Journal of Operational and Strategic Analytics
https://www.acadlore.com/journals/JOSA

Hierarchical Efficiency in Banking: Decentralized Bi-Level Programming with Stackelberg Dynamics

Rita De Fátima Muniz[1], Omar Mar Cornelio[2], Eisa Abdolmaleki[3*]

[1] Programa de Pós-graduação em Educação Núcleo de Avaliação Educacional, Federal University of Ceará, 60430-160 Fortaleza, Brazil
[2] Centro de Estudio de Matemática Computacional, Universidad de las Ciencias Informáticas, 19370 La Habana, Cuba
[3] Department of Mathematics, Tonekabon Branch, Islamic Azad University, 1477893780 Tonekabon, Iran

[*] Correspondence: Eisa Abdolmaleki (eabdolmaleki@aihe.ac.ir)

Received: 09-14-2024 **Revised:** 11-01-2024 **Accepted:** 11-12-2024

Citation: R. D. F. Muniz, O. M. Cornelio, and E. Abdolmaleki, "Hierarchical efficiency in banking: Decentralized bi-level programming with Stackelberg dynamics," *J. Oper. Strateg Anal.*, vol. 2, no. 4, pp. 225–234, 2024. https://doi.org/10.56578/josa020402.

Abstract: The inherent hierarchical and decentralized nature of decision-making within banking systems presents significant challenges in evaluating operational efficiency. This study introduces a novel bi-level programming (BLP) framework, incorporating Stackelberg equilibrium dynamics, to assess the performance of bank branches. By combining with data envelopment analysis (DEA), the proposed BLP-DEA model captures the leader-follower relationship that characterizes banking operations, wherein the leader focuses on marketability and the follower prioritizes profitability. A case study involving 15 Iranian bank branches was employed to demonstrate the model's capacity to evaluate performance comprehensively at both decision-making levels. The results underscore the model's effectiveness in identifying inefficiencies, analyzing cost structures, and providing actionable insights for performance optimization. This approach offers a robust tool for addressing the complexities associated with decentralized decision-making in hierarchical organizations. The findings have significant implications for both theoretical development and practical application, especially in the context of improving the operational efficiency of banking institutions.

Keywords: Bank branch performance; Stackelberg equilibrium; Bi-level programming; Hierarchical decision-making; Cost efficiency; Leader-follower model

1 Introduction

In the swiftly evolving and technologically advanced realm of computers and telecommunications, where competition is fierce, banks, as one of the most intricate sectors globally, must possess the agility to respond promptly to changes. In a highly competitive landscape, ongoing enhancement is essential for any prosperous firm. Consequently, enhancing performance is universally acknowledged as crucial for achieving a competitive edge. The banking chain possesses a multifaceted and intricate structure due to its operation in a competitive business environment. The banking chain is pivotal in the economic cycle of each nation, and to achieve or keep a competitive advantage, performance evaluation has become a crucial responsibility for the management of every financial institution. Consequently, in evaluating bank branch performance, the primary emphasis of this study has grown increasingly complex due to variations in size, the provision of diverse services to distinct clientele, and a multi-faceted structure. Two methodologies were employed in this study to assess the operational effectiveness of bank branches: parametric and non-parametric. The disadvantage of parametric approaches lies in several intrinsic restrictions that render them inadequate for accurately representing the increasingly intricate nature of banking networks.

Regression analysis, a premier parametric methodology, serves as a method of central tendency and is exclusively applicable for modeling single input-multiple outputs or multiple inputs-single output systems. DEA, a non-parametric methodology, serves as a superior tool for efficiency analysis by establishing an efficient production frontier to evaluate the Decision-Making Units (DMUs) in relation to the optimal ones operating under identical conditions [1]. The capability to manage many inputs and outputs is a compelling advantage that distinguishes DEA from other analytical instruments. The standard DEA assesses the relative efficiency of DMUs considering numerous inputs and outputs without adequate details for managerial decision-making [2]. In practical situations, corporations (DMUs) consist of multiple divisions or levels, characterized by significant inter-level interactions that are interconnected and adhere to multi-tiered Stackelberg linkages. The intricate hierarchical framework of banking institutions presents obstacles for their managers. Managers face significant issues related to fluctuating costs and demand within multi-level Stackelberg relationships, necessitating the coordination of these interactions to deliver high-quality services to clients. Bank managers can enhance cost efficiency by sustaining or elevating service quality to establish a competitive edge.

The cost efficiency of banking chains assesses their capacity to generate current outputs at minimal costs, offering insights into the managerial oversight of overall operational expenses. The evaluation of banking branch performance inherently presents a hierarchical multi-level decision modeling challenge. In multi-level decentralized decision-making organizations, each level often governs its own set of decision variables, which frequently have conflicting objectives. Consequently, the assessment of banking chain performance is conducted from several perspectives. Performance assessment within this specific multi-tiered decentralized decision framework can be represented by a BLP-DEA methodology [3]. This methodology operates as a black box. It represents the internal framework and internal dynamics of a system with a hierarchical organization. BLP-DEA can offer useful insights and comprehensive information to managers when assessing the operation of a system characterized by Stackelberg-game dynamics [4]. This study aims to demonstrate a rarely employed non-parametric analytical method known as BLP-DEA, which tackles the issue of cost efficiency assessment within Stackelberg leader-follower dynamics in the banking sector [5, 6].

The subsequent sections of this study are structured as follows: Section 2 presents a review of the literature. Section 3 presents the concepts of BLP and DEA cost efficiency. Section 4 presents the BLP-DEA model introduced by Wu [4]. Section 5 illustrates the practical use of the BLP-DEA model through a case study involving an Iranian bank. Conclusions are delineated in Section 6.

2 Research Background

Compared to methods for evaluating organizational performance, DEA introduced by Charnes et al. [7] offers a superior approach for data organization and analysis, as it accommodates variations in efficiency over time and does not necessitate prior assumptions regarding the specification of the efficient frontier. Consequently, DEA serves as a superior method for performance evaluation inside the banking sector. In numerous practical situations, DMUs exhibit a two-stage network process [1, 8, 9]. Consequently, DEA has been expanded to assess the efficiency of two-step processes, wherein all outputs from the initial stage serve as intermediate measures, constituting the inputs for the subsequent stage. Wang [10] delineated a two-step process within the banking sector, wherein banks utilize inputs such as fixed assets, manpower, and information technology (IT) investments in the initial stage to create deposits. The banks utilize the deposits as an intermediary measure to produce loans and profits as outputs. Bhattacharya et al. [11] employed a two-stage DEA methodology to analyze the influence of liberalization on the efficiency of the Indian banking sector. A technical efficiency score was computed in the initial step.

Conversely, in the subsequent step, a stochastic frontier analysis was employed to ascribe variations in efficiency scores to the following sources: ownership, temporal, and noise components. Seiford and Zhu [12] analyzed the performance of the leading 55 US banks employing a two-stage DEA methodology. Findings revealed that larger banks demonstrate superior profitability, whereas smaller banks excel in marketability. Sexton and Lewis [13] presented a bifurcated methodology for assessing Major League Baseball performance. Kao and Hwang [14] proposed an alternative method in which the complete two-stage process can be expressed as the product of the efficiencies of the two subprocesses.

Consequently, the overall efficiency and the efficiency of each stage can be determined. Tone and Tsutsui [15] expanded the Slacks-Based Measure (SBM) model into a network architecture to address intermediate products formally. Avkiran [16], as the inaugural empirical investigation of Network Slacks-Based Measure (NSBM), utilized actual aggregate data from domestic commercial banks in the United Arab Emirates and employed the non-oriented NSBM to assess profit efficiency. Fukuyama and Weber [17] expanded the slacks-based inefficiency metric to assess a two-stage system with undesirable outputs at a Japanese bank. Paradi et al. [18] devised a two-stage DEA methodology for concurrently evaluating the performance of operational units. Li et al. [19] introduced a non-cooperative, centralized model to assess the efficiency of a two-stage process, aiming to deconstruct the overall efficiency of intricate network topologies.

Bi-level decision-making, or BLP techniques, initially described by Von Stackelberg et al. [20], have been primarily developed to address decentralized decision processes involving decision-makers inside a hierarchical organization. The individual responsible for decision-making in the upper echelon is referred to as the leader, whereas at the lower tier, the followers possess their own potentially conflicting aims. BLP has been utilized in numerous domains. Ryu et al. [21] introduced a BLP paradigm to address the competing interests of several constituents in supply chain planning issues. Sun et al. [22] introduced a BLP model to address the location problem, considering the interests of both customers and logistics planners. Sakawa et al. [23] addressed a transportation issue in a housing material manufacturing firm and obtained an acceptable resolution to the problem. Roghanian et al. [24] examined a probabilistic bi-level linear multi-objective programming issue and its application in comprehensive supply chain planning within enterprises. Arora and Gupta [25] introduced an interactive fuzzy goal programming methodology for BLP issues exhibiting dynamic programming traits. Lan et al. [26] developed two inventory control models for degrading items, utilizing time- and quantity-based integrated delivery strategies for suppliers within the Vendor-Managed Inventory (VMI) paradigm, grounded in BLP.

Wu [4] devised a novel quantitative methodology to assess the efficacy of multi-level decision network structures by incorporating cost DEA into a BLP framework and formulating a BLP-DEA model. To illustrate the model's applicability, Wu [4] presented its use in two real cases: a banking chain and a manufacturing supply chain.

3 Fundamentals of BLP and DEA Cost Efficiency

3.1 Cost Efficiency Model

Diverse DEA models with varying objectives have been established [27]. The primary objective of most DEA models is to analyze the technical and physical dimensions of production in contexts where unit pricing and unit cost data are either unknown or constrained due to fluctuations in the relevant prices and costs. The cost efficiency model demonstrates the capacity of DMUs to generate current outputs at minimal costs and illustrates how DEA can detect potential inefficiencies when precise cost information is available [28, 29].

Let there be n DMUs under assessment, denoted as $j = 1, \ldots, n$; the input vector is denoted as $X = (x_1, \ldots, x_m)^T$ which generates the output vector $Y = (y_1, \ldots, y_s)^T$ within the production possibility set. Accordingly, the cost efficiency model for the 0^{th} DMU $(0 \in \{1, \ldots, n\})$ can be articulated as follows:

$$
\begin{aligned}
&\boldsymbol{cx}^* = \min \sum_{i=1}^{m} c_i x_i \\
&\text{s.to} \\
&\sum_{j=1}^{n} x_{ij}\lambda_j \le x_i, \quad i = 1, \ldots, m; \\
&\sum_{j=1}^{n} y_{rj}\lambda_j \ge y_{r0}, \quad r = 1, \ldots, s; \\
&\lambda_j \ge 0, x_i \ge 0.
\end{aligned}
\tag{1}
$$

where, x_i and λ_j represent decision variables; and c_i is the unit cost of input i, which may differ among several DMUs. This model permits substitutions in inputs. The model's goal function is to reduce the total cost of the 0^{th} DMU.

Based on an optimal solution (x^*, λ^*) of the above linear programming, the cost efficiency of DMU_0 is defined as:

$$E_c = \frac{CX^*}{CX_0} \tag{2}$$

where, X_0 is the input vector of DMU_0.

3.2 BLP

BLP, inspired by Stackelberg's game theory [20], pertains to scenarios involving two decision-makers inside an organization who are linked in a hierarchical framework [30]. In these circumstances, the individual who initially renders a decision is referred to as the leader, while the one who, aware of the opponent's decision, then makes a decision is designated as the follower. These two decision-makers possess independent, potentially conflicting, objectives. Within the framework of BLP, the leader initially delineates a decision. The follower, fully aware of the leader's decision, chooses to optimize their objective function.

Consequently, the leader also makes decisions to optimize their objective function. The solution derived from the approach mentioned above is a Stackelberg equilibrium solution [31]. A bi-level linear programming (BLLP) problem is formulated to derive the Stackelberg solution as follows:

$$
\begin{aligned}
&\underset{x}{\text{minimize}}\ z_1(x, y) = c_1 x + d_1 y \\
&\text{where, } y \text{ solves} \\
&\text{minimize } z_2(x, y) = c_2 x + d_2 y \\
&\text{subject to } Ax + By \le b; x \ge 0, y \ge 0.
\end{aligned}
\tag{3}
$$

where, $c_i(i=1,2)$ denote the n_1-dimensional row coefficient vector, while $d_i(i=1,2)$ represent the n_2-dimensional vector. Furthermore, coefficient matrices A and B are $m \times n_1$ and $m \times n_2$, and b is an m-dimensional column constant vector. In addition, $z_1(x,y)$ and $z_2(x,y)$ represent the leader's and follower's objective functions, respectively; x and y represent a collection of decision variables governed by the leader and the follower [31].

Definition 1. S represents the BLLP problem's feasible region [31]:

$$S = \{(x,y) \mid Ax + By \leq b, x \geq 0, y \geq 0\}$$

Definition 2. $S(x)$ is the follower decision space after x is specified by the leader [31]:

$$S(x) = \{y \geq 0 \mid By \leq b - Ax, x \geq 0\}$$

Definition 3. S_X is the leader decision space [31]:

$$S_X = \{x \geq 0 \mid \text{ there is a } y \text{ such that } Ax + By \leq b, y \geq 0\}$$

Definition 4. $R(x)$ denotes the set of follower rational responses for x designated by the leader [31]:

$$R(x) = \left\{ y \geq 0 \mid y \in \arg \min_{y \in S(x)} z_2(x,y) \right\}$$

Definition 5. Inducible region [31]:

$$IR = \{(x,y) \mid (x,y) \in S, y \in R(x)\}$$

Definition 6. Stackelberg solution [31]:

$$\left\{ (x,y) \mid (x,y) \in \arg \min_{(x,y) \in IR} z_1(x,y) \right\}$$

In BLP, the follower optimization issue is regarded as a constraint within a bi-level optimization framework. By employing the Kuhn-Tucker methodology, a prevalent technique for addressing BLP, the follower's problem can be substituted with the Kuhn-Tucker conditions pertinent to the follower's issue. The leader's issue with limitations related to the optimality criteria of the follower's problem is subsequently resolved [31]. Utilizing Kuhn-Tucker conditions, the BLLP problem (3) can be reformulated as an equivalent single-level nonlinear programming (SLNLP) problem:

$$\begin{aligned} &\text{minimize } z_1(x,y) = c_1x + d_1y \\ &\text{subject to } uB - v = -d_2; \\ &u(Ax + By - b) - vy = 0; \\ &Ax + By \leq b; \\ &x \geq 0, y \geq 0, u^T \geq 0, v^T \geq 0 \end{aligned} \tag{4}$$

where, u denotes an m-dimensional row vector, and v represents an n_2-dimensional row vector. In addition, u and v stand for the dual variables related to constraints $Ax + By \leq b$ and $y \geq 0$.

By employing zero-one vectors $w_1 = (w_{11}, \ldots, w_{1m})$ and $w_2 = (w_{21}, \ldots, w_{2n_2})$, the NLP model can be converted into a linear mixed zero-one programming issue, which may be addressed using a zero-one mixed integer solution:

$$\begin{aligned} &\text{minimize } z_1(x,y) = c_1x + d_1y \\ &\text{subject to } 0 \leq u^T \leq Mw_1^T; \\ &\qquad 0 \leq b - Ax - By \leq M\left(e - w_1^T\right); \\ &\qquad 0 \leq (uB + d_2)^T \leq Mw_2^T; \\ &\qquad 0 \leq y \leq M\left(e - w_2^T\right); \\ &\qquad x \geq 0 \end{aligned} \tag{5}$$

where, e represents an m-dimensional vector of ones, and M denotes a large positive constant. For more information, please refer to the study by Nishizaki and Sakawa [31].

4 The Proposed BLP-DEA Model

The evaluation of banking branch performance presents inherent challenges in two-level hierarchical decision modeling. In the banking hierarchy, the primary level gathers funds from consumers as deposits, while the secondary level utilizes these deposits to generate profit. The performance evaluation of banking branches can be treated as a leader-follower Stackelberg problem, as the money received at the first level influences the investment decisions at the second level [4]. Figure 1 illustrates the notional BLP-DEA model using shared resources.

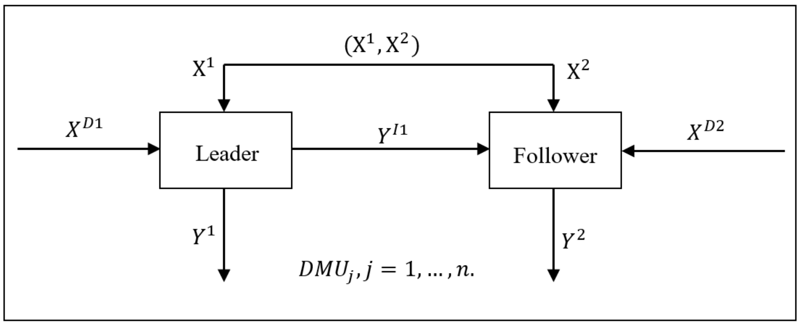

Figure 1. BLP-DEA model [4]

Consider n banking branches $(j = 1, \ldots, n)$ comprising L_1 and L_2 levels, the $L_1 - L_2$ chain was analyzed utilizing a BLP framework, with the initial level designated as the leader and the subsequent level as the follower. The performance evaluation issues of these two bank branch chains for a specific DMU_0 can be mathematically represented using the BLP-DEA model, which accounts for the hierarchical structure of the bank branches, incorporating the decision maker at each level who independently manages a set of decision variables, as follows:

$$
\begin{aligned}
&\underset{\bar{X}^1, \bar{X}^{D1}, \lambda}{\mathrm{Min}} \left(C^{1^T} \bar{X}^1 + C^{2^T} \bar{X}^{D1} \right) + \left(C^{1^T} \bar{X}^2 + D^{1^T} \bar{X}^{D2} + D^{2^T} \bar{Y}^{I1} \right) \\
&\text{s.to} \\
&\sum_{j=1}^{n} X_j^1 \lambda_j \leq \bar{X}^1; \\
&\sum_{j=1}^{n} X_j^{D1} \lambda_j \leq \bar{X}^{D1}; \\
&\sum_{j=1}^{n} Y_j^1 \lambda_j \geq Y_0^1; \\
&\sum_{j=1}^{n} Y_j^{I1} \lambda_j \geq Y_0^{I1}; \\
&\bar{X}^1 + \bar{X}^2 \leq E \ (\text{const.}); \\
&\underset{\bar{X}^2, \bar{X}^{D2}, \bar{Y}^{I1}, \pi}{\mathrm{Min}} \left(C^{1^T} \bar{X}^2 + D^{1^T} \bar{X}^{D2} + D^{2^T} \bar{Y}^{I1} \right) \\
&\text{s.to} \\
&\sum_{j=1}^{n} X_j^2 \pi_j \leq \bar{X}^2; \\
&\sum_{j=1}^{n} X_j^{D2} \pi_j \leq \bar{X}^{D2}; \\
&\sum_{j=1}^{n} Y_j^{I1} \pi_j \leq \bar{Y}^{I1}; \\
&\sum_{j=1}^{n} Y_j^2 \pi_j \geq Y_0^2; \\
&\bar{X}^1, \bar{X}^2, \bar{X}^{D1}, \bar{X}^{D2}, \bar{Y}^{I1}, \lambda, \pi \geq 0
\end{aligned}
\tag{6}
$$

where, m_1-dimensional X^1 denotes row vectors of the leader's shared input; m_1-dimensional X^2 denotes row vectors of the follower's shared input; m_2-dimensional X^{D1} represents the row vector of the leader's direct input; m_3-dimensional X^{D2} denotes the row vector of the follower's direct input; m_4-dimensional Y^{I1} represents a row vector, serving as the intermediate output to the leader and the intermediate input to the follower; m_5-dimensional Y^1 denotes the row vector of the leader's direct output; m_6-dimensional Y^2 stands for the row vector of the follower's direct output; $C^{1^T}, C^{2^T}, D^{1^T}$, and D^{2^T} denote the input unit cost vectors associated with the shared input, the direct input to the leader, the direct input to the follower, and the intermediate input to the follower, respectively; and λ and π represent the nonnegative multiplier used to aggregate existing leader and follower activities [4].

The BLP-DEA model was converted into the mixed integer single-level linear programming DEA (SLLP-DEA) model as outlined below:

$$
\begin{aligned}
&\min \left(C^{1^T}\bar{X}^1 + C^{2^T}\bar{X}^{D1}\right) + \left(C^{1^T}\bar{X}^2 + D^{1^T}\bar{X}^{D2} + D^{2^T}\overline{\boldsymbol{Y}}^{\boldsymbol{I}}\right)\\
&\text{s.to} \qquad \sum_{j=1}^{n} X_j^1\lambda_j \le \bar{X}^1; \qquad \sum_{j=1}^{n} X_j^{D1}\lambda_j \le \bar{X}^{D1};\\
&\qquad\qquad \sum_{j=1}^{n} Y_j^1\lambda_j \ge Y_0^1; \qquad \sum_{j=1}^{n} Y_j^{I1}\lambda_j \ge Y_0^{I1};\\
&0 \le \bar{X}^2 - \left(\sum_{j=1}^{n} X_j^2\pi_j\right) \le Mw_1^T;\\
&0 \le U^1 \le M\left(e - w_1^T\right);\\
&0 \le \bar{X}^{D2} - \left(\sum_{j=1}^{n} X_j^{D2}\pi_j\right) \le Mw_2^T;\\
&0 \le U^2 \le M\left(e - w_2^T\right);\\
&0 \le \bar{Y}^{I1} - \left(\sum_{j=1}^{n} Y_j^{I1}\pi_j\right) \le Mw_3^T;\\
&0 \le U^3 \le M\left(e - w_3^T\right);\\
&0 \le \sum_{j=1}^{n} Y_j^2\pi_j - Y_0^2 \le Mw_4^T;\\
&0 \le U^4 \le M\left(e - w_4^T\right);\\
&0 \le E - \bar{X}^1 - \bar{X}^2 \le Mw_5^T;\\
&0 \le U^5 \le M\left(e - w_5^T\right);\\
&U^1 - U^5 + V^1 = C^1;\\
&U^2 + V^2 = D^1;\\
&U^3 + V^3 = D^2;\\
&-X_j^2U^1 - X_j^{D2}U^2 - Y_j^{I1}U^3 + Y_j^2U^4 + V^4 = 0;\\
&0 \le \bar{X}^2 \le Mw_6^T;\\
&0 \le V^1 \le M\left(e - w_6^T\right);\\
&0 \le \bar{X}^{D2} \le Mw_7^T;\\
&0 \le V^2 \le M\left(e - w_7^T\right);\\
&0 \le \bar{Y}^{I1} \le Mw_8^T;\\
&0 \le V^3 \le M\left(e - w_8^T\right);\\
&0 \le \pi_j \le Mw_9^T;\\
&0 \le V^4 \le M\left(e - w_9^T\right);\\
&\bar{X}^1, \bar{X}^{D1}, \lambda \ge 0
\end{aligned}
\tag{7}
$$

where, U^1 and V^1 denote the m_1-dimensional dual vectors corresponding to the follower's shared input constraints and variables; U^2 and V^2 represent the m_3-dimensional dual vectors concerning the follower's direct input constraints and variables; U^3 and V^3 denote the m_4-dimensional dual vectors corresponding to the follower's intermediate input constraints and variables; U^4 and V^4 represent the n-dimensional dual vectors; U^5 is an m_1-dimensional dual vector corresponding to the constrained resource constraint; $W_i^T (i = 1, \ldots, 9)$ denotes the zero-one vectors; and e and M represent the vector of ones and the large positive constant.

By solving the BLP-DEA model, the optimal solutions of $\bar{X}^{1*}, \bar{X}^{2*}, \bar{X}^{D1*}, \bar{X}^{D2*}, \bar{Y}^{I1*}, \lambda_j{}^*$, and $\pi_j{}^*$ were obtained. Based on the optimal solutions, the cost efficiency of the leader of DMU_0 is defined as:

$$CE_0^L = \frac{c^{1^T} \bar{X}^{1*} + C^{2^T} \bar{X}^{D1*}}{C^{1^T} X_0{}^1 + C^{2^T} X_0^{D1}} \tag{8}$$

Furthermore, for DMU_0, the follower's cost efficiency is calculated as follows:

$$CE_0^F = \frac{C^{1^T} \bar{X}^{2*} + D^{1^T} \bar{X}^{D2*} + D^{2^T} \bar{Y}^{I1*}}{C^{1^T} X_0^2 + D^{1^T} X_0^{D2} + D^{2^T} Y_0^{I1}} \tag{9}$$

In addition, the total cost efficiency can be computed as follows:

$$CE_0^S = \frac{\left(C^{1^T} \bar{X}^{1*} + C^{2^T} \bar{X}^{D1*}\right) + \left(C^{1^T} \bar{X}^{2*} + D^{1^T} \bar{X}^{D2*} + D^{2^T} \bar{Y}^{I1^*}\right)}{\left(C^{1^T} X_0{}^1 + C^{2^T} X_0{}^{D1}\right) + \left(C^{1^T} X_0^2 + D^{1^T} X_0^{D2} + D^{2^T} \mathbf{Y}_0^{I1}\right)} \tag{10}$$

5 Empirical Study

Banks can be regarded as organizations where two decision-makers within a hierarchical framework alternate in making decisions to enhance their performance. The proposed BLP-DEA model was employed to assess some Iranian banks' performance in 2011. Each branch consists of marketability and profitability, with marketability as the primary factor and profitability as the secondary factor. The marketability level reflects the branch's capacity to attract deposits from clients by utilizing bank resources for marketing purposes. The profitability level reflects the branch's capacity to generate profit by allocating deposit values to other operations. The performance evaluation index system for the bank branch is presented in Table 1.

Table 1. The performance index system

Indicators	Name Leader	Unit	Indicators	Name Follower	Unit
Shared input	Employees	Person	Shared input	Employees	person
Direct input	Fixed assets	1,000,000,000 Riyal	Direct input	IT cost	100,000,000 Riyal
	Space	m^2			
Intermediate output	Deposit	10,000,000,000 Riyal	Intermediate output	Deposit	10,000,000,000 Riyal
Output	Non-invest deposit	100,000,000 Riyal	Output	Profit	10,000,000,000 Riyal
Input costs	Employees	1,000,000 Riyal	Input cost	Employees	1,000,000 Riyal
	Space	1,000,000 Riyal			

Each branch's performance is represented by several variables: fixed assets $\left(X^{D11}\right)$, space $\left(X^{D12}\right)$, noninvest deposit $\left(Y^1\right)$ at the leader level, IT costs $\left(X^{D2}\right)$ and profit $\left(Y^2\right)$ at the follower level. Deposit from leader to follower level $\left(Y^{I1}\right)$ serves as an intermediate variable. Marketability $\left(X^1\right)$ and profitability $\left(X^2\right)$ are resource-shared variables. The data for these factors is shown in Table 2, presenting the costs associated with shared staff and space. Due to the fixed nature of assets, deposits, and IT expenses, these costs are assumed to be unit-based.

Utilizing the mixed integer SLLP-DEA model, cost efficiency scores of branches and followers and leaders were derived. Table 3 presents the cost efficiency scores and the reference units for both the leader and the follower.

Table 3 indicates that no banks are cost-efficient, as they do not operate effectively at both decision levels. The 10th and 11th banks exhibit cost efficiency at the leader tier, while the 1st and 3rd banks demonstrate cost efficiency at the follower tier. Due to the inefficiency of the other players in these banks, these banks are consistently classified as inefficient. Table 3 demonstrates that the 2nd bank exhibits greater efficiency than the 3rd, 10th, and 11th banks, which operate at a singular level of efficiency, highlighting the potential advantage of coordination among participants.

Table 2. Problem data

DMUs	Leader Employee	Fixed Asset	Space	Non-Invest Deposit	Deposit	Follower Employee	IT Cost	Profit	Employees Cost	Space Cost
1	23	4.93	110	10.57	15.78	14	4.93	3.15	5.97	5.97
2	34	3.64	167.50	14.82	43.90	28	3.64	3.31	6.25	4.41
3	14	2.87	150	7.60	7.74	25	2.87	3.53	7.86	6.02
4	28	1.78	366.6	14.80	45.95	31	1.78	1.98	5.70	10.64
5	33	3.61	555	13.39	38.70	35	3.61	2.69	9.65	11.78
6	34	2.24	690	14.15	36.81	28	2.24	1.93	7.59	8.25
7	25	1.41	750.92	13.08	61.11	28	1.41	2.12	8.53	16.02
8	35	2.64	368.3	20.31	26.33	18	2.64	2.55	12.37	9.92
9	27	2.97	205	16.16	11.74	40	2.97	1.62	6.87	12.96
10	35	2.65	195	16.21	70.42	11	2.65	2.00	7.90	7.21
11	33	3.76	210	38.24	34.25	15	3.76	2.90	11.24	12.97
12	28	2.47	481	17.94	37.03	26	2.47	2.25	5.61	7.33
13	20	2.29	200	13.73	11.66	24	2.29	2.45	8.51	12.05
14	28	3.55	506	14.17	10.31	36	3.55	4.19	100.03	11.75
15	35	1.97	814	14.41	31.98	30	1.97	1.96	7.53	10.45

Table 3. Cost efficiency scores

DMUs	Bank Cost Efficiency	Leader Cost Efficiency	Follower Cost Efficiency	Reference Set for the Leader	Reference Set for the Follower
1	0.66	0.61	1.00	10, 11	1
2	0.50	0.90	0.70	10, 11	3
3	0.48	0.37	1.00	10, 11	3
4	0.14	0.41	0.28	10, 11	1
5	0.08	0.24	0.35	10, 11	1
6	0.10	0.20	0.31	10, 11	1
7	0.05	0.24	0.31	10	1
8	0.14	0.41	0.92	10, 11	3
9	0.19	0.54	0.29	11	3
10	0.32	1.00	0.52	10	1
11	0.18	1.00	0.79	11	1
12	0.15	0.33	0.52	10, 11	3
13	0.21	0.44	0.71	11	3
14	0.09	0.20	0.83	11	3
15	0.07	0.15	0.30	10, 11	1

6 Conclusions

In reality, banks operate with a decentralized framework where numerous decision-makers inside a hierarchical system make decisions sequentially or concurrently to optimize their goal function. In this swiftly evolving world, effective response to changes necessitates management's capacity to pinpoint inefficiencies. As a result, efficiency analysis can become a key tool for gaining a competitive edge. DEA is a superior method for assessing efficiency as it does not necessitate any preconceptions regarding the configuration of the efficient frontier. This study employs a BLP-DEA model, including two interconnected decision-makers inside a decentralized framework, to assess the performance of 15 Iranian bank branches, with one level representing a leader and the other a follower. The BLP-DEA model offers valuable insights and comprehensive information to bank management for assessing the efficiency of a bank inside Stackelberg-game dynamics. The outcomes derived from the BLP-DEA model exhibit significant discriminative capability by incorporating internal operations within the banking chain.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] M. Maghbouli and A. Pourhabib Yekta, "Managerial and natural disposability in two-stage network structure: A DEA-based approach," *Big Data Comput. Vis.*, vol. 1, no. 2, pp. 101–110, 2021. https://doi.org/10.22105/bdcv.2021.142087

[2] S. M. Muniz, W. B. Andriola, R. de Fátima Muniz, and S. A. Edalatpanah, "Impactos da Covid-19 na pós-graduação: Avaliação diagnóstica empregando-se o data envelopment analysys (DEA)," *Rev. Eletrônica Acta Sapientia*, vol. 9, no. 1, 2022. http://actasapientia.com.br/index.php/acsa/article/view/48

[3] K. Ghaziyani, S. F. Bagheri, and B. Ejlali, "Performance evaluation of mutual funds using the value-based DEA method," *Innov. Manag. Oper. Strateg.*, vol. 1, no. 1, pp. 82–97, 2020. https://doi.org/10.22105/imos.2021.261345.1009

[4] D. D. Wu, "Bilevel programming data envelopment analysis with constrained resource," *Eur. J. Oper. Res.*, vol. 207, no. 2, pp. 856–864, 2010. https://doi.org/10.1016/j.ejor.2010.05.008

[5] R. Fallah, M. Kouchaki Tajani, M. Maranjory, and R. Alikhani, "Comparison of banks and ranking of bank loans types on based of efficiency with DEA in Iran," *Big Data Comput. Vis.*, vol. 1, no. 1, pp. 36–51, 2021. https://doi.org/10.22105/bdcv.2021.142086

[6] S. A. Edalatpanah, R. Godarzi Karim, B. Khalilian, and S. Partouvi, "Data envelopment analysis and efficiency of firms: A goal programming approach," *Innov. Manag. Oper. Strateg.*, vol. 1, no. 1, pp. 1–16, 2020. https://doi.org/10.22105/imos.2021.266007.1024

[7] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision making units," *Eur. J. Oper. Res.*, vol. 2, no. 6, pp. 429–444, 1978. https://doi.org/10.1016/0377-2217(78)90138-8

[8] M. J. Goleij, "A new model of fuzzy two-stage data envelopment analysis with variable-return to scale," *J. Decis. Oper. Res.*, vol. 2, no. 2, pp. 130–146, 2018. https://doi.org/10.22105/dmor.2018.54145

[9] M. Kachouei, A. Ebrahimnejad, and H. Bagherzadeh Valami, "Efficiency evaluation in fuzzy two-stage data envelopment analysis based on fuzzy arithmetic approach," *J. Decis. Oper. Res.*, vol. 7, no. 1, pp. 143–159, 2022. https://doi.org/10.22105/dmor.2021.267206.1303

[10] H. Wang, "Contingent valuation of environmental resources: A stochastic perspective," Ph.D. dissertation, The University of North Carolina at Chapel Hill, 1997.

[11] A. Bhattacharyya, C. K. Lovell, and P. Sahay, "The impact of liberalization on the productive efficiency of Indian commercial banks," *Eur. J. Oper. Res.*, vol. 98, no. 2, pp. 332–345, 1997. https://doi.org/10.1016/S0377-2217(96)00351-7

[12] L. M. Seiford and J. Zhu, "Profitability and marketability of the top 55 US commercial banks," *Manag. Sci.*, vol. 45, no. 9, pp. 1270–1288, 1999. https://doi.org/10.1287/mnsc.45.9.1270

[13] T. R. Sexton and H. F. Lewis, "Two-stage DEA: An application to major league baseball," *J. Productivity Anal.*, vol. 19, pp. 227–249, 2003. https://doi.org/10.1023/A:1022861618317

[14] C. Kao and S. N. Hwang, "Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan," *Eur. J. Oper. Res.*, vol. 185, no. 1, pp. 418–429, 2008. https://doi.org/10.1016/j.ejor.2006.11.041

[15] K. Tone and M. Tsutsui, "Network DEA: A slacks-based measure approach," *Eur. J. Oper. Res.*, vol. 197, no. 1, pp. 243–252, 2009. https://doi.org/10.1016/j.ejor.2008.05.027

[16] N. K. Avkiran, "Opening the black box of efficiency analysis: An illustration with UAE banks," *Omega*, vol. 37, no. 4, pp. 930–941, 2009. https://doi.org/10.1016/j.omega.2008.08.001

[17] H. Fukuyama and W. L. Weber, "A slacks-based inefficiency measure for a two-stage system with bad outputs," *Omega*, vol. 38, no. 5, pp. 398–409, 2010. https://doi.org/10.1016/j.omega.2009.10.006

[18] J. C. Paradi, S. Rouatt, and H. Zhu, "Two-stage evaluation of bank branch efficiency using data envelopment analysis," *Omega*, vol. 39, no. 1, pp. 99–109, 2011. https://doi.org/10.1016/j.omega.2010.04.002

[19] Y. Li, Y. Chen, L. Liang, and J. Xie, "Dea models for extended two-stage network structures," *Omega*, vol. 40, no. 5, pp. 611–618, 2012. https://doi.org/10.1016/j.omega.2011.11.007

[20] H. Von Stackelberg, A. T. Peacock, E. Schneider, and T. W. Hutchison, "The theory of the market economy," *Economica*, vol. 20, no. 80, p. 384, 1953. https://doi.org/10.2307/2550609

[21] J. H. Ryu, V. Dua, and E. N. Pistikopoulos, "A bilevel programming framework for enterprise-wide process networks under uncertainty," *Comput. Chem. Eng.*, vol. 28, no. 6-7, pp. 1121–1129, 2004. https://doi.org/10.1016/j.compchemeng.2003.09.021

[22] H. Sun, Z. Gao, and J. Wu, "A bi-level programming model and solution algorithm for the location of logistics distribution centers," *Appl. Math. Model.*, vol. 32, no. 4, pp. 610–616, 2008. https://doi.org/10.1016/j.apm.2007.02.007

[23] M. Sakawa, I. Nishizaki, and Y. Uemura, "A decentralized two-level transportation problem in a housing material

manufacturer: Interactive fuzzy programming approach," *Eur. J. Oper. Res.*, vol. 141, no. 1, pp. 167–185, 2002. https://doi.org/10.1016/S0377-2217(01)00273-9

[24] E. Roghanian, S. J. Sadjadi, and M. B. Aryanezhad, "A probabilistic bi-level linear multi-objective programming problem to supply chain planning," *Appl. Math. Comput.*, vol. 188, no. 1, pp. 786–800, 2007. https://doi.org/10.1016/j.amc.2006.10.032

[25] S. R. Arora and R. Gupta, "Interactive fuzzy goal programming approach for bilevel programming problem," *Eur. J. Oper. Res.*, vol. 194, no. 2, pp. 368–376, 2009. https://doi.org/10.1016/j.ejor.2007.12.019

[26] H. Lan, R. Li, Z. Liu, and R. Wang, "Study on the inventory control of deteriorating items under VMI model based on bi-level programming," *Expert Syst. Appl.*, vol. 38, no. 8, pp. 9287–9295, 2011. https://doi.org/10.1016/j.eswa.2011.01.034

[27] F. Z. Montazeri, "The survey of data envelopment analysis models in fuzzy stochastic environments," *Int. J. Res. Ind. Eng.*, vol. 8, no. 4, pp. 366–383, 2019. https://doi.org/10.22105/riej.2020.219518.1121

[28] W. W. Cooper, L. M. Seiford, and K. Tone, *Data Envelopment Analysis: A Comprehensive Text with Models, Applications, References and Dea-Solver Software*. Springer, 2007. https://doi.org/10.1007/b109347

[29] J. Montazeri, R. Yazdani, and M. Kaviani, "Analysis of relationship between the performance and efficiency based on CCR and BCC models in the Iranian insurance industry: Tobit's approach," *Innov. Manag. Oper. Strateg.*, vol. 1, no. 1, pp. 57–72, 2020. https://doi.org/10.22105/imos.2021.265758.1023

[30] E. D. Gurmu and T. T. Fikadu, "Fuzzy programming approach to bi-level linear programming problems," *J. Fuzzy Extension Appl.*, vol. 1, no. 4, pp. 252–271, 2020. https://doi.org/10.22105/jfea.2020.262326.1069

[31] I. Nishizaki and M. Sakawa, *Cooperative and Noncooperative Multi-Level Programming*. Springer-Verlag US, 2009. https://doi.org/10.1007/978-1-4419-0676-2