alpha_1) = A(\alpha_2)$, this implies that $\alpha_1 = \alpha_2$.

3 Complex Polytopic Fuzzy Techniques Under Confidence Level

The introduction of innovative aggregation operators such as CCPoFWA, CCPoFOWA, and CCPoFHA marks a significant advancement in decision-making processes across various domains. These operators play a crucial role in refining analytical frameworks and enabling effective strategies. In data analysis, they are instrumental in condensing large datasets into meaningful summaries, thereby facilitating easier interpretation and comprehension of complex information. Moreover, in decision-making scenarios, these operators aid in synthesizing multiple criteria or attributes to reach optimal choices, ensuring comprehensive consideration of all relevant factors. Furthermore, in optimization tasks, aggregation operators streamline problem-solving processes by effectively aggregating objectives or constraints. Their versatility makes them indispensable tools for enhancing efficiency and effectiveness across diverse domains, offering valuable insights and aiding in informed decision-making. Through their application, organizations can navigate complexities with greater clarity, ultimately driving better outcomes and achieving strategic objectives.

Definition 5: Let $\alpha_j = \left(\mu_j e^{ix_j}, \eta_j e^{iy_j}, v_j e^{iz_j}\right)$ $(1 \le j \le n)$ be a finite family of CPoFNs, with their weighted vector $w = (w_1, w_2, \ldots, w_n)$ and their confidence level $\lambda_j (1 \le j \le n)$ under conditions: $w_j \in [0,1], \sum_{j=1}^{n} w_j = 1$ and $\lambda_j \in [0,1]$, respectively. Then the CCPoFWA operator can be expressed mathematically as follows:

$$\begin{aligned}&\mathrm{CCPoFWA}_\omega\left((\alpha_1,\lambda_1),(\alpha_2,\lambda_2),\ldots,(\alpha_n,\lambda_n)\right)\\ &= \left(\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} \exp i2\pi \left(1-\prod_{j=1}^{n}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}},\\ \prod_{j=1}^{n}\left(\eta_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{n}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{n}\left(v_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{n}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j}\end{array}\right)\end{aligned}$$

Example 1: An example was constructed to improve Definition 5 by considering the following four values with weights $\omega = (0.1, 0.2, 0.3, 0.4)$ and $q = 3$.

$$\begin{aligned}\alpha_1 &= \left\langle \left(0.60e^{i2\pi(0.80)}, 0.40e^{i2\pi(0.60)}, 0.80e^{i2\pi(0.60)}\right), 0.50\right\rangle\\ \alpha_2 &= \left\langle \left(0.50e^{i2\pi(0.70)}, 0.70e^{i2\pi(0.40)}, 0.40e^{i2\pi(0.70)}\right), 0.40\right\rangle\\ \alpha_3 &= \left\langle \left(0.80e^{i2\pi(0.40)}, 0.50e^{i2\pi(0.50)}, 0.70e^{i2\pi(0.50)}\right), 0.60\right\rangle\\ \alpha_4 &= \left\langle \left(0.70e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.70)}, 0.50e^{i2\pi(0.80)}\right), 0.30\right\rangle\end{aligned}$$

Before proceeding, the necessary figures were determined as follows:

$$\begin{aligned}&\left(1-\prod_{j=1}^{4}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} = \left(\begin{array}{c}1-\left(1-(0.60)^3\right)^{0.50\times0.1}\left(1-(0.50)^3\right)^{0.40\times0.2}\\ \left(1-(0.80)^3\right)^{0.60\times0.3}\left(1-(0.70)^3\right)^{0.30\times0.4}\end{array}\right)^{\frac{1}{3}} = 0.50\\ &\left(1-\prod_{j=1}^{4}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} = \left(\begin{array}{c}1-\left(1-(0.80)^3\right)^{0.50\times0.1}\left(1-(0.70)^3\right)^{0.40\times0.2}\\ \left(1-(0.40)^3\right)^{0.60\times0.3}\left(1-(0.60)^3\right)^{0.30\times0.4}\end{array}\right)^{\frac{1}{3}} = 0.47\\ &\prod_{j=1}^{4}\left(\eta_j\right)^{\lambda_j\omega_j} = (0.40)^{0.50\times0.1}(0.70)^{0.60\times0.2}(0.50)^{0.60\times0.3}(0.60)^{0.30\times0.4} = 0.77\\ &\prod_{j=1}^{4}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j} = (0.60)^{0.50\times0.1}(0.40)^{0.60\times0.2}(0.50)^{0.60\times0.3}(0.70)^{0.30\times0.4} = 0.76\\ &\prod_{j=1}^{4}\left(v_j\right)^{\lambda_j\omega_j} = (0.80)^{0.50\times0.1}(0.40)^{0.60\times0.2}(0.70)^{0.60\times0.3}(0.50)^{0.30\times0.4} = 0.70\\ &\prod_{j=1}^{4}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j} = (0.60)^{0.50\times0.1}(0.70)^{0.60\times0.2}(0.50)^{0.60\times0.3}(0.80)^{0.30\times0.4} = 0.81\end{aligned}$$

The following was obtained using the CCPoFWA operator:

$$\begin{aligned}&\mathrm{CCPoFWA}_\omega\left((\alpha_1,\lambda_1),(\alpha_2,\lambda_2),(\alpha_3,\lambda_3),(\alpha_4,\lambda_4)\right)\\ &= \left(\begin{array}{c}\left(1-\prod_{j=1}^{4}\left(1-\mu_j^q\right)^{\lambda_{j}\omega_j}\right)^{\frac{1}{q}} \exp i2\pi \left(1-\prod_{j=1}^{4}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}},\\ \prod_{j=1}^{4}\left(\eta_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{4}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{4}\left(v_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{4}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j}\end{array}\right)\\ &= (0.50\exp i2\pi(0.47), 0.77\exp i2\pi(0.76), 0.70\exp i2\pi(0.81))\end{aligned}$$

Theorem 1: Let $\alpha_j = \left(\mu_j e^{ix_j}, \eta_j e^{iy_j}, v_j e^{iz_j}\right)$ $(1 \le j \le n)$ be a family of CPoFNs. The application of the CCPoFWA operator still yields a CPoFN as a result.

$$\begin{aligned}&\mathrm{CCPoFWA}_\omega\left((\alpha_1,\lambda_1),(\alpha_2,\lambda_2),\ldots,(\alpha_n,\lambda_n)\right)\\ &= \left(\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} \exp i2\pi \left(1-\prod_{j=1}^{n}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}},\\ \prod_{j=1}^{n}\left(\eta_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{n}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{n}\left(v_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{n}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j}\end{array}\right)\end{aligned} \tag{2}$$

Proof: The following was obtained by considering $n = 2$:

$$\lambda_1\omega_1\left(\alpha_1\right) = \left(\left(1-\left(1-\mu_1^q\right)^{\lambda_1\omega_1}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\left(1-\left(\frac{x_1}{2\pi}\right)^q\right)^{\lambda_1\omega_1}\right)^{\frac{1}{q}}}, \left(\eta_1\right)^{\lambda_1\omega_1} e^{i2\pi\left(\frac{y_1}{2\pi}\right)^{\lambda_1\omega_1}}, \left(v_1\right)^{\lambda_1\omega_1} e^{i2\pi\left(\frac{z_1}{2\pi}\right)^{\lambda_1\omega_1}}\right)$$

$$\lambda_2\omega_2\left(\alpha_2\right) = \left(\left(1-\left(1-\mu_2^q\right)^{\lambda_2\omega_2}\right)^{\frac{1}{q}} e^{i2\pi\left(1-\left(1-\left(\frac{x_2}{2\pi}\right)^q\right)^{\lambda_2\omega_2}\right)^{\frac{1}{q}}}, \left(\eta_2\right)^{\lambda_2\omega_2} e^{i2\pi\left(\frac{y_2}{2\pi}\right)^{\lambda_2\omega_2}}, \left(v_2\right)^{\lambda_2\omega_2} e^{i2\pi\left(\frac{z_2}{2\pi}\right)^{\lambda_2\omega_2}}\right)$$

By Definition 5, the following was obtained:

$$\begin{aligned}&\mathrm{CCPoFWA}_\omega\left((\alpha_1,\lambda_1),(\alpha_2,\lambda_2)\right)\\ &= \left(\begin{array}{c}\left(1-\prod_{j=1}^{2}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} \exp i2\pi \left(1-\prod_{j=1}^{2}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}},\\ \prod_{j=1}^{2}\left(\eta_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{2}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{2}\left(v_j\right)^{\lambda_j\omega_j} \exp i2\pi \prod_{j=1}^{2}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j}\end{array}\right)\end{aligned}$$

Then the following was obtained if Eq. (2) is true for $n = k$:

$$\begin{aligned}
&\text{CCPoFWA}_\omega\left(\left(\alpha_1, \lambda_1\right), \left(\alpha_2, \lambda_2\right) \ldots, \left(\alpha_k, \lambda_k\right)\right) \\
&= \begin{pmatrix} \left(1-\prod_{j=1}^{k}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} \exp i2\pi\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}, \\ \prod_{j=1}^{k}\left(\eta_j\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{k}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{k}\left(v_j\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{k}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j} \end{pmatrix}
\end{aligned}$$

If Eq. (2) is true for $n = k$, then it was proved that it is true for $n = k + 1$.

$$\begin{aligned}
&\text{CCPoFWA}_\omega\left(\left(\alpha_1, \lambda_1\right), \left(\alpha_2, \lambda_2\right) \ldots, \left(\alpha_k, \lambda_k\right), \left(\alpha_{k+1}, \lambda_{k+1}\right)\right) \\
&= \begin{pmatrix} \left(1-\prod_{j=1}^{k}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} \exp i2\pi\left(1-\prod_{j=1}^{k}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}, \\ \prod_{j=1}^{k}\left(\eta_j\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{k}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{k}\left(v_j\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{k}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j} \end{pmatrix} \\
&\oplus \begin{pmatrix} \left(1-\left(1-\mu_{k+1}^q\right)^{\lambda_{k+1}\omega_{k+1}}\right)^{\frac{1}{q}} \exp i2\pi\left(1-\left(1-\left(\frac{x_{k+1}}{2\pi}\right)^q\right)^{\lambda_{k+1}\omega_{k+1}}\right)^{\frac{1}{q}}, \\ \left(\eta_{k+1}\right)^{\lambda_{k+1}\omega_{k+1}} \exp i2\pi\left(\frac{y_{k+1}}{2\pi}\right)^{\lambda_{k+1}\omega_{k+1}}, \left(v_{k+1}\right)^{\lambda_{k+1}\omega_{k+1}} \exp^{i2\pi\left(\frac{z_{k+1}}{2\pi}\right)^{\lambda_{k+1}\omega_{k+1}}} \end{pmatrix} \\
&= \begin{pmatrix} \left(1-\prod_{j=1}^{k+1}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} \exp i2\pi\left(1-\prod_{j=1}^{k+1}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}, \\ \prod_{j=1}^{k+1}\left(\eta_j\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{k+1}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{k+1}\left(v_j\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{k+1}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j} \end{pmatrix}
\end{aligned}$$

Therefore, $n = k + 1$ also holds true. Thus, Eq. (2) is valid for n. The proof was successfully completed.

Definition 6: Let $\alpha_j = \left(\mu_j e^{ix_j}, \eta_j e^{iy_j}, v_j e^{iz_j}\right)$ $(1 \leq j \leq n)$ be a finite family of CPoFNs, $w = (w_1, w_2, \ldots, w_n)$ be their weighted vector, and $\lambda_j (1 \leq j \leq n)$ be their confidence level with conditions: $w_j \in [0, 1]$, $\sum_{j=1}^{n} w_j = 1$ and $\lambda_j \in [0, 1]$, respectively, then the CCPoFOWA operator can be expressed mathematically as follows:

$$\begin{aligned}
&\text{CCPoFOWA}_{\ \omega}\left(\left(\alpha_1, \lambda_1\right), \left(\alpha_2, \lambda_2\right), \ldots, \left(\alpha_n, \lambda_n\right)\right) \\
&= \begin{pmatrix} \left(1-\prod_{j=1}^{n}\left(1-\mu_{\sigma(j)}^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}} \exp i2\pi\left(1-\prod_{j=1}^{n}\left(1-\left(\frac{x_{\sigma(j)}}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}, \\ \prod_{j=1}^{n}\left(n_{\sigma(j)}\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{n}\left(\frac{y_{\sigma(j)}}{2\pi}\right)^{\lambda_j\omega_j}, \prod_{j=1}^{n}\left(v_{\sigma(j)}\right)^{\lambda_j\omega_j} \exp i2\pi\prod_{j=1}^{n}\left(\frac{z_{\sigma(j)}}{2\pi}\right)^{\lambda_j\omega_j} \end{pmatrix}
\end{aligned}$$

where, $\alpha_{\sigma(j-1)} \succ \alpha_{\sigma(j)}$ under the condition of $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is any rearrangement of the given numbers, such as $(1, 2, \ldots, n)$.

Example 2: An example was supposed to develop the above novel approach by considering the following four CPoFNs, with $v = (0.10, 0.20, 0.30, 0.40)^T$ and $q = 4$.

$$\begin{aligned}
\alpha_1 &= \left\langle\left(0.50e^{i2\pi(0.40)}, 0.80e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.70)}\right), 0.7\right\rangle \\
\alpha_2 &= \left\langle\left(0.70e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.50)}, 0.50e^{i2\pi(0.70)}\right), 0.5\right\rangle \\
\alpha_3 &= \left\langle\left(0.80e^{i2\pi(0.50)}, 0.60e^{i2\pi(0.40)}, 0.50e^{i2\pi(0.80)}\right), 0.4\right\rangle \\
\alpha_4 &= \left\langle\left(0.70e^{i2\pi(0.50)}, 0.60e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.60)}\right), 0.6\right\rangle
\end{aligned}$$

The scores were computed for further processing using Definition 3:

$$\begin{aligned}
\text{scor}\left(\alpha_1\right) &= \frac{1}{3}\left[\left(1+(0.50)^4+(0.80)^4-(0.60)^4\right)+\left(1+(0.40)^4+(0.60)^4-(0.70)^4\right)\right] = 0.75 \\
\text{scor}\left(\alpha_2\right) &= \frac{1}{3}\left[\left(1+(0.70)^4+(0.60)^4-(0.40)^4\right)+\left(1+(0.60)^4+(0.50)^4-(0.70)^4\right)\right] = 0.76 \\
\text{scor}\left(\alpha_3\right) &= \frac{1}{3}\left[\left(1+(0.80)^4+(0.60)^4-(0.50)^4\right)+\left(1+(0.50)^4+(0.40)^4-(0.80)^4\right)\right] = 0.71 \\
\text{scor}\left(\alpha_4\right) &= \frac{1}{3}\left[\left(1+(0.70)^4+(0.60)^4-(0.60)^4\right)+\left(1+(0.50)^4+(0.60)^4-(0.60)^4\right)\right] = 0.77
\end{aligned}$$

Thus, the following was obtained with the help of score functions:

$$\begin{aligned}
\alpha_{\sigma(1)} &= \left\langle\left(0.70e^{i2\pi(0.50)}, 0.60e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.60)}\right), 0.6\right\rangle \\
\alpha_{\sigma(2)} &= \left\langle\left(0.70e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.50)}, 0.40e^{i2\pi(0.70)}\right), 0.5\right\rangle \\
\alpha_{\sigma(3)} &= \left\langle\left(0.50e^{i2\pi(0.40)}, 0.80e^{i2\pi(0.60)}, 0.60e^{i2\pi(0.70)}\right), 0.7\right\rangle \\
\alpha_{\sigma(4)} &= \left\langle\left(0.80e^{i2\pi(0.50)}, 0.60e^{i2\pi(0.40)}, 0.50e^{i2\pi(0.80)}\right), 0.4\right\rangle
\end{aligned}$$

Then the following required values were found:

$$\left(1-\prod_{j=1}^{4}\left(1-\mu_j^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}=\left(\begin{array}{c}1-\left(1-(0.70)^4\right)^{0.6\times0.1}\left(1-(0.70)^4\right)^{0.5\times0.2}\\ \left(1-(0.50)^4\right)^{0.7\times0.3}\left(1-(0.80)^4\right)^{0.3\times0.4}\end{array}\right)^{\frac{1}{4}}=0.60$$

$$\left(1-\prod_{j=1}^{4}\left(1-\left(\frac{x_j}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}=\left(\begin{array}{c}1-\left(1-(0.50)^4\right)^{0.6\times0.1}\left(1-(0.60)^4\right)^{0.5\times0.2}\\ \left(1-(0.40)^4\right)^{0.7\times0.3}\left(1-(0.50)^4\right)^{0.3\times0.4}\end{array}\right)^{\frac{1}{4}}=0.42$$

$$\prod_{j=1}^{4}(\eta_j)^{\lambda_j\omega_j}=(0.60)^{0.6\times0.1}(0.60)^{0.5\times0.2}(0.80)^{0.7\times0.3}(0.60)^{0.4\times0.4}=0.81$$

$$\prod_{j=1}^{4}\left(\frac{y_j}{2\pi}\right)^{\lambda_j\omega_j}=(0.60)^{0.6\times0.1}(0.50)^{0.5\times0.2}(0.60)^{0.7\times0.3}(0.40)^{0.4\times0.4}=0.70$$

$$\prod_{j=1}^{4}(v_j)^{\lambda_j\omega_j}=(0.60)^{0.6\times0.1}(0.40)^{0.5\times0.2}(0.60)^{0.7\times0.3}(0.50)^{0.4\times0.4}=0.71$$

$$\prod_{j=1}^{4}\left(\frac{z_j}{2\pi}\right)^{\lambda_j\omega_j}=(0.60)^{0.6\times0.1}(0.70)^{0.5\times0.2}(0.70)^{0.7\times0.3}(0.80)^{0.4\times0.4}=0.83$$

The following was obtained using the CCPoFOWA operator:

$$\begin{aligned}&\mathrm{CCPoFWA}_{\omega}\left((\alpha_1,\lambda_1),(\alpha_2,\lambda_2),(\alpha_3,\lambda_3),(\alpha_4,\lambda_4)\right)\\&=\left(\begin{array}{c}\left(1-\prod_{j=1}^{4}\left(1-\mu_{\sigma(j)}^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}\exp i2\pi\left(1-\prod_{j=1}^{4}\left(1-\left(\frac{x_{\sigma(j)}}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}},\\ \prod_{j=1}^{4}\left(\eta_{\sigma(j)}\right)^{\lambda_j\omega_j}\exp i2\pi\prod_{j=1}^{4}\left(\frac{y_{\sigma(j)}}{2\pi}\right)^{\lambda_j\omega_j},\prod_{j=1}^{4}\left(v_{\sigma(j)}\right)^{\lambda_j\omega_j}\exp i2\pi\prod_{j=1}^{4}\left(\frac{z_{\sigma(j)}}{2\pi}\right)^{\lambda_j\omega_j}\end{array}\right)\\&=\left(0.60e^{i2\pi(0.42)},0.81e^{i2\pi(0.70)},0.71e^{i2\pi(0.83)}\right)\end{aligned}$$

The CCPoFWA operator focuses on weighting the complex polytopic fuzzy values (CPoFVs) themselves, while the CCPoFOWA operator weights the ordered positions of these values within a certain confidence level. To overcome the limitations of these methods, the confidence complex polytopic fuzzy hybrid averaging aggregation (CCPoFHAA) operator was developed. This new operator simultaneously considers both the CPoFVs and their ordered positions, offering a more comprehensive and balanced weighting mechanism, thus improving decision-making processes with complex polytopic fuzzy data. Hybrid aggregation operators provide increased flexibility and robustness by merging different techniques, enabling them to manage complex and uncertain data more effectively. By integrating multiple criteria and accommodating various data types, these operators enhance decision-making accuracy. Their adaptability is particularly valuable in fields that demand detailed and comprehensive assessments, improving the overall quality and reliability of decisions.

Definition 7: Let $\alpha_j=\left(\mu_j e^{ix_j},\eta_j e^{iy_j},\nu_j e^{iz_j}\right)(1\leq j\leq n)$ be a finite family of CPoFNs. Let their associated vector and weighted vector be $\varpi=(\varpi_1,\varpi_2,...,\varpi_n)$ and $\omega=(\omega_1,\omega_2,...,\omega_n)^T$, respectively, and both vectors satisfy the conditions of belonging to the closed interval [0,1] with a sum equal to one. Furthermore, $\dot{\alpha}_{\sigma(j)}=n\varpi_j(\alpha_j)$, where $\dot{\alpha}_{\sigma(j)}$ is the maximum value in the equation and is determined by the parameter n, which represents a positive number known as the balancing coefficient. This coefficient plays a crucial role in ensuring a balanced and effective equation by influencing the overall outcome and magnitude of the maximum value. If $\varpi=(\varpi_1,\varpi_2,...,\varpi_n)$ tends to $\left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)$, then $(n\varpi_1\alpha_1,n\varpi_2\alpha_2,...,n\varpi_n\alpha_n)$ approaches $(\alpha_1,\alpha_2,...,\alpha_n)$. Let $_j$ $(1\leq j\leq n)$ be their confidence level with $\ell\in[0,1]$, then the CCPoFHA operator can be mathematically expressed as follows:

$$\begin{aligned}&\mathrm{CCPoFWA}_{\varpi,\omega}\left((\alpha_1,\lambda_1),(\alpha_2,\lambda_2),\ldots,(\alpha_n,\lambda_n)\right)\\&=\left(\begin{array}{c}\left(1-\prod_{j=1}^{n}\left(1-\dot{\mu}_{\sigma(j)}^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}}\exp i2\pi\left(1-\prod_{j=1}^{n}\left(1-\left(\frac{\dot{x}_{\sigma(j)}}{2\pi}\right)^q\right)^{\lambda_j\omega_j}\right)^{\frac{1}{q}},\\ \left(\prod_{j=1}^{n}\left(\dot{\eta}_{\sigma(j)}\right)^{\lambda_j\omega_j}\right)\exp i2\pi\prod_{j=1}^{n}\left(\frac{\dot{y}_{\sigma(j)}}{2\pi}\right)^{\lambda_j\omega_j},\prod_{j=1}^{n}\left(\dot{v}_{\sigma(j)}\right)^{\lambda_j\omega_j}\exp i2\pi\prod_{j=1}^{n}\left(\frac{\dot{z}_{\sigma(j)}}{2\pi}\right)^{\lambda_j\omega_j}\end{array}\right)\end{aligned}$$

4 An Application of the Proposed Approaches

In the realm of decision-making, the DMP framework stands out as a robust method for selecting optimal alternatives amidst various choices. It is a fundamental and essential aspect of human life and is also crucial in business, government, and various other fields. This is the point at which one option is chosen over the others. While personal judgment is critical, it is advisable to seek advice or input from others if necessary. CPoFSs can be used in multi-criteria group decision making (MCGDM) to evaluate and rank alternatives based on multiple, interrelated criteria. This is especially useful in situations where decision-makers need to consider complex, multidimensional factors. A decision-making process was developed using the complex polytopic fuzzy information, and its corresponding operators, namely CCPoFWA, CCPoFOWA, and CCPoFHA, to aggregate the results and select the best alternative.

Algorithm: In the multi-attribute group decision making (MAGDM) process, a group of attributes, alternatives and decision-makers are considered. A fixed group of m alternatives is presented by $\top = \{\top_1, \top_2, ..., \top_m\}$, and a finite group of n options is presented by $V = \{V_1, V_2, ..., V_n\}$, whose weighted is $\omega = (\omega_1, \omega_2, ..., \omega_n)$. Let a finite group of k decision-makers be $D = \{D_1, D_2, ..., D_k\}$, whose weight is $\varpi = (\varpi_1, \varpi_2, ..., \varpi_k)$. The major steps are as follows in Figure 1:

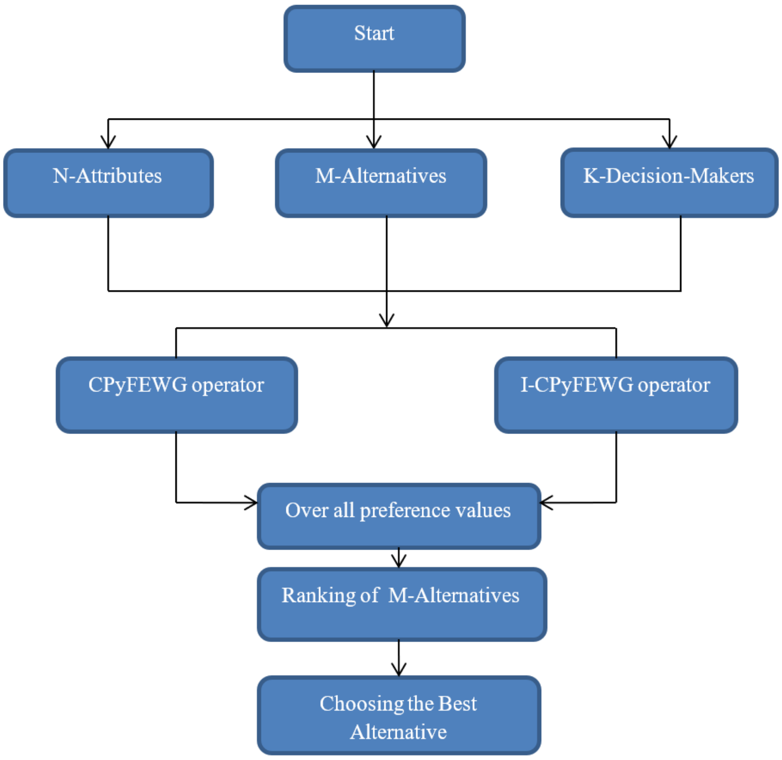

Figure 1. Flowchart

Step 1: This step involves gathering all relevant data pertaining to each alternative within the proposed criteria. This data was provided by k experts and was structured within a complex polytopic fuzzy environment, reflecting the multifaceted nature of the decision-making context.

$$D_l(1 \leq l \leq k) = \begin{array}{c} \top_1 \\ \top_2 \\ . \\ . \\ . \\ \top_m \end{array} \begin{bmatrix} \alpha_{11}^k & \alpha_{12}^k & \alpha_{13}^k & \cdots & \alpha_{1n}^k \\ \alpha_{21}^k & \alpha_{22}^k & \alpha_{23}^k & \cdots & \alpha_{2n}^k \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \alpha_{m1}^k & \alpha_{m2}^k & \alpha_{m3}^k & \cdots & \alpha_{mn}^k \end{bmatrix}$$

Step 2: This step involves consolidating individual decision matrices into a unified collective decision matrix using the CCPoFWA operator. Thus, all the individual complex polytopic fuzzy decision matrices $M_l \; (l = 1, 2, ..., k)$ were aggregated into a single collective decision matrix $M = (\S_{pq})_{mn}$ using the proposed method. This process enables the aggregation of various perspectives, weighting each individual's contributions appropriately to derive a comprehensive collective decision.

Step 3: In this step, the proposed techniques were applied again to compute all preference values, such as $f_i \; (i = 1, 2, ..., m)$ from the collective decision matrix. Thus, the proposed methods were utilized to derive comprehensive preference scores, integrating diverse perspectives for informed decision-making for subsequent processes.

Step 4: In this step, the score function was derived from the preference values, providing a quantitative representation of the ranking or evaluation criteria. It quantifies the relative importance or desirability of each option based on the provided preferences.

Step 5: In this step, options were prioritized based on their respective scores, selecting the one with the highest value. This ensures the selection of the most favorable choice according to the established criteria.

5 Illustrative Example

Selecting the optimal site for a nuclear power plant in Pakistan requires a comprehensive evaluation of various critical criteria to ensure the facility's safety, efficiency, and long-term sustainability. The government has identified four potential locations for further assessment:

Lahore (T_1): For Lahore, the panel should assess the availability of a suitable radioactive waste disposal facility, considering the city's economic cost of land for the plant, and the proximity to adequate water resources for cooling, prioritizing these factors in the decision-making process.

Karachi (T_2): Karachi, being a coastal city, offers significant shipping resources for efficient transportation of equipment and fuel to the proposed nuclear power plant site. However, careful consideration of radioactive waste disposal facilities and storage space for fuel remains imperative for decision-makers.

Quetta (T_3): For Quetta, the panel should assess the availability of suitable facilities for radioactive waste disposal, ensuring maximum capacity. They should also consider the economic cost of land, aiming for the least expensive option for land acquisition. Additionally, they should evaluate water availability and transportation resources to ensure the efficient operation of the nuclear power plant.

Peshawar (T_4): Peshawar's suitability for the nuclear power plant project should be assessed based on its capacity for radioactive waste disposal, economic cost of land, and availability of water resources. The panel should consider Peshawar's proximity to transportation infrastructure and its potential for providing ample storage space for fuel to ensure efficient and sustainable operation of the facility.

A panel of three decision-makers has been tasked with thoroughly examining each site based on the following criteria:

Radioactive waste disposal facility (V_1): This criterion emphasizes the availability of suitable facilities for the safe disposal of radioactive waste generated by the nuclear power plant. The preference is for a location that offers maximum capacity for waste disposal, ensuring long-term management of radioactive materials with minimal environmental impact.

Economic cost of land (V_2): The cost of acquiring land for the construction and operation of the nuclear power plant is a significant consideration.

Water availability (V_3): Adequate water resources are essential for cooling purposes in nuclear power plants. The panel prioritizes locations with maximum availability of water resources, such as proximity to rivers, lakes, or oceans, to ensure reliable and sustainable operation of the plant without imposing undue strain on local water supplies.

Storage space for fuel (V_4): The availability of adequate storage space for fuel is essential to ensuring continuous and uninterrupted operation of the nuclear power plant. The panel seeks locations that offer maximum space for fuel storage, enabling the plant to maintain sufficient fuel reserves for its energy production needs.

By thoroughly evaluating each potential site based on these criteria, the panel aims to identify the most suitable location for the establishment of the new nuclear power plant in Pakistan. This decision-making process requires careful consideration of technical feasibility, economic viability, environmental impact, and logistical considerations to ensure the successful implementation and operation of the facility. Tables 1-4 show the information of decision-makers.

Step 1: All pertinent data regarding decision-makers was organized into matrices, and diverse information was condensed into a structured format, facilitating comprehensive analysis and informed decision-making processes (shown in Tables 1-4).

Step 2: All individual matrices were combined into a unified matrix using the CCPoFWA technique, where $\varpi = (0.3, 0.2, 0.1, 0.4)$ and $q = 4$, as shown in Table 5.

This process ensures equitable accounting of individual contributions, culminating in a comprehensive matrix.

Step 3: Then the following was obtained using the CCPoFWA operator, with $\omega = (0.3, 0.3, 0.2, 0.2)$:

$$f_1 = \left(0.74e^{i2\pi(0.51)}, 0.78e^{i2\pi(0.66)}, 0.81e^{i2\pi(0.58)}\right)$$
$$f_2 = \left(0.69e^{i2\pi(0.58)}, 0.86e^{i2\pi(0.66)}, 0.64e^{i2\pi(0.70)}\right)$$
$$f_3 = \left(0.75e^{i2\pi(0.66)}, 0.79e^{i2\pi(0.77)}, 0.78e^{i2\pi(0.71)}\right)$$
$$f_4 = \left(0.74e^{i2\pi(0.62)}, 0.82e^{i2\pi(0.64)}, 0.70e^{i2\pi(0.61)}\right)$$

Table 1. Assessment of expert D_1

	$\mathbf{V}_1$	$\mathbf{V}_2$	$\mathbf{V}_3$	$\mathbf{V}_4$
T_1	$\langle (0.57e^{i2\pi(0.68)}, 0.54e^{i2\pi(0.63)}, 0.52e^{i2\pi(0.64)}), 0.6 \rangle$	$\langle (0.51e^{i2\pi(0.46)}, 0.47e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.5 \rangle$	$\langle (0.70e^{i2\pi(0.46)}, 0.67e^{i2\pi(0.45)}, 0.46e^{i2\pi(0.72)}), 0.8 \rangle$	$\langle (0.48e^{i2\pi(0.46)}, 0.44e^{i2\pi(0.45)}, 0.59e^{i2\pi(0.72)}), 0.9 \rangle$
T_2	$\langle (0.71e^{i2\pi(0.51)}, 0.55e^{i2\pi(0.46)}, 0.56e^{i2\pi(0.68)}), 0.4 \rangle$	$\langle (0.58e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.9 \rangle$	$\langle (0.66e^{i2\pi(0.46)}, 0.57e^{i2\pi(0.45)}, 0.54e^{i2\pi(0.72)}), 0.3 \rangle$	$\langle (0.77e^{i2\pi(0.60)}, 0.73e^{i2\pi(0.40)}, 0.40e^{i2\pi(0.30)}), 0.5 \rangle$
T_3	$\langle (0.66e^{i2\pi(0.60)}, 0.46e^{i2\pi(0.40)}, 0.56e^{i2\pi(0.50)}), 0.8 \rangle$	$\langle (0.44e^{i2\pi(0.46)}, 0.67e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.6 \rangle$	$\langle (0.54e^{i2\pi(0.46)}, 0.56e^{i2\pi(0.45)}, 0.36e^{i2\pi(0.72)}), 0.8 \rangle$	$\langle (0.55e^{i2\pi(0.46)}, 0.57e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.7 \rangle$
T_4	$\langle (0.63e^{i2\pi(0.51)}, 0.68e^{i2\pi(0.46)}, 0.46e^{i2\pi(0.68)}), 0.6 \rangle$	$\langle (0.64e^{i2\pi(0.46)}, 0.54e^{i2\pi(0.45)}, 0.36e^{i2\pi(0.72)}), 0.8 \rangle$	$\langle (0.48e^{i2\pi(0.46)}, 0.37e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.5 \rangle$	$\langle (0.48e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.9 \rangle$

Table 2. Assessment of expert D_2

	$\mathbf{V}_1$	$\mathbf{V}_2$	$\mathbf{V}_3$	$\mathbf{V}_4$
T_1	$\langle (0.53e^{i2\pi(0.46)}, 0.54e^{i2\pi(0.45)}, 0.46e^{i2\pi(0.72)}), 0.5 \rangle$	$\langle (0.55e^{i2\pi(0.62)}, 0.38e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.45)}), 0.5 \rangle$	$\langle (0.63e^{i2\pi(0.51)}, 0.56e^{i2\pi(0.46)}, 0.56e^{i2\pi(0.68)}), 0.5 \rangle$	$\langle (0.65e^{i2\pi(0.46)}, 0.35e^{i2\pi(0.45)}, 0.53e^{i2\pi(0.72)}), 0.4 \rangle$
T_2	$\langle (0.65e^{i2\pi(0.51)}, 0.68e^{i2\pi(0.46)}, 0.46e^{i2\pi(0.68)}), 0.6 \rangle$	$\langle (0.64e^{i2\pi(0.46)}, 0.54e^{i2\pi(0.45)}, 0.36e^{i2\pi(0.72)}), 0.8 \rangle$	$\langle (0.48e^{i2\pi(0.46)}, 0.37e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.5 \rangle$	$\langle (0.48e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.9 \rangle$
T_3	$\langle (0.65e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.53e^{i2\pi(0.72)}), 0.7 \rangle$	$\langle (0.44e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.66e^{i2\pi(0.74)}), 0.4 \rangle$	$\langle (0.67e^{i2\pi(0.46)}, 0.57e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.7 \rangle$	$\langle (0.51e^{i2\pi(0.46)}, 0.47e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.5 \rangle$
T_4	$\langle (0.48e^{i2\pi(0.46)}, 0.47e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.4 \rangle$	$\langle (0.61e^{i2\pi(0.46)}, 0.47e^{i2\pi(0.45)}, 0.53e^{i2\pi(0.72)}), 0.5 \rangle$	$\langle (0.63e^{i2\pi(0.51)}, 0.48e^{i2\pi(0.46)}, 0.46e^{i2\pi(0.68)}), 0.9 \rangle$	$\langle (0.58e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.9 \rangle$

Table 3. Assessment of expert D_3

	$\mathbf{V}_1$	$\mathbf{V}_2$	$\mathbf{V}_3$	$\mathbf{V}_4$
T_1	$\langle (0.58e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.9 \rangle$	$\langle (0.66e^{i2\pi(0.46)}, 0.57e^{i2\pi(0.45)}, 0.54e^{i2\pi(0.72)}), 0.3 \rangle$	$\langle (0.67e^{i2\pi(0.46)}, 0.57e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.7 \rangle$	$\langle (0.51e^{i2\pi(0.46)}, 0.47e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.5 \rangle$
T_2	$\langle (0.44e^{i2\pi(0.46)}, 0.67e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.6 \rangle$	$\langle (0.54e^{i2\pi(0.46)}, 0.56e^{i2\pi(0.45)}, 0.36e^{i2\pi(0.72)}), 0.8 \rangle$	$\langle (0.77e^{i2\pi(0.46)}, 0.44e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.8 \rangle$	$\langle (0.44e^{i2\pi(0.68)}, 0.55e^{i2\pi(0.63)}, 0.57e^{i2\pi(0.64)}), 0.5 \rangle$
T_3	$\langle (0.64e^{i2\pi(0.46)}, 0.54e^{i2\pi(0.45)}, 0.36e^{i2\pi(0.72)}), 0.8 \rangle$	$\langle (0.48e^{i2\pi(0.46)}, 0.37e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.5 \rangle$	$\langle (0.45e^{i2\pi(0.51)}, 0.54e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.68)}), 0.9 \rangle$	$\langle (0.58e^{i2\pi(0.46)}, 0.55e^{i2\pi(0.45)}, 0.56e^{i2\pi(0.72)}), 0.9 \rangle$
T_4	$\langle (0.55e^{i2\pi(0.62)}, 0.38e^{i2\pi(0.67)}, 0.72e^{i2\pi(0.45)}), 0.5 \rangle$	$\langle (0.63e^{i2\pi(0.51)}, 0.56e^{i2\pi(0.46)}, 0.56e^{i2\pi(0.68)}), 0.5 \rangle$	$\langle (0.65e^{i2\pi(0.46)}, 0.35e^{i2\pi(0.45)}, 0.53e^{i2\pi(0.72)}), 0.4 \rangle$	$\langle (0.63e^{i2\pi(0.46)}, 0.54e^{i2\pi(0.45)}, 0.36e^{i2\pi(0.72)}), 0.8 \rangle$

Step 4: The score functions were calculated as follows:

$$\operatorname{scor}(f_1) = \frac{1}{3}\left[\left(1 + (0.74)^4 + (0.78)^4 - (0.81)^4\right) + \left(1 + (0.51)^4 + (0.65)^4 - (0.58)^4\right)\right] = 0.79$$

$$\operatorname{scor}(f_2) = \frac{1}{3}\left[\left(1 + (0.69)^4 + (0.86)^4 - (0.64)^4\right) + \left(1 + (0.58)^4 + (0.66)^4 - (0.70)^4\right)\right] = 0.88$$

$$\operatorname{scor}(f_3) = \frac{1}{3}\left[\left(1 + (0.75)^4 + (0.79)^4 - (0.78)^4\right) + \left(1 + (0.66)^4 + (0.77)^4 - (0.71)^4\right)\right] = 0.86$$

$$\operatorname{scor}(f_4) = \frac{1}{3}\left[\left(1 + (0.74)^4 + (0.82)^4 - (0.70)^4\right) + \left(1 + (0.62)^4 + (0.64)^4 - (0.61)^4\right)\right] = 0.89$$

Table 4. Assessment of expert D_4

	$\mathbf{V}_1$	$\mathbf{V}_2$	$\mathbf{V}_3$	$\mathbf{V}_4$
T_1	$\left\langle \begin{pmatrix} 0.48e^{i2\pi(0.46)}, \\ 0.47e^{i2\pi(0.45)}, \\ 0.56e^{i2\pi(0.72)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle \begin{pmatrix} 0.61e^{i2\pi(0.46)}, \\ 0.45e^{i2\pi(0.45)}, \\ 0.53e^{i2\pi(0.72)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle \begin{pmatrix} 0.63e^{i2\pi(0.51)}, \\ 0.48e^{i2\pi(0.46)}, \\ 0.46e^{i2\pi(0.68)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle \begin{pmatrix} 0.58e^{i2\pi(0.46)}, \\ 0.55e^{i2\pi(0.45)}, \\ 0.56e^{i2\pi(0.72)} \end{pmatrix}, 0.9 \right\rangle$
T_2	$\left\langle \begin{pmatrix} 0.71e^{i2\pi(0.51)}, \\ 0.55e^{i2\pi(0.46)}, \\ 0.56e^{i2\pi(0.68)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle \begin{pmatrix} 0.58e^{i2\pi(0.46)}, \\ 0.55e^{i2\pi(0.45)}, \\ 0.56e^{i2\pi(0.72)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle \begin{pmatrix} 0.66e^{i2\pi(0.46)}, \\ 0.57e^{i2\pi(0.45)}, \\ 0.54e^{i2\pi(0.72)} \end{pmatrix}, 0.3 \right\rangle$	$\left\langle \begin{pmatrix} 0.77e^{i2\pi(0.60)}, \\ 0.73e^{i2\pi(0.40)}, \\ 0.40e^{i2\pi(0.30)} \end{pmatrix}, 0.5 \right\rangle$
T_3	$\left\langle \begin{pmatrix} 0.54e^{i2\pi(0.46)}, \\ 0.56e^{i2\pi(0.45)}, \\ 0.36e^{i2\pi(0.72)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle \begin{pmatrix} 0.44e^{i2\pi(0.46)}, \\ 0.55e^{i2\pi(0.45)}, \\ 0.66e^{i2\pi(0.74)} \end{pmatrix}, 0.4 \right\rangle$	$\left\langle \begin{pmatrix} 0.44e^{i2\pi(0.51)}, \\ 0.53e^{i2\pi(0.46)}, \\ 0.76e^{i2\pi(0.68)} \end{pmatrix}, 0.7 \right\rangle$	$\left\langle \begin{pmatrix} 0.49e^{i2\pi(0.46)}, \\ 0.59e^{i2\pi(0.45)}, \\ 0.49e^{i2\pi(0.72)} \end{pmatrix}, 0.6 \right\rangle$
T_4	$\left\langle \begin{pmatrix} 0.64e^{i2\pi(0.46)}, \\ 0.54e^{i2\pi(0.45)}, \\ 0.36e^{i2\pi(0.72)} \end{pmatrix}, 0.8 \right\rangle$	$\left\langle \begin{pmatrix} 0.48e^{i2\pi(0.46)}, \\ 0.37e^{i2\pi(0.45)}, \\ 0.56e^{i2\pi(0.72)} \end{pmatrix}, 0.5 \right\rangle$	$\left\langle \begin{pmatrix} 0.45e^{i2\pi(0.51)}, \\ 0.54e^{i2\pi(0.46)}, \\ 0.55e^{i2\pi(0.68)} \end{pmatrix}, 0.9 \right\rangle$	$\left\langle \begin{pmatrix} 0.58e^{i2\pi(0.46)}, \\ 0.55e^{i2\pi(0.45)}, \\ 0.56e^{i2\pi(0.72)} \end{pmatrix}, 0.9 \right\rangle$

Table 5. Collective assessment of all experts

	V_1	V_2	V_3	V_4
T_1	$\begin{pmatrix} 0.89e^{i2\pi(0.54)}, \\ 0.75e^{i2\pi(0.68)}, \\ 0.96e^{i2\pi(0.45)} \end{pmatrix}$	$\begin{pmatrix} 0.89e^{i2\pi(0.77)}, \\ 0.75e^{i2\pi(0.55)}, \\ 0.96e^{i2\pi(0.68)} \end{pmatrix}$	$\begin{pmatrix} 0.96e^{i2\pi(0.65)}, \\ 0.83e^{i2\pi(0.42)}, \\ 0.75e^{i2\pi(0.37)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.46)}, \\ 0.47e^{i2\pi(0.45)}, \\ 0.56e^{i2\pi(0.72)} \end{pmatrix}$
T_2	$\begin{pmatrix} 0.68e^{i2\pi(0.68)}, \\ 0.55e^{i2\pi(0.49)}, \\ 0.90e^{i2\pi(0.59)} \end{pmatrix}$	$\begin{pmatrix} 0.57e^{i2\pi(0.55)}, \\ 0.68e^{i2\pi(0.61)}, \\ 0.81e^{i2\pi(0.70)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.69)}, \\ 0.56e^{i2\pi(0.50)}, \\ 0.89e^{i2\pi(0.60)} \end{pmatrix}$	$\begin{pmatrix} 0.73e^{i2\pi(0.451)}, \\ 0.58e^{i2\pi(0.46)}, \\ 0.56e^{i2\pi(0.68)} \end{pmatrix}$
T_3	$\begin{pmatrix} 0.68e^{i2\pi(0.75)}, \\ 0.59e^{i2\pi(0.64)}, \\ 0.86e^{i2\pi(0.48)} \end{pmatrix}$	$\begin{pmatrix} 0.96e^{i2\pi(0.58)}, \\ 0.72e^{i2\pi(0.67)}, \\ 0.68e^{i2\pi(0.45)} \end{pmatrix}$	$\begin{pmatrix} 0.68e^{i2\pi(0.75)}, \\ 0.59e^{i2\pi(0.64)}, \\ 0.86e^{i2\pi(0.48)} \end{pmatrix}$	$\begin{pmatrix} 0.45e^{i2\pi(0.46)}, \\ 0.57e^{i2\pi(0.45)}, \\ 0.53e^{i2\pi(0.72)} \end{pmatrix}$
T_4	$\begin{pmatrix} 0.96e^{i2\pi(0.65)}, \\ 0.83e^{i2\pi(0.42)}, \\ 0.75e^{i2\pi(0.37)} \end{pmatrix}$	$\begin{pmatrix} 0.90e^{i2\pi(0.68)}, \\ 0.72e^{i2\pi(0.42)}, \\ 0.65e^{i2\pi(0.84)} \end{pmatrix}$	$\begin{pmatrix} 0.89e^{i2\pi(0.54)}, \\ 0.75e^{i2\pi(0.68)}, \\ 0.96e^{i2\pi(0.45)} \end{pmatrix}$	$\begin{pmatrix} 0.72e^{i2\pi(0.63)}, \\ 0.65e^{i2\pi(0.81)}, \\ 0.51e^{i2\pi(0.56)} \end{pmatrix}$

Step 5: Thus, the most suitable location is Peshawar.

6 Comparative and Sensitivity Analyses

CPoFSs are specialized extensions of traditional FSs, designed to model complex, higher-dimensional fuzzy information. These sets excel in applications requiring complex and multidimensional data handling, such as decision-making, control, and pattern recognition, using complex numbers to represent both magnitude and phase of uncertainty. In this section, the proposed model was compared with existing models like FSs, IFSs, PyFSs, CFSs, CIFSs, and CPyFSs. While FSs handle simple membership degrees, IFSs and PyFSs add non-membership and hesitation degrees without using complex numbers. CFSs incorporate complex numbers for membership but lack non-membership handling, whereas CIFSs and CPyFSs combine complex numbers with hesitation and non-membership for richer uncertainty representation. CPoFSs can be tailored for polytopic fuzzy data by setting the phase terms to zero, and for q-Rung orthopair fuzzy data by setting both the neutral and phase terms to zero. This adaptability demonstrates the flexibility and versatility of the proposed model compared to existing models. Figure 2 and Tables 6-7 illustrate how this model accommodates various types of fuzzy data, showcasing its broader applicability. By effectively managing different fuzzy data scenarios, CPoFSs enhance the precision and robustness of decision-making processes, making them superior to traditional models.

Table 6. Function scores of all methods

	CCPoFWA	**CCPoFOWA**	**CCPoFHA**
Lahore	0.79	0.82	0.77
Karachi	0.88	0.87	0.84
Quetta	0.86	0.84	0.81
Peshawar	0.89	0.91	0.88

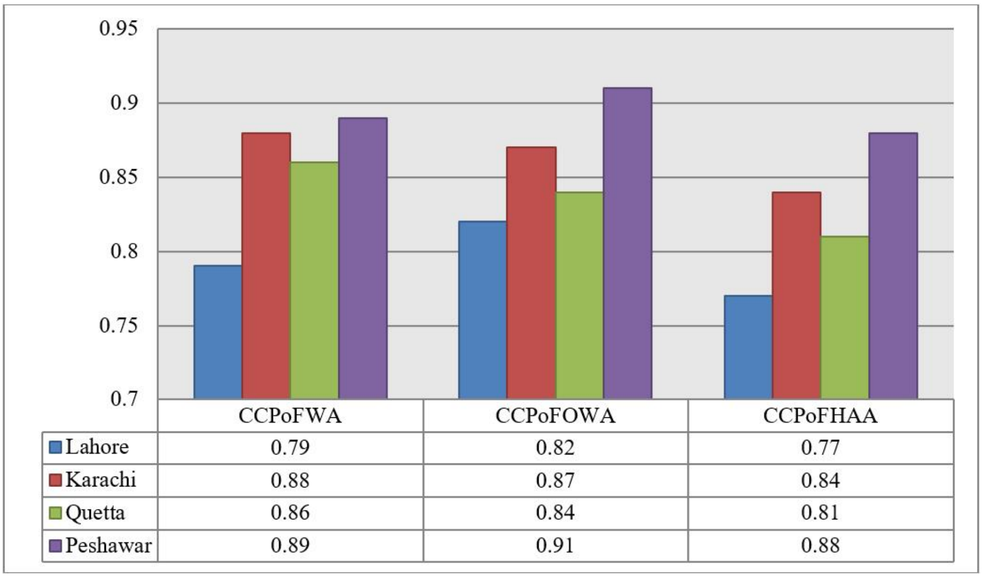

Figure 2. Ranking of all methods

Table 7. Sensitivity analysis

Sets	Uncertainty	Falsity	Indeterminacy	Periodicity	2D Information	qth-Power
FSs	✓	×	×	×	×	×
IFSs	✓	✓	✓	×	×	×
PyFSs	✓	✓	✓	×	×	×
FFSs	✓	✓	✓	×	×	×
CFSs	✓	×	×	✓	✓	×
CIFSs	✓	✓	✓	✓	✓	×
CPyFSs	✓	✓	✓	✓	✓	×
CPoFSs	✓	✓	✓	✓	✓	✓

7 Conclusion

This study introduces the innovative concept of CPoFSs, which significantly extends traditional CFSs. These sets combine fuzzy membership grades across various subregions within the domain of discourse, marking a significant advancement in FS theory. The fundamental principles and operational laws were established for CPoFSs and numbers, forming a solid foundation for their application in diverse fields. Additionally, novel averaging techniques were proposed, including CCPoFWA, CCPoFOWA, and CCPoFHA operators, each offering unique benefits and enhancing the versatility of the proposed model. Compelling examples demonstrated the practical efficacy of the proposed approach, particularly in decision-making scenarios where it is crucial to select the optimal choice from a range of alternatives.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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