Journal of Operational and Strategic Analytics
https://www.acadlore.com/journals/JOSA

Ismail's Ratio Conquers New Horizons: The Non-Stationary *M/D/1* Queue's State Variable Closed Form Expression with Queueing Applications to Traffic Management Optimization

Ismail A Mageed*

School of Computer Science, AI, and Electronics, University of Bradford, BD7 1DP Bradford, UK

* Correspondence: Ismail A Mageed (iammoham@bradford.ac.uk; drismail664@gmail.com)

Received: 07-19-2024 **Revised:** 08-27-2024 **Accepted:** 09-16-2024

Citation: I. A. Mageed, "Ismail's ratio conquers new horizons The non-stationary *M/D/1* queue's state variable closed form expression with queueing applications to traffic management optimization," *J. Oper. Strateg Anal.*, vol. 2, no. 3, pp. 144–159, 2024. https://doi.org/10.56578/josa020302.

Abstract: This paper investigates the search for an exact analytic solution to a temporal first-order differential equation that represents the number of customers in a non-stationary or time-varying $M/D/1$ queueing system. Currently, the only known solution to this problem is through simulation. However, a study proposes a constant ratio, β (Ismail's ratio), that relates the time-dependent mean arrival and mean service rates, offering an exact analytical solution. The stability dynamics of the time-varying $M/D/1$ queueing system are then examined numerically in relation to time, β, and the queueing parameters. On another note, many potential queueing-theoretic applications to traffic management optimization are provided. The paper concludes with a summary, combined with open problems and future research pathways.

Keywords: Time varying $M/D/1$ queueing system; Number of customers; Mean arrival rate; Time; The pointwise stationary fluid flow approximation (PSFFA); Time varying queueing systems (TVQSs); Traffic management; Optimization

1 Introduction

There is a lack of literature in the topic of transient/non-stationary analysis [1–5], which falls into the categories of analysis, applications, simulation, and transient analysis. These categories include several methods of examining dynamic systems, such as using simulations, assessing transient behaviour, and investigating non-stationary phenomena. Analysing non-stationary queueing systems can sometimes be done using closed-form expressions and mathematical transformations. It can be computationally challenging to evaluate these expressions, though.

Zhao et al. [1] have highlighted the importance of air transport as a key driver of global economic growth, emphasizing the challenges posed by increasing air traffic demand and frequent flight delays. On another important note, they discussed the application of queuing theory in modelling and estimating flight delays in multi-airport systems, showcasing the development of a queuing model for a multiple airport system to improve operational efficiency and alleviate airspace congestion. The model considers factors like airport and airspace operations, aiming to provide decision support tools for traffic managers to enhance the overall performance of the air traffic management system.

Modelling and estimating flight delays in a Multiple Airport System (MAS) is complex due to interdependent airport operations, as depicted by Figure 1. A queuing theory-based model is developed to estimate delays by considering airport and airspace characteristics, using stable fluid flow to assess queue lengths and track server delays. The model, validated with real data from the Guangdong-Macao-Hong Kong Great Bay Area, aims to aid traffic managers in mitigating airspace congestion and enhancing MAS operational efficiency.

Figure 2 illustrates the model's flow chart, with the dashed line box representing the input section where parameters are derived from input flight data. The model calculates the queue changes at each server in five-minute intervals starting from the initial time, t_0. This process allows for the dynamic tracking of flight queues within the system based on the input data and time intervals.

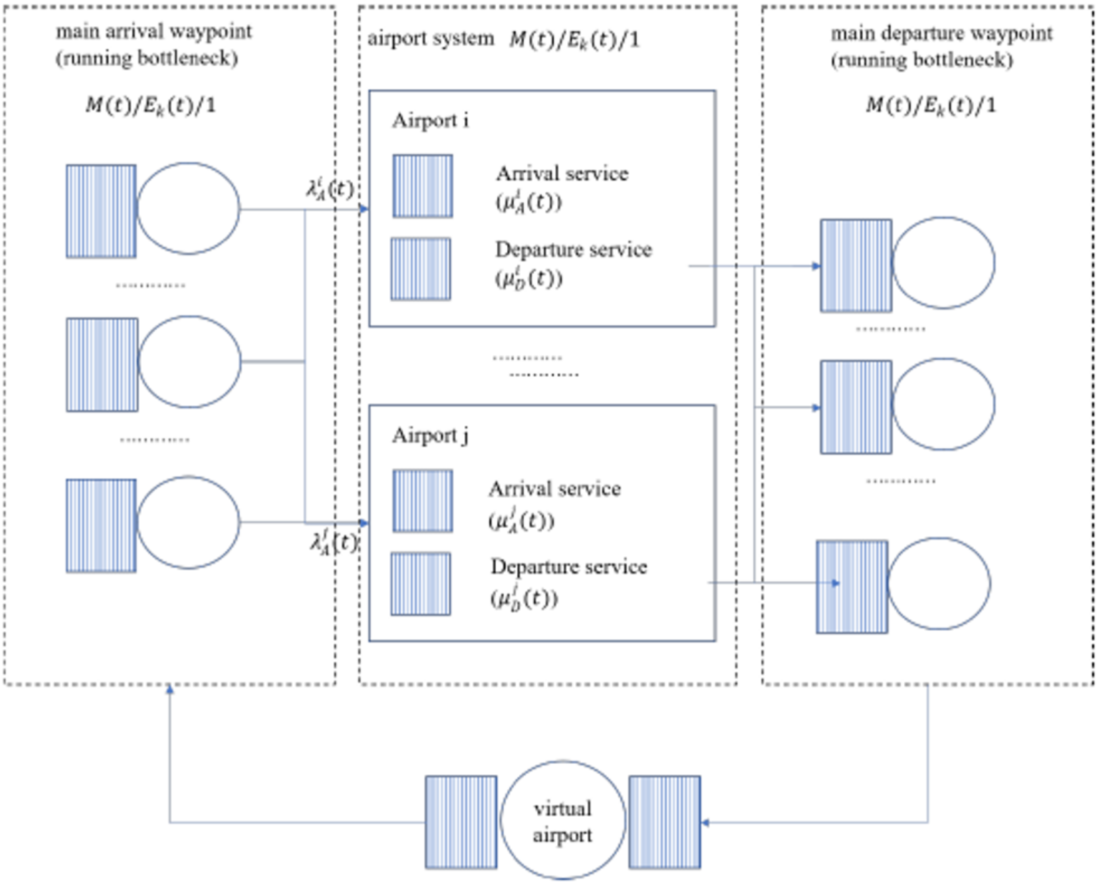


Figure 1. A model designed to analyze and optimize the flow of flights within the MAS, considering factors such as flight congestion and airspace operations, to improve overall operational efficiency and reduce delays [1]

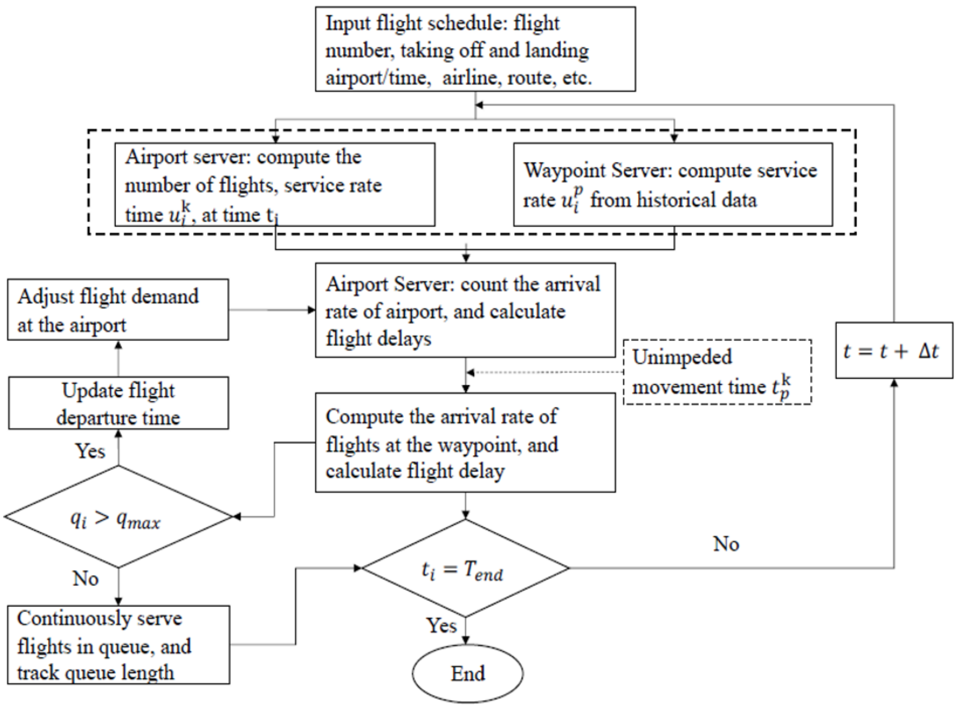


Figure 2. A model to capture the dynamic nature of flights entering and leaving queues by considering various parameters and historical data to simulate the flow of air traffic efficiently [1]

As a visual descriptor of a queuing network model in the context of a Multi-Agent System (MAS) with multiple servers representing airports and waypoints, Figure 3 explains how flights move through the system, including the flow of flights from different airports to specific servers, the time it takes for flights to travel between locations, and how external traffic impacts the queuing process at each server. This model helps analyze and predict the delays and interactions within the system to optimize air traffic management.

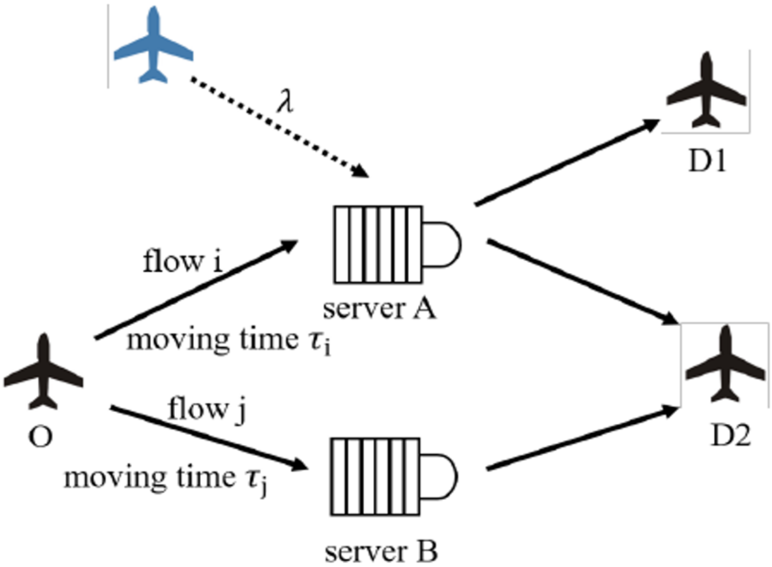


Figure 3. An example showing traffic flow between servers [1]

Figure 3 likely illustrates the movement of data or tasks between different server entities in a system. This visualization could demonstrate how data or requests are processed and transferred between servers A and B, showcasing the flow of information within a networked environment. The depiction may help in understanding the dynamics of server interactions and the distribution of workload in a server-based system.

As shown by Figure 4, the queuing network model was used to analyze the Multi-Airport System (MAS) in the Guangdong–Hong Kong–Macao Greater Bay Area, focusing on five airports: Guangzhou, Shenzhen, Zhuhai, Huizhou, and Macau. The study excluded Hong Kong airport due to separate traffic flow considerations.

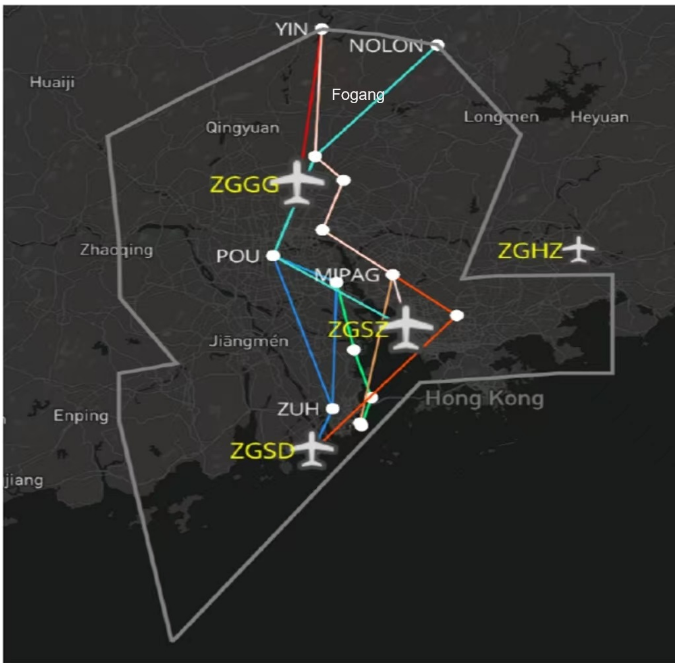


Figure 4. Regulating air traffic, and guaranteeing effective connectivity throughout the Greater Bay area [1]

2 Pointwise Stationary Fluid Flow Approximation (PSFFA)

Let $f_{\text{in}}(t)$ and $f_{\text{out}}(t)$ serve as the temporal flow [1–5], and flow out, respectively. Therefore, by Eq. (1):

$$\frac{dx(t)}{dt} = x^{\cdot}(t) = -f_{\text{out}}(t) + f_{\text{in}}(t), x(t) \text{ as the state variable} \tag{1}$$

$f_{\text{out}}(t)$ links server utilization, $\rho(t)$ and the time-dependent mean service rate, $\mu(t)$ by Eq. (2):

$$f_{\text{out}}(t) = \mu(t)\rho(t) \tag{2}$$

For an infite queue waiting space, defined by Eq. (3):

$$f_{\text{in}}(t) = \text{ Mean arrival rate } = \lambda(t) \tag{3}$$

Thus Eq. (1) rewrites to Eq. (4):

$$x^{\cdot}(t) = -\mu(t)\rho(t) + \lambda(t), 1 > \rho(t) = \frac{\lambda(t)}{\mu(t)} > 0 \tag{4}$$

In stability, $x^{\cdot}(t) = 0$ holds by Eq. (5):

$$x = G_1(\rho) \tag{5}$$

The numerical invertibility of $G_1(\rho)$, yields Eq. (6):

$$\rho = G_1^{-1}(x) \tag{6}$$

Hence, by Eq. (7):

$$x(t) = -\mu(t)\left(G_1^{-1}(x(t))\right) + \lambda(t) \tag{7}$$

Thus, $M/D/1$ queueing system's $-G_1$ [1–5] reads by Eq. (8):

$$G_1(x) = \left((x+1) - \sqrt{(x^2+1)}\right) \tag{8}$$

Thus, $M/D/1$ queue's -PSFFA model reads as in Eq. (9):

$$x^{\cdot} = -\mu\left((x+1) - \sqrt{(x^2+1)}\right) + \lambda \tag{9}$$

TVQSs' life example [6] is depicted by Figure 5.

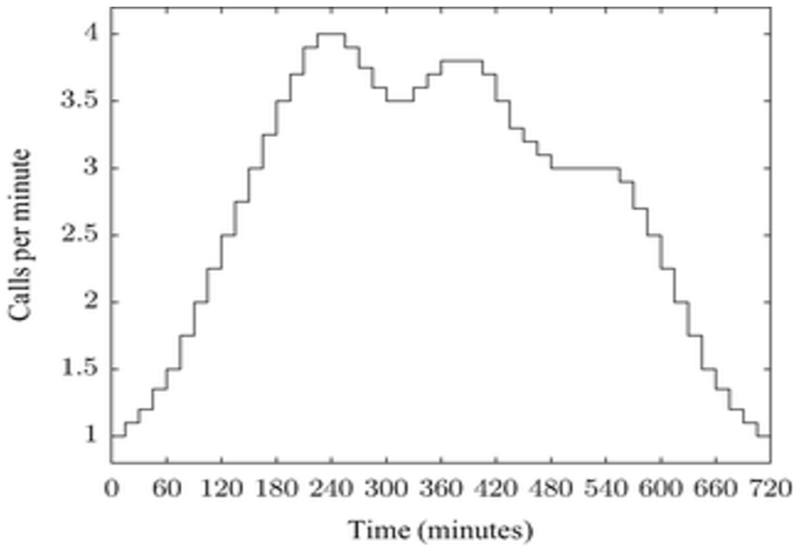


Figure 5. A schematic showcasing how time impacts the calls trend [6]

The current paper contributes to:

- Proposing a constant ratio β (Ismail's ratio) that relates the time-dependent mean arrival and mean service rates, offering an exact analytical solution of the temporal first-order differential equation that represents the number of customers in a non-stationary or time-varying $M/D/1$ queueing system.
- Highlighting the influential role of queueing theory to optimize traffic management.
- The exposition of several potential open problem to expand on for future research.

3 Solving the Non-Stationary *M/D/1* Queueing System's PSFFA

Theorem 1. Eq. (9) analytic solution reads as in Eq. (10):

$$(x-\beta(x+1))^{\frac{1}{(1-\beta)^2}} = \gamma e^{\left[\frac{(x+1)}{(1-\beta)} - \int \mu(t)dt\right]}, \beta = \frac{\lambda(t)}{\mu(t)} \tag{10}$$

Proof

$$x^{\cdot} = -\mu\left((x+1)-\sqrt{(x^2+1)}\right)+\lambda$$

Let, then $x' = -y'$ cothy cschy. Setting, $\beta = \frac{\lambda(t)}{\mu(t)}$. Thus, we have by Eq. (11):

$$-y\cdot\coth y\,\mathrm{csch}\, y = -\mu((1+\mathrm{csch}\, y)-\coth y)+\lambda = -\mu[(1+\mathrm{cschy})-\coth y)-\beta] \tag{11}$$

Therefore, we have

$$\frac{-\coth y\,\mathrm{cschydy}}{[(1+\mathrm{csch}\, y)-\coth y)-\beta]} = -\mu dt$$

This rewrites to Eq. (12):

$$\begin{aligned}
&\frac{\mathrm{coshydy}}{[-(1+\sinh y-\cosh y)+\beta\sinh y]\sinh y} = -\mu dt\\
&\frac{\frac{2}{\beta}\left(e^{3y}+e^{y}\right)dy}{\left[e^{2y}-\frac{e^{y}}{\beta}+\left(\frac{2}{\beta}-1\right)\right]\left(e^{2y}-1\right)} = -\mu dt\\
&e^{2y}-\frac{e^{y}}{\beta}+\left(\frac{2}{\beta}-1\right) = 0 \Rightarrow e^{y} = a, b
\end{aligned} \tag{12}$$

where,

$$a = \frac{\left(\frac{1}{\beta}+\sqrt{\left(\frac{1}{\beta^2}-\frac{8}{\beta}+4\right)}\right)}{2}, b = \frac{\left(\frac{1}{\beta}-\sqrt{\left(\frac{1}{\beta^2}-\frac{8}{\beta}+4\right)}\right)}{2}$$

Let Eq. (13) be:

$$\frac{\frac{2}{\beta}\left(e^{3y}+e^{y}\right)}{\left[e^{2y}-\frac{e^{y}}{\beta}+\left(\frac{2}{\beta}-1\right)\right]\left(e^{2y}-1\right)} = \frac{A}{(e^{y}-a)}+\frac{B}{(e^{y}-b)}+\frac{C}{(e^{y}-1)}+\frac{D}{(e^{y}+1)} \tag{13}$$

Hence, it is implied by Eq. (14) that:

$$\begin{aligned}
&A+B+C+D = \frac{2}{\beta} \Rightarrow A = \frac{2}{\beta}-(B+C+D)\\
&\therefore B = \frac{(1-a)(C-D)}{\left(\frac{2}{\beta}+a\right)}
\end{aligned} \tag{14}$$

Thus,

$$\begin{aligned}
&C = \frac{\frac{4}{\beta}\left(\frac{2}{\beta}+a\right)-\left[\left(1+\frac{2}{\beta}+2a\right)-(ab-(a+b))\left(\frac{2}{\beta}+a\right)\right]D}{\left(\frac{2}{\beta}\right)+a+[ab-(a+b)]\left(\frac{2}{\beta}+a\right)}\\
&bA+aB+abC-abD = 0
\end{aligned}$$

Implies the obtained complicated formula of Eq. (15):

$$bD = \frac{\frac{2b}{\beta}+a\left(ab-\frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)}+a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)\left[\frac{\frac{4}{\beta}\left(\frac{2}{\beta}+a\right)}{\left(\frac{2}{\beta}\right)+a+[ab-(a+b)]\left(\frac{2}{\beta}+a\right)}\right]}{\left(ab+b\left[\frac{\left(1+\frac{2}{\beta}+2a\right)}{\left(\frac{2}{\beta}+a\right)}\right]+a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)+a\left(ab-\frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)}+a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)\left[\frac{\left(1+\frac{2}{\beta}+2a\right)-(ab-(a+b))\left(\frac{2}{\beta}+a\right)}{\left(\frac{2}{\beta}\right)+a+[ab-(a+b)]\left(\frac{2}{\beta}+a\right)}\right]} \tag{15}$$

Thus, it holds that by Eq. (16), that we can calculate the super complicated formula for C.

$C =$

$$\frac{\left[\left(1+\frac{2}{\beta}+2a\right)-\left(ab-(a+b)\right)\left(\frac{2}{\beta}+a\right)\right]\dfrac{\frac{4}{\beta}\left(\frac{2}{\beta}+a\right)-\frac{2b}{\beta}\left[\left(1+\frac{2}{\beta}+2a\right)-\left(ab-(a+b)\right)\left(\frac{2}{\beta}+a\right)\right]+\frac{4a}{\beta}\left(\frac{2}{\beta}+a\right)\left(ab-\frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)}+a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)}{\left(ab+b\left[\frac{b\left(1+\frac{2}{\beta}+2a\right)+a(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)\frac{2}{\beta}+a+[ab-(a+b)]\left(\frac{2}{\beta}+a\right)+a\left(ab-\frac{b\left(1+\frac{2}{\beta}\right)}{\left(\frac{2}{\beta}+a\right)}+a\left[\frac{(1-a)}{\left(\frac{2}{\beta}+a\right)}\right]\right)\left[\left(1+\frac{2}{\beta}+2a\right)-\left(ab-(a+b)\right)\left(\frac{2}{\beta}+a\right)\right]}}{\left(\frac{2}{\beta}\right)+a+[ab-(a+b)]\left(\frac{2}{\beta}+a\right)} \tag{16}$$

Eventually, this showcases Eq. (17):

$$A=\frac{2}{\beta}-\frac{\left(1+\frac{2}{\beta}\right)c}{\left(\frac{2}{\beta}+a\right)}-\frac{\left(1+\frac{2}{\beta}+2a\right)D}{\left(\frac{2}{\beta}+a\right)}, B=\frac{(1-a)(C-D)}{\left(\frac{2}{\beta}+a\right)} \tag{17}$$

Thus, integrating both sides:

$$\left[A\ln\left|\left(1-ae^{-y}\right)\right|+B\ln\left|\left(1-be^{-y}\right)\right|+C\ln\left|\left(1-e^{-y}\right)\right|-D\ln\left(1+e^{-y}\right)\right]=-\int\mu dt+\ln\eta,\eta>0$$

This rewrites to Eq. (18):

$$\frac{\left|(1-ae^{-y})\right|^{A}\left|(1-be^{-y})\right|^{B}\left|(1-e^{-y})\right|^{C}}{(1+e^{-y})^{D}}=\eta e^{-\int\mu dt} \tag{18}$$

This transforms to the final required closed form solution, given by Eq. (19):

$$\frac{\left|\left(1-ae^{-\operatorname{csch}^{-1}(x)}\right)\right|^{A}\left|\left(1-be^{-\operatorname{csch}^{-1}(x)}\right)\right|^{B}\left|\left(1-e^{-\operatorname{csch}^{-1}(x)}\right)\right|^{C}}{\left(1+e^{-\operatorname{csch}^{-1}(x)}\right)^{D}}=\eta e^{-\int\mu dt} \tag{19}$$

Moreover, by Eq. (20):

$$\operatorname{csch}^{-1}(x)=\ln\left(\frac{1+\sqrt{1+x^{2}}}{x}\right) \tag{20}$$

with the domain of real line with zero removed Thus, one gets Eq. (21):

$$\frac{\left(\left|\left(1-\frac{ax}{1+\sqrt{1+x^{2}}}\right)\right|\right)^{A}\left(\left|\left(1-\frac{bx}{1+\sqrt{1+x^{2}}}\right)\right|\right)^{B}\left(\left|\left(1-\frac{x}{1+\sqrt{(1+x^{2})}}\right)\right|\right)^{C}}{\left(1+\frac{x}{1+\sqrt{(1+x^{2})}}\right)^{D}}=\eta e^{-\int\mu dt} \tag{21}$$

Colloray 1. As $x(t)\to 0$, we have

$$\eta e^{-\int\mu dt}=\lim_{x(t)\to 0}\frac{\left(\left|1-\frac{ax}{1+\sqrt{1+x^{2}}}\right|\right)^{A}\left(\left|\left(1-\frac{bx}{1+\sqrt{(1+x^{2})}}\right)\right|\right)^{B}\left(\left|1-\frac{x}{1+\sqrt{1+x^{2}}}\right|\right)^{C}}{\left(1+\frac{x}{1+\sqrt{(1+x^{2})}}\right)^{D}}=1$$

Implying Eq. (22)

$$\int\mu dt=\ln\gamma \tag{22}$$

Colloray 2. As $x(t) \to \infty$, we have the obtained result given by Eq. (23)

$$\eta e^{-\int \mu dt} = \frac{(|1-a|)^A (|(1-b)|)^B \left(\left|1 - \frac{1}{\frac{1}{x}+\sqrt{1+\left(\frac{1}{x}\right)^2}}\right|\right)^C}{\left(1 + \frac{1}{\frac{1}{x}+\sqrt{\left(1+\left(\frac{1}{x}\right)^2\right)}}\right)^D} = 0 \to \int \mu dt \to \infty \tag{23}$$

Numerical Experiments

Numerical Experiment One

Let $\beta = 2$, then $a = 0.5, b = 0, \eta = 1, \mu(t) = t,$

$$A = -7.923076923, B = 1.846153846, C = 6, D = 0.4615384615.$$

We have

$$\sqrt{2}\left[\frac{\left(\left|\left(1 - \frac{0.5x}{1+\sqrt{1+x^2}}\right)\right|\right)^{-7.923076923}\left(\left|\left(1 - \frac{x}{1+\sqrt{(1+x^2)}}\right)\right|\right)^6}{\left(1 + \frac{x}{1+\sqrt{(1+x^2)}}\right)^{0.4615384615}}\right] = t$$

Figure 6 shows a new phenomenon to queuing theorist. The possibility that time will converge to a certain value for sufficiently large number in the time varying $M/D/1$ queuing system. This is for an increasing temporal mean service rate.

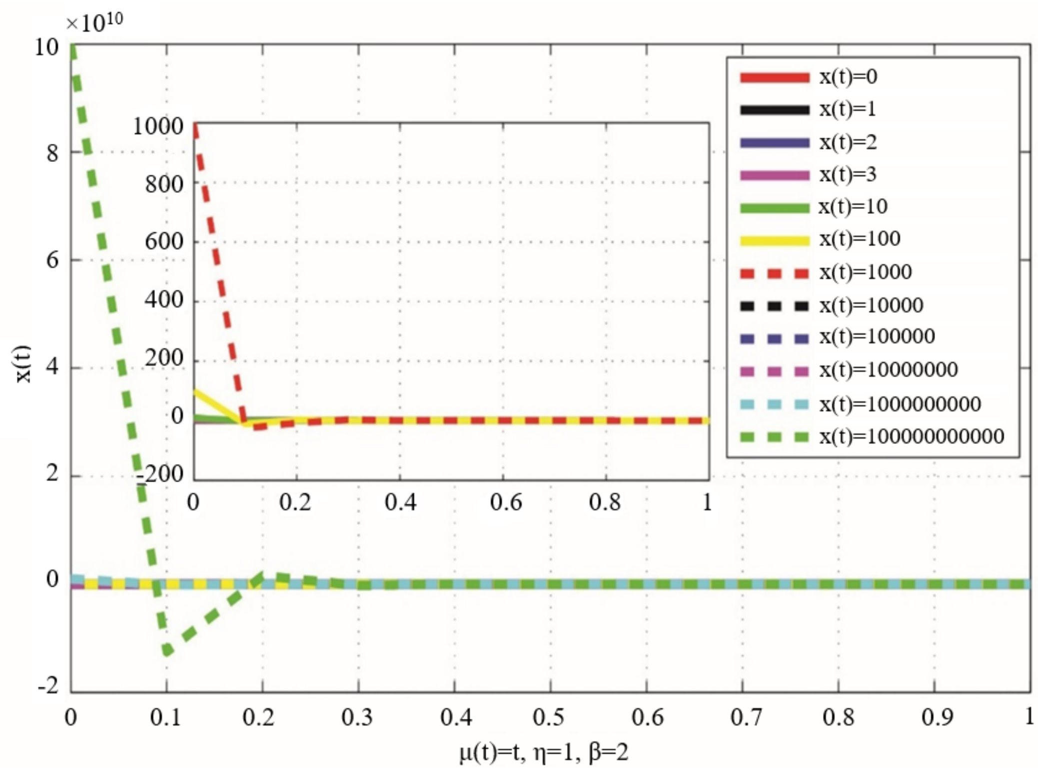


Figure 6. Communicating numerical experiment one, visually

This shows that as the time varying $M/D/1$ queuing system's state variable becomes sufficiently large, time vanishes.

Numerical Experiment Two

Let $\beta = 2$, then $a = 0.5, b = 0, \eta = 1, \mu(t) = \frac{1}{t},$

$$A = -7.923076923, B = 1.846153846, C = 6, D = 0.4615384615.$$

Figure 7 visualizes a new phenomenon to queuing theorist. The possibility that time will converge to a certain value for sufficiently large number in the time varying $M/D/1$ queuing system. This is for a decreasing temporal mean service rate.

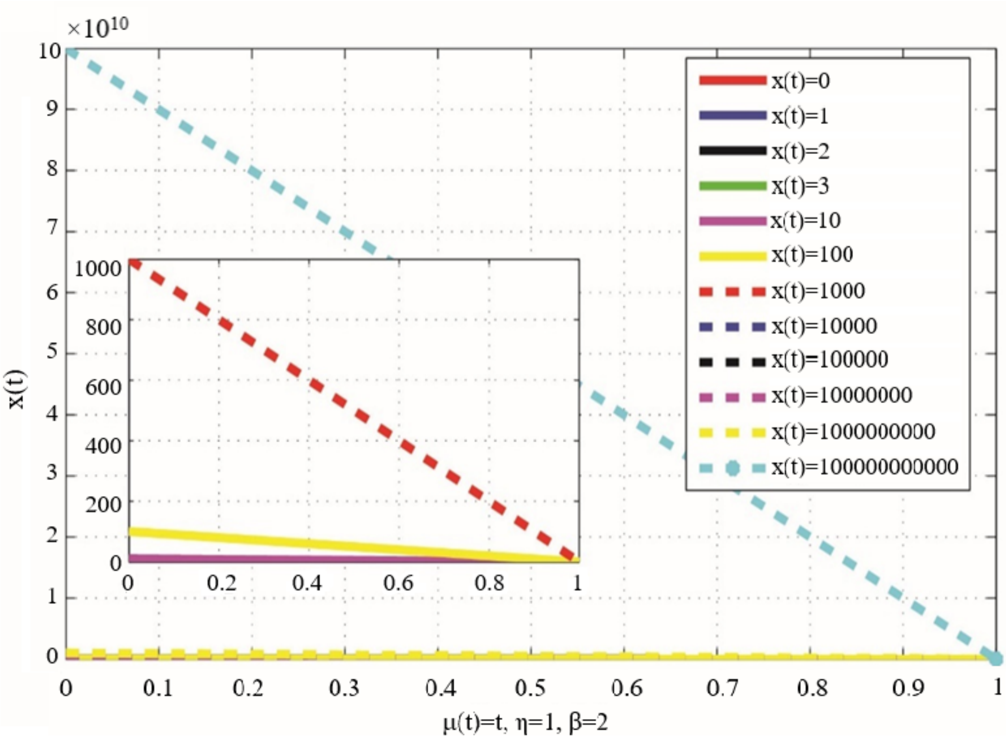


Figure 7. Communicating numerical experiment two, visually

4 Queueing-Theoretic Applications to Traffic Management Optimization

The work [7] showcases how traffic congestion impacts various aspects of society, including economic costs, travel discomfort, road accidents, and environmental pollution. It highlights the importance of using stochastic queuing models in traffic management to address congestion issues and optimize solutions for common road congestion problems. By applying techniques like analytical queuing models and stochastic optimization, the article aims to improve traffic and crowd management systems to alleviate daily traffic blockages.

An exposition of the importance of stochastic modelling approaches in modern traffic and crowd management research, highlighting the complexity of optimizing traffic systems under various uncertainties like safety management, uneven traffic distribution, and weather conditions. More briefly, the undertaken research [7] aimed to use stochastic queuing models to address congestion and crowding issues by applying predictive traffic handling computations for optimal results, emphasizing the need for robust decision-making support in traffic and crowd management through the analysis of various stochastic traffic management models.

The Static Traffic Assignment with Queuing (STAQ) model [8] presents a viable alternative to traditional STA and DTA approaches due to its accuracy, robustness, and accountability in strategic transport planning for congested networks. In the context of macroscopic Dynamic Traffic Assignment (DTA) models, STAQ employs a route sub model like other DTA models, as shown by Figure 8.

Figure 9 showcases the comparison between STA and STAQ models in transportation planning, highlighting how STAQ requires more precise input but offers increased model accuracy through capacity constraints. STAQ incorporates turn capacities from junction modelling as strict constraints, enhancing model accuracy by considering flow metering and spill-back effects. This results in a more accountable model with improved accuracy, although requiring more detailed input compared to STA models.

It calculates reduction factors for turning movements based on the capacities of the turns and downstream links, aiming to optimize the flow of traffic through the network. This phase iterates until consistent reduction factors are determined for all turns, helping improve the network's efficiency and traffic flow.

In Figure 10, STAQ and DTA successfully identify the second bottleneck (Voorburg) but fail to recognize the first bottleneck (Centrum Zuid) due to the absence of spillback modelling from outside the network.

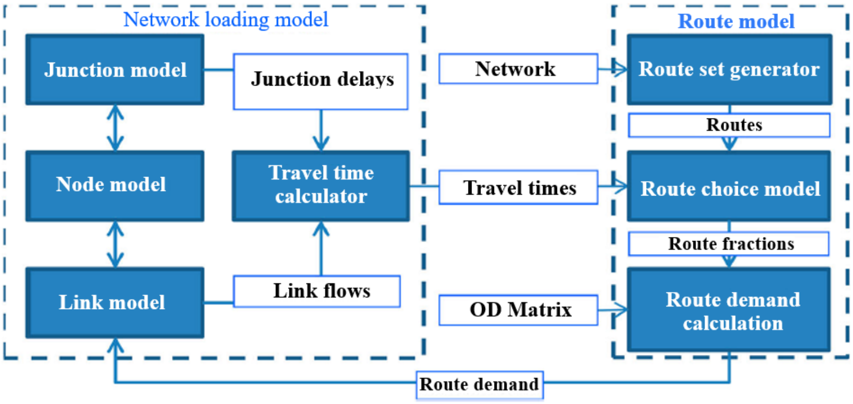


Figure 8. A framework used to analyze traffic patterns and optimize transportation systems for efficiency and effectiveness [8]

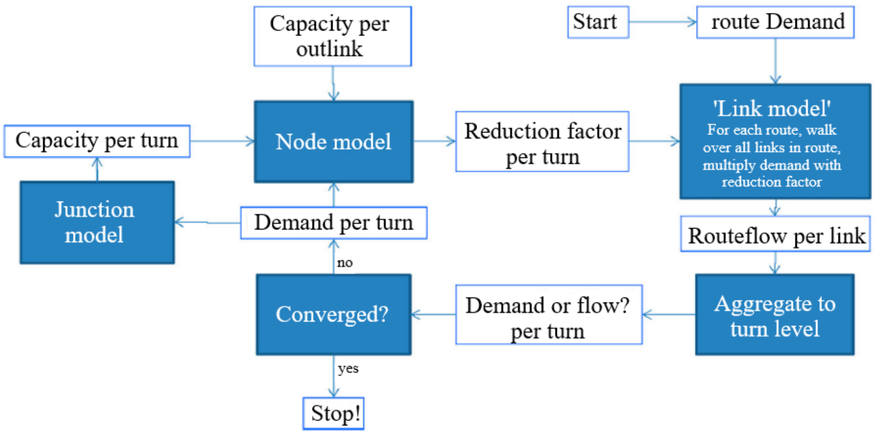


Figure 9. A flowchart of the squeezing phase algorithm [8]

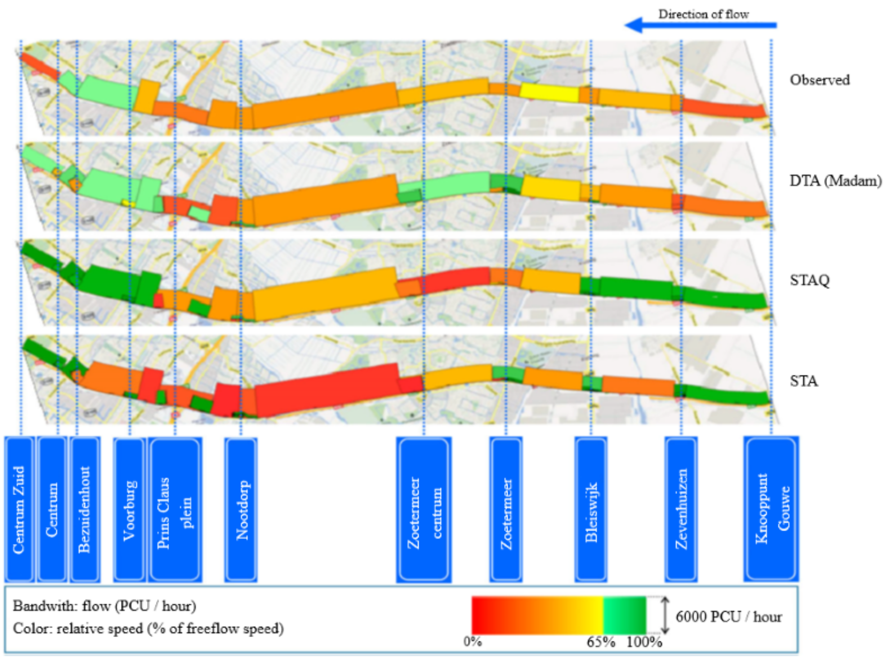
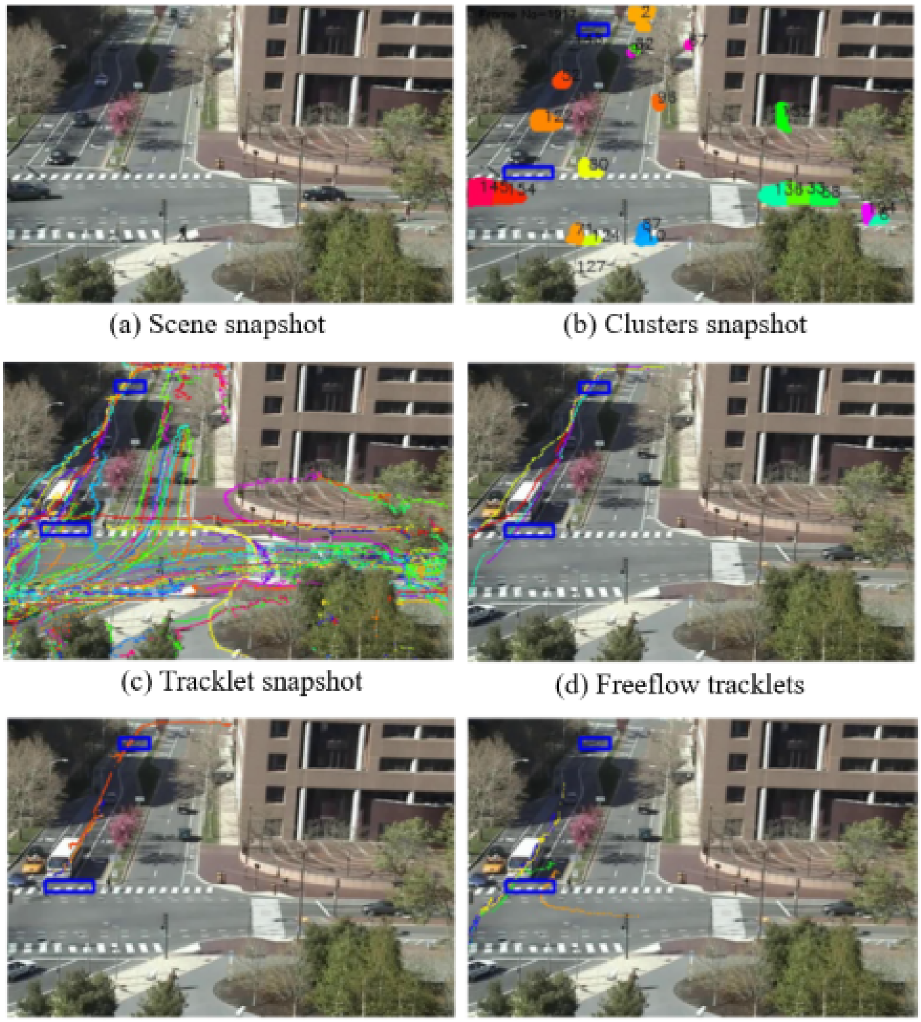


Figure 10. Congestion patterns on the A12 motorway between Gouda and Den Haag are compared between observations and models [8]

During tracklet analysis in traffic flow [9], three types of tracklets are identified: free-flow tracklets occur when vehicles can pass without stopping at a clear signal, queuing tracklets form when vehicles are stopped at the signal, and queue clearing tracklets are created when the signal changes and vehicles start moving. Figure 11 visually illustrates these different types of tracklets for better understanding and analysis of traffic behavior at signals.



(a) Scene snapshot (b) Clusters snapshot

(c) Tracklet snapshot (d) Freeflow tracklets

(e) Queuing tracklets (f) Queue clearing tracklets

Figure 11. Several characteristics acquired from the videos in the MIT dataset: (a) A scene's snippets; (b) Corresponding scene groupings; (c) Tracklets discovered up until this frame; (d)–(f) Several tracklet types flowing in a single direction [8]

The application of queuing theory to optimize transportation operations at Lafarge WAPCO in Nigeria was interestingly addressed [10], focusing on the analysis of arrival times, loading processes, and truck productivity. By developing a queuing model M/M/1: FCFS/∞//∞/, the study calculated key queueing parameters to enhance productivity and reduce idle time. The findings suggest that increasing the number of trucks can effectively utilize idle time, improve productivity, and minimize production costs in surface mining operations. Notably, the study [10] has overviewed queuing theory to analyze traffic patterns during peak hours in Bhopal, Indore, and Ujjain cities in India. By observing the number of vehicles waiting and being served at different times of the day, this study has identified peak traffic intensity during morning and evening rush hours. The findings suggest that adjusting traffic light timings and implementing strategies like increasing road capacity and providing separate lanes can help minimize congestion and reduce delays during peak traffic periods.

In metropolitan areas worldwide [11], traffic congestion is a common issue caused by various factors like high traffic volume and poor infrastructure. Queuing theory, originally developed for telephone systems, is now applied in transportation to optimize waiting times and service efficiency during congestion, helping to reduce delays and frustration for commuters and improve overall traffic management. By using queuing theory principles, transportation systems can better handle the demands of modern civilizations and mitigate the negative impacts of traffic congestion on daily activities and goods delivery.

The research conducted by Koko et al. [12] focused on applying queuing theory to analyze bus services at Federal Polytechnic transport system, Kaura Namoda, using single server and multiple servers' models. By comparing parameters like traffic intensity and average customer numbers, the study found that the multiple servers' model is more efficient in minimizing these factors, suggesting the need for additional school buses to alleviate traffic congestion based on the research results.

Transport logistics involves managing the movement of goods and passengers within a transportation network [13], from entry to exit points. The demand for parking spaces, especially in urban areas like České Budějovice, is crucial for meeting EU regulations and enhancing competition in the transport industry. Efforts to address parking shortages include modernizing parking systems and implementing park and ride facilities based on queuing theory to optimize car park occupancy and alleviate traffic congestion.

The significance of congestion in urban traffic networks and the various techniques proposed to alleviate it, including traffic signal control was deeply explored in study [14], through explaining the difference between pre-timed and traffic-actuated signal control methods, highlighting the advantages of adaptive signal control in responding to real-time traffic conditions. Also, the use of queueing models in traffic signal design research to optimize signal timing based on traffic volume and congestion levels was equally explained [14].

Queueing theory involves developing mathematical models to analyze waiting and queueing scenarios, determining various performance metrics like queue length [15], waiting time, and service rates. These models are crucial for optimizing systems such as road structures, intersections, and traffic signal timing. By examining arrival and departure patterns and making certain assumptions, queueing models can be created and refined to improve system performance [15].

Queueing theory [16], a branch of mathematics, focuses on analyzing waiting lines or queues in various scenarios like customers waiting at supermarkets, banks, or hospitals. It is a crucial tool for evaluating system performance and has applications in diverse fields such as banking, healthcare, traffic control, and computer science. The theory helps optimize processes by studying queue lengths, waiting times, and service rates, making it valuable for designing efficient systems like traffic signal timing and parking lot management.

Reinforcement learning [17] is a technique commonly used in traffic signal control, where a traffic signal agent learns to optimize traffic lights by testing different actions based on the current traffic conditions. While these methods can outperform traditional control systems, they initially require an exploration phase involving trial-and-error actions, leading to unstable performance and high computational costs. To address this, a bootstrapping method is proposed to reduce the computation time needed for learning-based traffic control methods to achieve satisfactory performance levels by modelling road lanes as queues and estimating the agent's policy based on the average service time of vehicles at intersections.

Simulation results of Figure 12 make it abundantly evident that the bootstrapped learning approach offers a notable benefit during the initial learning rounds. But when the non-bootstrapped agent learns, this early advantage gradually wanes until both approaches achieve comparable performance levels, both falling short of the constant signal cycle strategy.

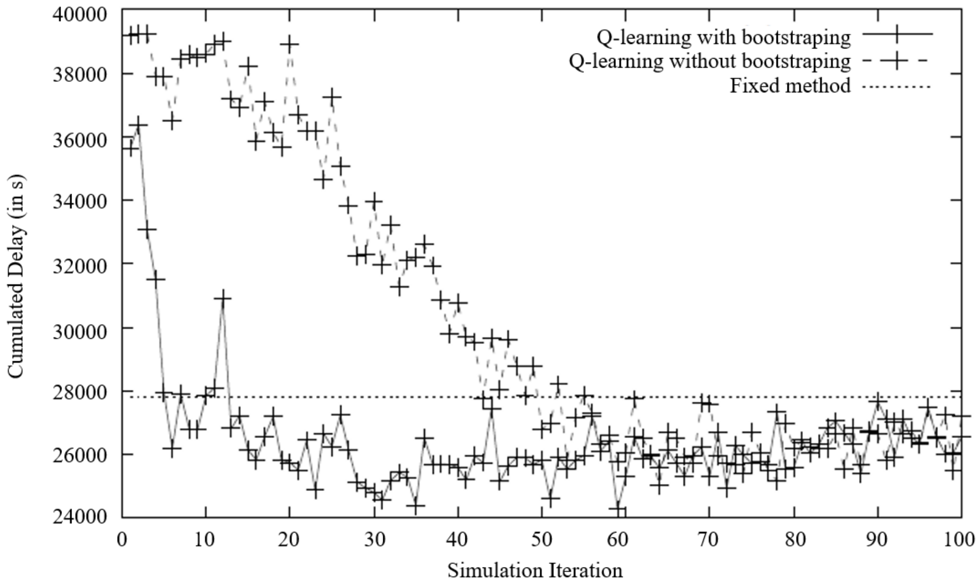


Figure 12. Performance comparison of a bootstrapped and non-bootstrapped Q learning agent [17]

Queueing theory is a field that uses mathematical models to study waiting lines and their applications, such as analyzing traffic delays [18]. By applying queueing-theoretic methods, researchers can understand the dynamics of congestion and optimize traffic management strategies. When dealing with intersections where multiple roads converge and various factors like different traffic streams, cyclists, and pedestrians come into play, more complex queueing models are needed, posing challenges in analysis compared to simpler single-dimensional queue models, as illustrated by Figure 13. The warning is that compared to single-dimensional models, such higher-dimensional stochastic models are far more difficult to analyse.

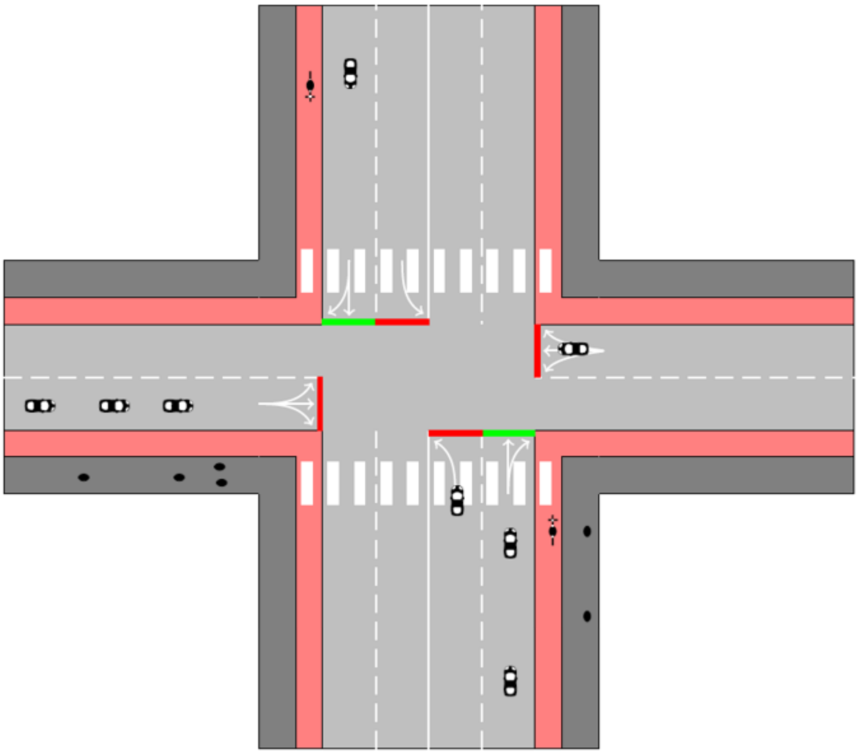


Figure 13. A graphical representation of a general intersection with vehicles, cyclists, and pedestrians

The importance of efficient transport networks for industrial development, emphasizing the impact of transportation delays on competitiveness and profitability is crucial [19]. The study focused on assessing the efficiency of transport connectivity between an Inland Container Depot (ICD) and its customers to identify factors causing delays and propose solutions for improving the existing transport system. By optimizing road usage and fleet management [19], the study aims to reduce congestion, fuel consumption, greenhouse gas emissions, and transportation costs. Study [20] discussed how fluid mechanics simulation theory is used to calculate the queue length of vehicles at urban road intersections. It introduces the Greenberg model to address issues with the traditional stop wave model and provides a method to determine queuing times based on different arrival scenarios, aiding in traffic management strategies and urban road traffic system evaluation. The study [20] aimed to improve the effectiveness of modelling vehicle queues at intersections by considering various queuing situations and optimizing queuing times based on different traffic conditions.

In this context, the simulation model discussed [21] is based on queue theory and implemented in Simulink software. This model is designed to efficiently test and evaluate Intelligent Transportation System (ITS) applications and protocols before real-world implementation. By considering factors like lane changing, reaction to road signs, and route generation, the model provided [21] valuable insights into system performance through accurate traffic simulations, even for complex scenarios.

Passenger transportation hubs play a crucial role in connecting large-capacity inter-city transportation with the intra-city network [22], impacting city traffic management significantly. By applying threshold queueing theory, researchers develop a fundamental diagram model to analyze traffic characteristics near passenger transportation hubs and identify bottlenecks. This model [22] considers passenger boarding as a service process in a queue system, adjusting service rates based on the congestion level to better understand human-vehicle interactions and optimize traffic flow.

The study [23] highlights the importance of considering transient effects, especially under varying station loads, and demonstrates the adaptability of queue models for predicting system behavior in different railway station scenarios, even with limited information available.

The Querétaro-Celaya highway [24] in Mexico is a crucial route due to new company establishments, leading to the need for studying traffic dynamics on this road. By utilizing queueing theory models that account for the random nature of traffic flow, researchers can analyze the vehicular flow on the highway using macroscopic approaches and analytical models to determine that the current traffic flow is non-congested. This method provides an alternative for institutions lacking specialized tools to conduct such studies effectively.

The study [15] introduced a method called "rank, detect, and quantify impacts" to identify illegal parking hotspots, detect curb occupancy, and quantify the impact of illegal parking on travel times. The research [25] proposed a model that considers the spatial locations of illegally parked commercial trucks to provide valuable insights for cities and transportation agencies.

In Figure 14, the results of detecting illegal parking at three different locations are shown. The detection accuracy varied due to factors like occlusion, image resolution, and vehicle types, leading to occasional over- or underestimations in the model's performance.

(a) Double-parked (left) and parked vehicles in a moving lane (right)

(b) Parked vehicle in a moving lane (left) and car parked in a bus lane (right)

(c) Parked vehicle in a moving lane—far side (right)

Figure 14. Parking/bus lane occupancy detection output [25]

5 Closing Remarks, Open Problems with Next Phase of Research

A difficult topic within queueing theory is investigated in this study; more specifically, the state variable of the underlying queue is found. The non-stationary $M/D/1$ queueing system is formulated in the article using a pointwise stationary fluid flow approximation (PSFFA) technique, which provides a solution to this problem. Several revolutionary spotlights on the significance of queueing theory to advance the contemporary theory of traffic management optimization are thoroughly explored. The impact of time and queueing factors on the underlying queue's stability dynamics is also investigated in this work.

The following are some emerging open problems arising from the current study:

• There exist some potential limitations in the study [1], related to airport operations modelling, specifically focusing on the sequential service assumption for nodes by a single server system. It points out that the model does not account for scenarios with multiple servers, like multiple runways, leading to discrepancies in flight operations simulation. Future research could explore queuing models incorporating airport surface operations and real-time calculations for more accurate delay predictions and decision-making.

• Following the study [8], enhancing a specific model to accommodate multiple time periods would bring it closer to Dynamic Traffic Assignment (DTA) models. However, this improvement may lead to challenges like increased input requirements, convergence issues, and longer computation times. To address these issues, a mechanism for transferring residual traffic between time periods is proposed, which can help manage traffic flow continuity for trips that extend beyond a single time period, especially in scenarios involving large networks or short time intervals.

• According to the study [9], it is crucial to maintain the camera at an elevated position to obtain a top view or nearly top view of the scene to use the suggested strategy. This guarantees that the queue is visible. Nevertheless, the criteria have not been met by the existing datasets. This can be resolved by arranging the cameras so that recordings can be made of the top or nearly top viewpoints. An additional observation is that the method's first features include optical flow. Robust optical flow estimation is therefore necessary. It should be mentioned that the DPMM learning is not limited to merely observing optical flow features. To enhance tracking, additional features may also be included for temporal clustering.

• Adapting the current models in study [14], queue capacities are considered infinite to avoid blockages. However, this assumption hinders the models' effectiveness when dealing with oversaturated traffic. To improve future models' accuracy, is it mathematically feasible to incorporate finite queue capacities to address the limitations observed during high traffic conditions.

• Using full state definitions does not significantly improve performance in many cases [17], even though it greatly increases the problem's state space and computation time. This proposes a complicated unsolved open problem.

• Halfin-Whitt scaling models. The Halfin-Whitt type of scaling rule [18], for the FCTL queue yields good asymptotic features, as is shown in study [18]. For the collection of cyclic queueing models taken into consideration in the first extension [18], the same kind of attributes can likely be obtained using the same scaling rule. Instead, the queueing models examined in study [18] are higher dimensional and behave fundamentally differently, but they have comparable Halfin-Whitt-type characteristics when scaling in line with the FCTL queue. Finding a more comprehensive, possibly all-encompassing set of queueing models that yields asymptotic conclusions like those of Halfin-Whitt would be quite intriguing. While simulation may be used to demonstrate the asymptotic qualities for this broader class of models, it is also interesting to identify the precise limiting process, as we did with the FCTL queue.

• Networks of intersections. Future research on applying the findings we found for isolated junctions to a network of intersections would be an intriguing area of study. Queueing models and networks of intersections are practically very relevant, although they are typically hard to analyse. We propose, based on concepts from study [18], to study control strategies for traffic light networks. This is because additional research in this area may yield structural insights from this analysis. The application of decomposition and aggregation-disaggregation techniques as well as simulation-based optimisation strategies are two examples. There are various issues with a network of intersections [18]. First, there is a dependency on the vehicle arrival processes. Furthermore, there is a lot of interaction between different intersections—think of spillover effects, for example.

• Multiple streams of vehicles receiving a green light. The need to adjust traffic-light strategies to account for scenarios where multiple non-conflicting streams of vehicles receive a green light simultaneously. This adjustment is necessary as existing models primarily focus on one stream receiving a green light at a time, leading to changes in delay characteristics and the need to modify the approximation scheme. The proposed algorithms [18] need further improvements to address this issue by extending the models to accommodate multiple streams of vehicles receiving a green light simultaneously, considering factors like conflicting streams and turning movements.

• When opposing streams of vehicles receive a green light simultaneously [18], the traffic model becomes more complex, especially if there are left-turning vehicles in both streams. This complexity may require a two-dimensional analysis of queue-length distribution due to potential blockages between vehicles from different streams. Overall, traffic-light models become significantly more complicated when dealing with multiple potentially conflicting streams of vehicles receiving a green light simultaneously, highlighting the need for further research to address such scenarios effectively.

The focus of future study will be on solving open research problems and examining non-stationary queue applications in various scientific domains, as well as the investigation of possible solutions to the proposed open problems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Acknowledgements

The author wishes to express his sincerest thanks to Dr. R. Raja, assistant professor Ramanujan Center for Higher Mathematics, Alagappa University, India, for the most professional drawing of the curves of Figures 7 and 8.

Conflicts of Interest

The author declears no conflicts of interest.

References

[1] X. Zhao, Y. Wang, L. Li, and D. Delahaye, "A queuing network model of a multi-airport system based on pointwise stationary approximation," *Aerospace*, vol. 9, no. 7, p. 390, 2022. https://doi.org/10.3390/aerospace9070390

[2] V. B. Iversen, *ITU Teletraffic Engineering Handbook*. Technical University of Denmark, 2021.

[3] I. A. Mageed and K. Q. Zhang, "Solving the open problem for GI/M/1 pointwise stationary fluid flow approximation model (PSFFA) of the non-stationary D/M/1 queueing system," *Elec. J. Comp. Sci. Inf. Tech.*, vol. 9, no. 1, pp. 1–6, 2023. https://doi.org/10.52650/ejcsit.v9i1.127

[4] L. Hu, B. Zhao, J. Zhu, and Y. Jiang, "Two time-varying and state-dependent fluid queuing models for traffic circulation systems," *Eur. J. Oper. Res.*, vol. 275, no. 3, pp. 997–1019, 2019. https://doi.org/10.1016/j.ejor.2019.01.020

[5] A. Roubos, S. Bhulai, and G. Koole, "Flexible staffing for call centers with non-stationary arrival rates," in *Markov Decision Processes in Practice*, R. Boucherie and N. van Dijk, Eds. Springer, Cham., 2017. https://doi.org/10.1007/978-3-319-47766-4_19

[6] N. Zychlinski, "Applications of fluid models in service operations management," *Queueing Syst.*, vol. 103, pp. 161–185, 2023. https://doi.org/10.1007/s11134-022-09868-2

[7] R. Rathore, "A study on application of stochastic queuing models for control of congestion and crowding," *Int. J. Glob. Acad. Sci. Res.*, vol. 1, no. 1, pp. 1–7, 2022. https://doi.org/10.55938/ijgasr.v1i1.6

[8] L. Brederode, A. Pel, L. Wismans, E. de Romph, and S. Hoogendoorn, "Static traffic assignment with queuing: Model properties and applications," *Transportmetrica A*, vol. 15, no. 2, pp. 179–214, 2019. https://doi.org/10.1080/23249935.2018.1453561

[9] S. K. Kumaran, D. P. Dogra, and P. P. Roy, "Queuing theory guided intelligent traffic scheduling through video analysis using Dirichlet process mixture model," *Expert Syst. Appl.*, vol. 118, pp. 169–181, 2019. https://doi.org/10.1016/j.eswa.2018.09.057

[10] V. N. Varghese, V. Verghese, and A. Chandran, "Application of Queuing Theory in transportation," *Int. J. Eng. Res. Techn*, vol. 9, no. 6, pp. 55–58, 2021.

[11] R. Sharma, "Application of Queuing Theory in traffic management system," *Int. J. Math. Stat. Invent.*, vol. 8, no. 7, pp. 59–64, 2020. https://doi.org/10.35629/4767-08075964

[12] M. A. Koko, M. S. Burodo, and S. Suleiman, "Queuing theory and its application analysis on bus services using single server and multiple servers model," *Am. J. Oper. Manag. Inf. Syst.*, vol. 3, no. 4, pp. 81–85, 2018. https://doi.org/10.11648/j.ajomis.20180304.12

[13] J. Cejka and J. Šedivý, "Discussion of operational transport analysis methods and the practical application of Queuing Theory to stationary traffic," *Transp. Res. Procedia*, vol. 53, pp. 196–203, 2021. https://doi.org/10.1016/j.trpro.2021.02.026

[14] H. Liu, "Traffic signal control using Queuing Theory," Ph.D. dissertation, The University of Texas at Austin, 2018.

[15] F. Gunes, S. Bayrakli, and A. H. Zaim, "Flow characteristics of traffic flow at signalized intersections and performance comparison using Queuing Theory," in *2020 4th International Symposium on Multidisciplinary Studies and Innovative Technologies (ISMSIT)*, Istanbul, Turkey, 2020, pp. 1–9. https://doi.org/10.1109/ISMSIT50672.2020.9255044

[16] N. T. Mai and D. M. Cuong, "The application of Queuing Theory in the parking lot: A literature review," in *Proceedings of the International Conference on Emerging Challenges: Business Transformation and Circular Economy (ICECH 2021)*, Hanoi, Vietnam, 2021, pp. 190–203. https://doi.org/10.2991/aebmr.k.211119.020

[17] M. Tréca, J. Garbiso, D. Barth, and M. Zargayouna, "Fast bootstrapping for reinforcement learning-based traffic signal control systems using Queuing Theory," in *2020 IEEE 92nd Vehicular Technology Conference (VTC2020-Fall)*, Victoria, BC, Canada, 2020, pp. 1–5. https://doi.org/10.1109/VTC2020-Fall49728.2020.9348821

[18] R. W. Timmerman, "Performance analysis at the crossroad of Queuing Theory and road traffic," Ph.D. dissertation, Eindhoven University of Technology, 2022.

[19] N. Malipatil, S. I. Avati, H. N. Vinay, and S. Sunil, "Application of queuing theory to a Toll Plaza-A

case study," in *Transportation Research: Proceedings of CTRG 2017*, Mumbai, India, 2020, pp. 343–354. https://doi.org/10.1016/j.tra.2022.05.024

[20] Z. Li, "Research on traffic optimization based on computer modeling of maximum queue length," in *2021 International Conference on Electronic Information Engineering and Computer Science (EIECS)*. Changchun, China: IEEE, 2021, pp. 354–357. https://doi.org/10.1109/EIECS53707.2021.9588003

[21] B. O. Jalel and V. Véronique, "Queuing theory based simulation model for vehicular mobility," in *ICC 2021-IEEE International Conference on Communications*. Montreal, QC, Canada: IEEE, 2021, pp. 1–6. https://doi.org/10.1109/ICC42927.2021.9500977

[22] H. Zheng, Y. Yang, G. Gao, K. Yang, and J. Chen, "Traffic stream characteristics analysis for roadway linking to pick-up zone of passenger transportation hub: A fundamental diagram derived from threshold queueing theory," *Appl. Sci.*, vol. 13, no. 1, p. 175, 2022. https://doi.org/10.3390/app13010175

[23] M. Zharkov, A. Kazakov, and A. Lempert, "On the application of queuing theory in the analysis of transients in the operation of a freight railway station," in *International Conference on Information Technologies and Mathematical Modelling*. Tomsk, Russia: Springer International Publishing, 2021, pp. 266–277. https://doi.org/10.1007/978-3-031-09331-9_21

[24] G. Guzmán-Sáenz, S. Hernández-González, J. A. Jiménez-García, V. Figueroa-Fernández, and M. A. Melchor-Navarro, "Analysis of Querétaro-Celaya Highway applying queueing theory," *J. Traff. Transp. Eng.*, vol. 6, pp. 97–107, 2018. https://doi.org/10.17265/2328-2142/2018.02.005

[25] J. Gao, F. Zuo, K. Ozbay, O. Hammami, and M. L. Barlas, "A new curb lane monitoring and illegal parking impact estimation approach based on queueing theory and computer vision for cameras with low resolution and low frame rate," *Transp. Res. Part A Policy Pract.*, vol. 162, pp. 137–154, 2022. https://doi.org/10.1016/j.tra.2022.05.024