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Analysis of Heat and Mass Transfer in Unsteady Magnetohydrodynamic MHD Casson Fluid Flow over Isothermal Inclined Plates with Thermal Diffusion and Heat Source Effects

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Abstract: In this study, we investigate the heat and mass transfer characteristics of an unsteady mixed convection magnetohydrodynamic (MHD) flow of Casson fluid through a porous medium in the presence of thermal diffusion and heat source effects. The flow is considered between isothermal inclined plates, incorporating the influences of Joule heating and viscous dissipation. Using dimensionless variables, the governing partial differential equations are transformed into their dimensionless form. The resulting dimensionless equations are solved empirically through the perturbation methodology. The effects of various critical parameters on the velocity, temperature, and concentration distributions within the boundary layer are analyzed with the aid of graphical representations. Additionally, numerical values of skin friction, Nusselt number, and Sherwood number near the plate are examined for different parameter values and presented in tabular form. The findings provide a deeper understanding of heat and mass transfer mechanisms in MHD flows through porous media, which are relevant to various industrial and engineering applications.

Keywords: Thermal diffusion; Chemical reaction; Casson fluid; Isothermal plates

1 Introduction

In modern engineering, many flow characteristics are not understandable with the Newtonian fluid model. Hence non-Newtonian fluid theory has become useful. Non- Newtonian fluid exerts non-linear relationships between the shear stress and rate of shear strain. It has an extensive variety of applications in engineering and industry, especially in the extraction of crude oil from petroleum products. Casson fluid is one of such fluids. Casson fluids classified as the most popular non-Newtonian fluid which has several applications in food processing, metallurgy, drilling operations and bio-engineering operations. Casson fluid model was introduced by Casson [1] for the prediction of the flow behavior of pigment-oil suspensions. Hari et al. [2] investigated the Soret and heat generation effects on MHD Casson fluid flow past an oscillating vertical plate embedded through porous medium. Rajakumar et al. [3] studied the Steady MHD Casson Ohmic heating and viscous dissipative fluid flow past an infinite vertical porous plate in the presence of Soret, Hall, and ion-slip current. Rajakumar et al. [4] discussed Unsteady MHD Casson Dissipative Fluid Flow past a Semi-infinite Vertical Porous Plate with Radiation Absorption and Chemical Reaction in Presence of Heat Generation. Rajakumar et al. [5] analyzed Radiation, dissipation and Dufour effects on MHD free convection Casson fluid flow through a vertical oscillatory porous plate with ion-slip current. In this regard a very recently Akhil et al. [6] have discovered Influence of thermophoresis and Brownian motion on mixed convection two dimensional MHD Casson fluid flow with non-linear radiation and heat generation. Harshad et al. [7] have analyzed MHD flow of micropolar nanofluid over a stretching/shrinking sheet considering radiation. Hari et al. [8] have possessed velocity, mass and temperature analysis of gravity-driven convection nanofluid flow past an oscillating vertical plate in the presence of magnetic field in a porous medium. Sheikholeslami et al. [9] have studied Radiation effects on heat transfer of three dimensional nanofluid flow considering thermal interfacial resistance and micro mixing in suspensions. Hari and Akhil [10] have studied Mathematical model for velocity and temperature of gravity-driven convective optically thick nanofluid flow past an oscillating vertical plate in presence of magnetic field and radiation.

MHD is the science of motion of electrically conducting fluids in presence of magnetic field. It concerns with the interaction of magnetic field with the fluid velocity of electrically conducting fluid. MHD generators, MHD pumps and MHD flow meters are some of the numerous examples of MHD principles. Dynamo and motor are classical examples of MHD principle. Convection problems of electrically conducting fluid in presence of magnetic field have got much importance because of its wide applications in Geophysics, Astrophysics, Plasma Physics, Missile technology, etc. MHD principles also find its applications in Medicine and Biology. Magnetohydrodynamics has many industrial applications such as physics, chemistry and engineering, crystal growth, metal casting and liquid metal cooling blankets for fusion reactors. Mishra et al. [11] have possessed a steady planar flow of an electrically conducting incompressible viscous fluid on a vertical plate with variable wall temperature and concentration in a doubly stratified micropolar fluid in the presence of a transverse magnetic field. Mishra et al. [12] have reviewed theoretical analysis of MHD unsteady free convection viscoelastic fluid flow through a porous medium. The medium is treated as incompressible and optically transparent. Mishra et al. [13] have investigated numerical approach to boundary layer stagnation-point flow past a stretching/shrinking sheet. Ramshankar et al. [14] have analysed Chemical reaction effect on MHD free convective surface over a moving vertical plate through porous medium. Swarnalatha et al. [15] have possessed combined effect of heat and mass transfer in Jeffrey fluid flow through porous medium over a stretching sheet subject to transverse magnetic field in the presence of heat source/sink. Mishra et al. [16] have investigated numerical approach to boundary layer stagnation-point flow past a stretching/shrinking sheet. Tharapatla et al. [17] have discussed heat alongside mass transport in a nonlinear free convection magnetohydrodynamics (MHD) non-Newtonian fluid flow with thermal radiation and heat generation deep-rooted in a thermally stratified penetrable medium.

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, and gas turbines and various propulsion devices for aircraft, missiles, satellites, and space vehicles are examples of such engineering applications. The effect of radiation on MHD flow, heat and mass transfer becomes more important industrially. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. The quality of the final product depends to a great extent on the heat controlling factors, and the knowledge of radiative heat transfer in the system can lead to a desired product with sought qualities. Different researches have been forwarded to analyze the effects of thermal radiation on different flows. Cortell [18] has reviewed Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. Ibrahim et al. [19] have discussed Effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction. Shateyi [20] has possessed thermal radiation and buoyancy effects on heat and mass transfer over a semi-infinite stretching surface with suction and blowing. Shateyi et al. [21] have investigated thermal radiation effects on heat and mass transfer over an unsteady stretching surface. Aliakbar et al. [22] have analysed the influence of thermal radiation on MHD flow of Maxwellian fluids above stretching sheets. Obulesu et al. [23] studied the radiation absorption effects on MHD Jeffrey fluid flow Past a vertical plate through a porous medium in conducting field.

Problems on fluid flow and mass transfer through porous media are the interest of not only mathematicians but also chemical engineers who have generally concerned with reacting and absorbing species, petroleum engineers, who have concerned with the miscible displacement process and civil engineers who are confronted with the problem of salt water encroachment of careful costal equiforce. Also, porous media are very widely used for heated body to maintain its temperature. To make the heat insulation of the surface more effective it is necessary to study the free convective effects under flow through porous media and to estimate effects on the heat transfer. Further, Gebhart and Pera [24] studied the laminar flows which arise in fluid due to the interaction of the gravity forces and density differences caused by the simultaneous diffusion of thermal energy and of chemical species neglecting the thermal diffusion and diffusion-thermo effects because the level of species concentration is very low. Raghunath et al. [25] have discussed Heat and mass transfer on an unsteady MHD flow through porous medium between two porous vertical plates. Raghunath et al. [26] have possessed heat and mass transfer on unsteady MHD Flow of a Visco-Elastic Fluid Past an Infinite Vertical Oscillating Porous Plate. Suresh et al. [27] have studied Finite Element Analysis of Free Convection Heat Transfer Flow in a Vertical Conical Annular Porous Medium.

Such effects are significant when density differences exist in the flow regime. For example when species are introduced at a surface in fluid domain, with different (lower) density than the surrounding fluid, Soret effects can be significant. Also, when heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more complex nature. It has been found that an energy flux can be generated not only by temperature gradients but also by composition gradients as well. The thermal diffusion (Soret) effect, for instance, has been utilized for isotope separation, and in mixture between gases with very light molecular weight (H_2, He) and of medium molecular weight (N_2, air). In fluid mechanics, fluid flow through porous media is the manner in which fluids behave when flowing through a porous medium, for example sponge or wood, or when filtering water using sand or another porous material. As commonly observed, some fluid flows through the media while some mass of the fluid is stored in the pores present in the media. The basic law governing the flow of fluids through porous media is Darcy's Law. Hari and Patel [28] have investigated Soret and heat generation effects on MHD Casson fluid flow past an oscillating vertical plate embedded through porous medium. Raju et al. [29] have studied Soret effects due to Natural convection between heated inclined plates with magnetic field. Turkyilmazoglu and Pop [30] examined the effects of Soret and heat source on unsteady MHD free convection flow from an impulsively started infinite vertical plate in presence of thermal radiation. Babu [31] has analyzed Soret and chemical reaction effects on an unsteady MHD free convection flow of radiative nanofluid through Porous Medium. Raju et al. [32] have reviewed Soret effect Due to Mixed convection on Unsteady Magnetohydrodynamic Flow past a Semi Infinite Vertical Permeable Moving Plate in Presence of Thermal Radiation. Chandrareddy et al. [33] have investigated Soret and Dufour effects on MHD free convection flow of rivlin-ericksen fluid past a semi-infinite vertical plate [34, 35]. Recently Raghunath et al. [36] have studied Investigation of MHD Casson fluid flow past a vertical porous plate under the influence of thermal diffusion and chemical reaction. Raghunath and Obulesu [37] have studied Unsteady MHD oscillatory Casson fluid flow past an inclined vertical porous plate in the presence of chemical reaction with heat absorption and Soret effects.

The chemical reaction is the process through which one set of chemical substances transforms into another. Additionally, chemical reactions occur between a fluid and foreign mass. The examination of heat and mass transfer issues, accounting for chemical reaction effects, has numerous applications in various chemical and hydrometallurgical industries, including glassware and ceramics production, food processing involving endothermic or exothermic reactions, and catalytic chemical reactors. Seth and Sarkar [38] examined hydromagnetic natural convection flow with an induced magnetic field and nth-order chemical reaction of a heat-absorbing fluid past an impulsively moving vertical plate with ramped temperature. Tripathy et al. [39] discussed the effects of chemical reactions on free convective surface over a moving vertical plate through a porous medium.

A thorough literature review motivated us to investigate the heat and mass transfer characteristics of unsteady mixed convection MHD flow of Casson fluid through a porous medium in the presence of thermal diffusion and heat source effects. By employing dimensionless variables, we transformed the governing partial differential equations into their dimensionless form and solved them using the perturbation methodology. Building upon the work of Ramachandra Reddy et al. [40], we incorporated additional effects such as Hall current, activation energy, and Brownian motion in MHD Darcy-Forchheimer Casson nanofluid flow. A comparative analysis with prior studies validated our findings, demonstrating substantial agreement with previously reported results and reinforcing the accuracy of our model. The study provides significant insights into heat and mass transfer mechanisms relevant to biomedical applications, such as blood flow analysis in the cardiovascular system, as well as industrial processes like glass manufacturing, paper production, and crude oil purification.

2 Formulation and Solution of the Problem

The unsteady MHD free convective, electrically conducting, chemically reactive, non-Newtonian (Casson) fluid flow over an incline porous plate with heat source and sink, diffusion thermo and radiation absorption is studied. The physical model of the problem is shown in Figure 1. The following assumptions are made:

The x-axis is oriented along the plate in the vertical upward direction, while the y-direction is perpendicular to it. The plate is inclined in the vertical direction at an angle α. The influence of magnetic fields is disregarded due to the extremely low magnetic Reynolds number of the flows. Initially, it is assumed that both the plate and the adjacent fluid are at uniform temperatures $\mathrm{T}\infty$ and concentrations $\mathrm{C}\infty$. Except for density, the fluid properties remain constant in the expression for buoyancy forces. Both chemical reactions and thermal radiation effects are taken into consideration. The fluid flow occurs along the x-axis direction; hence, all physical quantities are functions of y and t only. Schematic diagram in Figure 1 illustrated the dynamics of unsteady magnetohydrodynamic (MHD) flow of Casson fluid between inclined porous plates under the influence of chemical reactions, diffusion thermo, and radiation absorption phenomena. Key variables include velocity (U(x)), temperature (T(x)), and concentration (C(x)), with external forces such as gravity (g) and a magnetic field (B_0).

Under the aforementioned conditions, the equations describing the conservation of mass (continuity), momentum, energy, and concentration can be expressed as follows. The primary equations and respective boundary conditions for the flow domain have been delineated by Ramachandra Reddy et al. [40].

Equation of continuity

$$\frac{\partial v^*}{\partial y^*} = 0 \rightarrow v^* = -v_0\,(v_0 > 0) \tag{1}$$

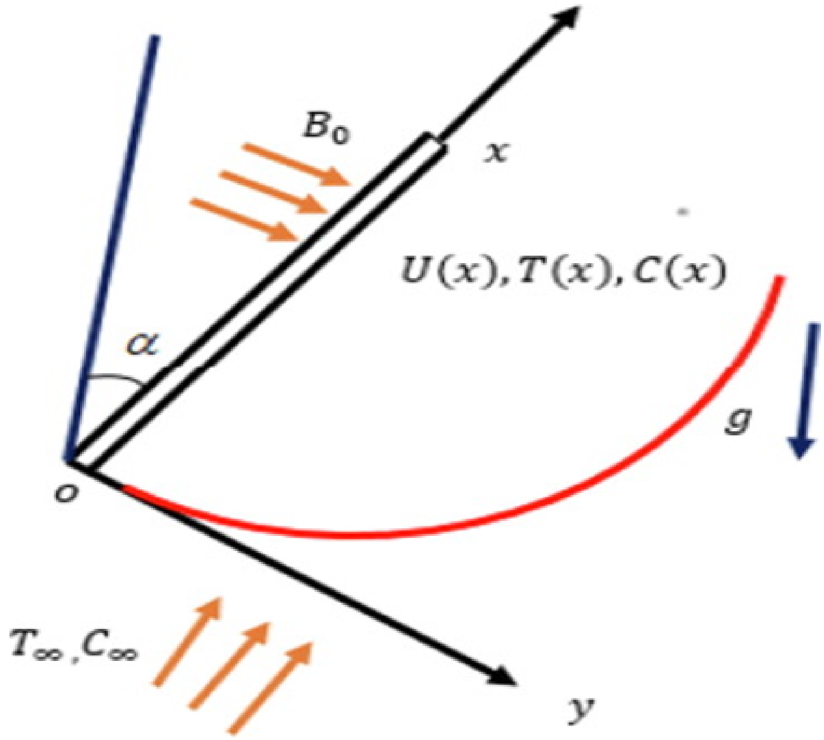

Figure 1. The dynamics of unsteady magnetohydrodynamic (MHD) flow

Momentum equation

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \vartheta \left(1 + \frac{1}{\lambda}\right) \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2}{\rho} u^* + g\left[\beta\left(T^* - T_\infty^*\right) + \beta^*\left(C^* - C_\infty\right)\right] \mathrm{Cos}\,\alpha - \frac{\vartheta u^*}{k^*} \tag{2}$$

Energy equation

$$\frac{\partial T*}{\partial t*} + v * \frac{\partial T*}{\partial y*} = k_1 \frac{\partial^2 T*}{\partial y*^2} + {Q^*}_0(T * - T_\infty^*) + \frac{\vartheta}{C_p}\left(\frac{\partial u*}{\partial y*}\right)^2 + \frac{\sigma B_0^2}{\rho} u*^2 \tag{3}$$

Species continuity equation

$$\frac{\partial C*}{\partial t*} + v * \frac{\partial C*}{\partial y*} = D \frac{\partial^2 C*}{\partial y*^2} + D_1 \frac{\partial^2 T*}{\partial y*^2} - K_C^*(C * - C_\infty^*) \tag{4}$$

Given the preceding assumptions, the suitable boundary conditions for the velocity, temperature, and concentration distributions can be expressed as follows:

$$\begin{array}{ll} t \leq 0, \quad u^* = 0, \quad v^* = 0, \quad T^* = T_\infty^*, \quad C^* = C_\infty^*, \forall z^* \\ t^* > 0, \quad u^* = u_0, \quad v^* = 0, T^* = T_w^*, C^* = {C^*}_w \text{ at } z^* = 0 \end{array} \tag{5}$$

$$u^* \to 0, v^* \to 0, \quad T^* \to T_\infty^* \quad C^* \to C_\infty^* \quad \text{as } z^* \to \infty \tag{6}$$

where, u^* and v^* are velocity components in x^* and y^* directions respectively, g is the acceleration due to gravity, β is the thermal expansion coefficient, β^* is the mass expansion coefficient, T^* is the temperature of the fluid, T_∞^* the temperature away from the plate, T_w^* is the temperature near the plate, C^* is the concentration of the fluid, C_∞^* is the concentration away from the Plate, C_w^* is the concentration near the plate, σ is the magnetic permeability of the fluid, B_0 is the Coefficient of magnetic field, ${q_r}^*$ is the Radiation heat flux density, ρ is the density of the fluid, α is the inclined angle, ϑ is the kinematic viscosity, k^* is the permeability of porous medium, k_1 is the thermal conductivity, C_p is the specific heat at constant pressure, D is the chemical molecular diffusivity, $K_C{}^*$ is the chemical reaction rate constant,ε is the scalar constant, n is the dimensionless exponential index.

To standardize the mathematical model of the physical problem, we introduce the subsequent dimensionless quantities and parameters:

$$
\begin{aligned}
& u=\frac{u^*}{v_0}, y=\frac{v_0 y^*}{\vartheta}, \theta=\frac{T^*-T_\infty^*}{T_w^*-T_\infty^*}, \phi=\frac{C^*-C_\infty^*}{C_w^*-C_\infty^*}, \operatorname{Pr}=\frac{\vartheta}{k_1}, Sc=\frac{\vartheta}{D}, M^2=\frac{\sigma B_0^2 \vartheta}{\rho v_0^2}, w=\frac{4 \vartheta w}{v_0^2} \\
& Gr=\frac{\vartheta g \beta_T\left(T_w^*-T_\infty^*\right)}{v_0^3}, Gm=\frac{\vartheta g \beta_c^*\left(C_w^*-C_\infty^*\right)}{v_0^3}, k=\frac{v_0^2 k^*}{\vartheta^2}, t=\frac{t^* v_0^2}{4 \vartheta}, \vartheta=\frac{n_0}{\rho}, \\
& Ec=\frac{v_0^2}{C_p\left(T_w-T_\infty\right)}, S_r=\frac{D_1\left(T_w-T_\infty\right)}{\vartheta\left(C_w-C_\infty\right)}, K_C^*=\frac{\vartheta K}{v_0^2}, Q=\frac{Q_{0 \vartheta}}{\rho C_p v_0^2}
\end{aligned} \tag{7}
$$

The non-dimensional form of the Eqs. (2) to (4) are presented as follows:

$$
\frac{1}{4} \frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\left(1+\frac{1}{\lambda}\right) \frac{\partial^2 u}{\partial y^2}-M^2 u-\frac{1}{k} u+(Gr \theta+Gm \phi) \operatorname{Cos} \alpha \tag{8}
$$

$$
\frac{1}{4} \frac{\partial \theta}{\partial t}-\frac{\partial \theta}{\partial y}=\frac{1}{\operatorname{Pr}} \frac{\partial^2 \theta}{\partial y^2}+\frac{1}{4} Q \theta+\operatorname{Pr} E_c\left(\frac{\partial u}{\partial y}\right)^2+\operatorname{Pr} Ec M^2 u^2 \tag{9}
$$

$$
\frac{1}{4} \frac{\partial \phi}{\partial t}-\frac{\partial \phi}{\partial y}=\frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2}+{ }_r S_c \frac{\partial^2 \theta}{\partial y^2}-K \phi \tag{10}
$$

The respective boundary conditions are defined as follows:

$$
u=0 \quad \theta=1, \quad \phi=1, \quad \text{at} \quad y=0 \tag{11}
$$

$$
u \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty \tag{12}
$$

3 Method of Solution

To address the non-linear partial differential Eqs. (8)-(10) while adhering to condition (11-12), we assume that the fluid velocity, temperature, spices can be expressed as

$$
F(y, t)=F_0(y)+E_c F_1(y)+O\left(E_c^2\right) \tag{13}
$$

where, F in terms of u, θ, φ.

By substituting Eq. (13) into Eqs. (8) to (10), and then equating the coefficients of terms with identical powers of ε while disregarding higher-order terms, we derive the following equations:

$$
\left(1+\frac{1}{\lambda}\right) u_0^{\prime \prime}+u_0^{\prime}-M^2 \mathbf{u}_0-\frac{1}{k} \mathbf{u}_0=-\left(\operatorname{Gr} \theta_0+\operatorname{Gm} \phi_0\right) \operatorname{Cos} \alpha \tag{14}
$$

$$
\theta_0^{\prime \prime}+\operatorname{Pr} \theta_0^{\prime}-\operatorname{Pr} \mathrm{Q} \theta_0=0 \tag{15}
$$

$$
\phi_0^{\prime \prime}+Sc \phi_0^{\prime}-\operatorname{ScK} \phi_0=-\mathrm{S}_{\mathrm{c}} S_r \theta_0^{\prime \prime} \tag{16}
$$

$$
\left(1+\frac{1}{\lambda}\right) u^{\prime \prime}{ }_1+u^{\prime}{ }_1-\left(M^2+\frac{1}{k}+\frac{1}{4} i w\right) \mathrm{u}_1=-\left(\operatorname{Gr} \theta_1+\operatorname{Gm} \varphi_1\right) \operatorname{Cos} \alpha \tag{17}
$$

$$
\theta^{\prime \prime}{ }_1+\operatorname{Pr} \theta^{\prime}{ }_1-\left(\frac{1}{4} i w-\frac{1}{4} Q\right) \mathrm{P}_{\mathrm{r}} \theta_1=0 \tag{18}
$$

$$\varphi''_1 + Sc\,\varphi'_1 - Sc\,K\,\varphi_1 = -\mathrm{Sc}\ \mathrm{S_r}\,\theta''_1$$

The boundary conditions associated with this are

$$\begin{array}{ll} u_0 = 0, u_1 = 0, \theta_0 = 1, \theta_1 = 1, \quad \phi_0 = 1, \quad \phi_1 = 1 & \text{at } y = 0 \\ u_0 = 1, u_1 = 0, \theta_0 \to 0, \theta_1 \to 0, \phi_0 \to 0, \phi \to 0 & \text{as } y \to \infty \end{array} \tag{19}$$

Upon solving Eqs. (14)-(18) with respect to the boundary conditions specified in Eq. (19), the resulting solutions are as follows:

$$\theta_0 = \exp\left(-l_1 y\right) \tag{20}$$

$$\phi_0 = b_1 \exp\left(-l_1 y\right) + b_2 \exp\left(-l_2 y\right) \tag{21}$$

$$u_0 = b_3 \exp\left(-l_1 y\right) + b_4 \exp\left(-l_2 y\right) + b_5 \exp\left(-l_3 y\right) \tag{22}$$

$$\begin{aligned} \theta_1 =& b_6 \exp\left(-2l_1 y\right) + b_7 \exp\left(-2l_2 y\right) + b_8 \exp\left(-2l_3 y\right) + b_9 \exp\left(-\left(l_1 + l_2\right) y\right) + \\ & b_{10} \exp\left(-\left(l_3 + l_2\right) y\right) + b_{11} \exp\left(-\left(l_1 + l_3\right) y\right) + b_{12} \exp\left(-l_4 y\right) \end{aligned} \tag{23}$$

$$\begin{aligned} \phi_1 =& b_{13} \exp\left(-l_4 y\right) + b_{14} \exp\left(-2l_1 y\right) + b_{15} \exp\left(-2l_2 y\right) + b_{16} \exp\left(-2l_3 y\right) + \\ & b_{17} \exp\left(-\left(l_1 + l_2\right) y\right) + b_{18} \exp\left(-\left(l_3 + l_2\right) y\right) + b_{19} \exp\left(-\left(l_1 + l_3\right) y\right) + b_{20} \exp\left(-l_5 y\right) \end{aligned} \tag{24}$$

$$\begin{aligned} u_1 = & b_{21} \exp\left(-l_4 y\right) + b_{22} \exp\left(-2l_1 y\right) + b_{23} \exp\left(-2l_2 y\right) + b_{24} \exp\left(-2l_3 y\right) + b_{25} \exp\left(-\left(l_1 + l_2\right) y\right) \\ & + b_{26} \exp\left(-\left(l_3 + l_2\right) y\right) + b_{27} \exp\left(-\left(l_1 + l_3\right) y\right) + b_{28} \exp\left(-l_5 y\right) + b_{29} \exp\left(-l_6 y\right) \end{aligned} \tag{25}$$

By substituting Eqs. (20)-(25) into Eq. (13), we get the velocity, temperature, and concentration distributions within the boundary layer as outlined below.

$$\begin{aligned} u(y,t) =& b_3 \exp\left(-l_1 y\right) + b_4 \exp\left(-l_2 y\right) + b_5 \exp\left(-l_3 y\right) + E_c \left[b_{21} \exp\left(-l_4 y\right) + b_{22} \exp\left(-2l_1 y\right) + \right. \\ & b_{23} \exp\left(-2l_2 y\right) + b_{24} \exp\left(-2l_3 y\right) + b_{25} \exp\left(\left(l_1 + l_2\right) y\right) + b_{26} \exp\left(\left(l_3 + l_2\right) y\right) + \\ & \left. b_{27} \exp\left(\left(l_1 + l_3\right) y\right) + b_{28} \exp\left(-l_5 y\right) + b_{29} \exp\left(-l_6 y\right)\right] \end{aligned} \tag{26}$$

$$\begin{aligned} \theta(y,t) =& \exp\left(-l_1 y\right) + E_c \left[b_6 \exp\left(-2l_1 y\right) + b_7 \exp\left(-2l_2 y\right) + b_8 \exp\left(-2l_3 y\right) + \right. \\ & \left. b_9 \exp\left(-\left(l_1 + l_2\right) y\right) + b_{10} \exp\left(-\left(l_3 + l_2\right) y\right) + b_{11} \exp\left(-\left(l_1 + l_3\right) y\right) + b_{12} \exp\left(-l_4 y\right)\right] \end{aligned} \tag{27}$$

$$\begin{aligned} \phi(y,t) =& b_1 \exp\left(-l_1 y\right) + b_2 \exp\left(-l_2 y\right) + E_c \left[b_{13} \exp\left(-l_4 y\right) + b_{14} \exp\left(-2l_1 y\right) + \right. \\ & b_{15} \exp\left(-2l_2 y\right) + b_{16} \exp\left(-2l_3 y\right) + b_{17} \exp\left(-\left(l_1 + l_2\right) y\right) + b_{18} \exp\left(-\left(l_3 + l_2\right) y\right) + \\ & \left. b_{19} \exp\left(-\left(l_1 + l_3\right) y\right) + b_{20} \exp\left(-l_5 y\right)\right] \end{aligned} \tag{28}$$

The skin friction, Nusselt and Sherwood numbers are imperative substantial parameters of border layer flow and are specified by

$$\begin{aligned} \tau =& -\left(b_3 l_1 + b_4 l_2 + b_5 l_3\right) - E_c \left[b_{21} l_4 + 2b_{22} l_1 y + 2b_{23} l_2 + 2b_{24} l_3 + b_{25}\left(l_1 + l_2\right) + \right. \\ & \left. b_{26}\left(l_3 + l_2\right) + b_{27}\left(l_1 + l_3\right) + b_{28} l_5 + b_{29} l_6 y\right)] \end{aligned} \tag{29}$$

$$N_u == -l_1 - E_c \left[2b_6 l_1 + 2b_7 l_2 + 2b_8 l_3 + b_9\left(l_1 + l_2\right) + b_{10}\left(l_3 + l_2\right) + b_{11}\left(l_1 + l_3\right) + b_{12} l_4\right] \tag{30}$$

$$\begin{aligned} Sh =& -\left(b_1 l_1 + b_2 l_2\right) - E_c \left[b_{13} l_4 + 2b_{14} l_1 + 2b_{15} l_2 + 2b_{16} l_3 + b_{17}\left(l_1 + l_2\right) + b_{18}\left(l_3 + l_2\right) + \right. \\ & \left. b_{19}\left(l_1 + l_3\right) + b_{20} l_5\right] \end{aligned} \tag{31}$$

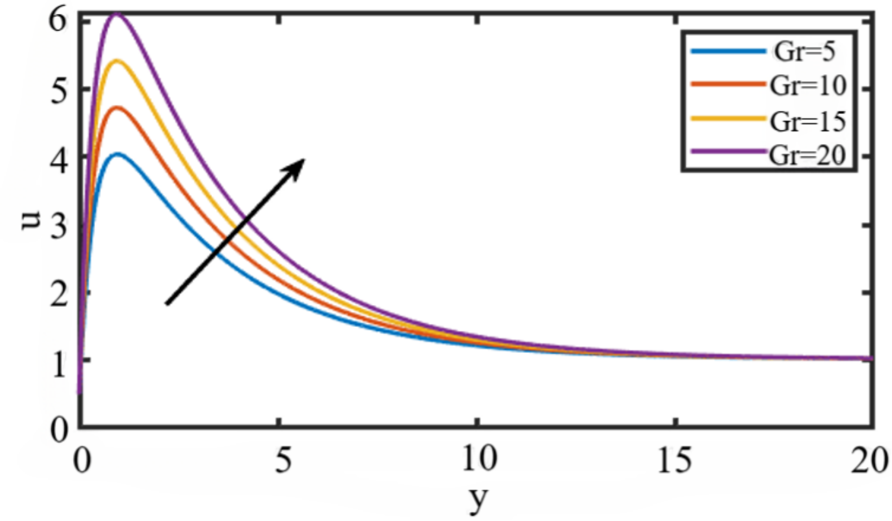

Figure 2. Influence of Gr on velocity profiles u

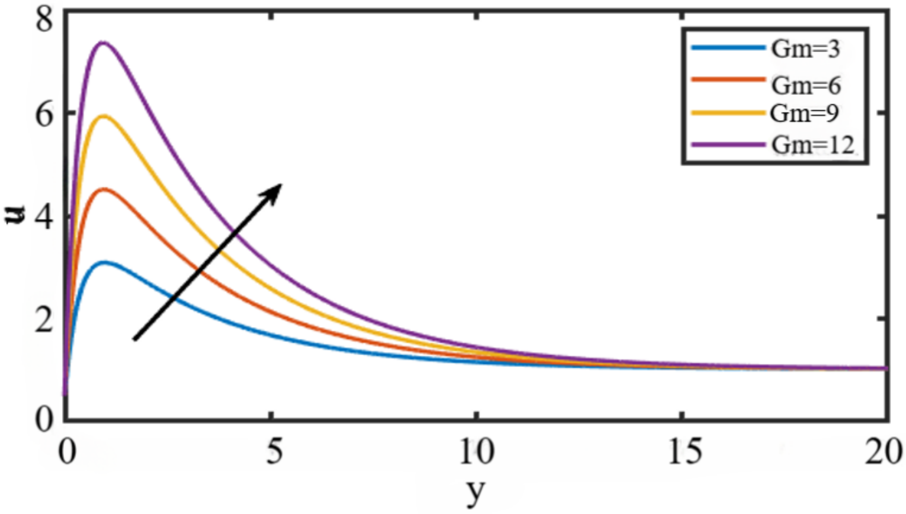

Figure 3. Influence of Gm on velocity profiles u

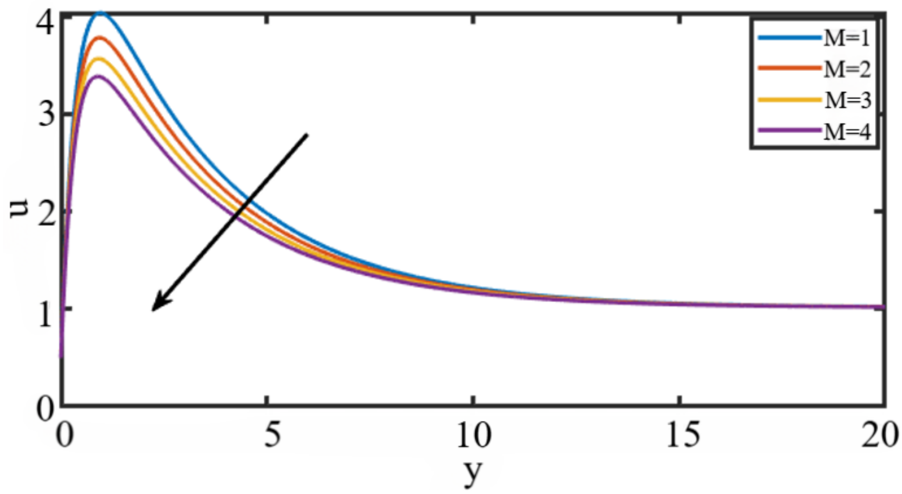

Figure 4. Influence of M on velocity profiles u

4 Results and Discussion

Figure 2 and Figure 3 exhibit the effect of Grashof number for heat transfer Gr and Grashof number for mass transfer Gm on the velocity profile with other parameters are fixed. The Grashof number for heat transfer Gr signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermal buoyancy force. Also, as Gr increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity. The Grashof number for mass transfer Gm defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value. It is noticed that the velocity increases with increasing values of the Grashof number for mass transfer Gm. The effect of the Magnetic field parameter M is shown in Figure 4. It is observed that the velocity of the fluid decreases with the increase of the magnetic field number values. The decrease in the velocity as the Magnetic field parameter M increases is because the presence of a magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present study. This resistive force slows down the fluid velocity component as shown in Figure 4. Figure 5 shows the effect of the permeability of the porous medium parameter on the velocity distribution. As shown, the velocity is increasing with the increasing dimensionless porous medium parameter. Physically, this result can be achieved when the holes of the porous medium may be neglected. The velocity profiles in Figure 6 shows that rate of motion is significantly reduced with increasing of Casson fluid parameter λ. Also, it is observed from Figure 6, the boundary layer momentum thickness decreases as increase of Casson fluid parameter λ. The effect of angle of inclination of the plate α effect on the velocity field has been illustrated in Figure 7. It is seen that as the angle of inclination of the plate α increases the velocity field is decreases.

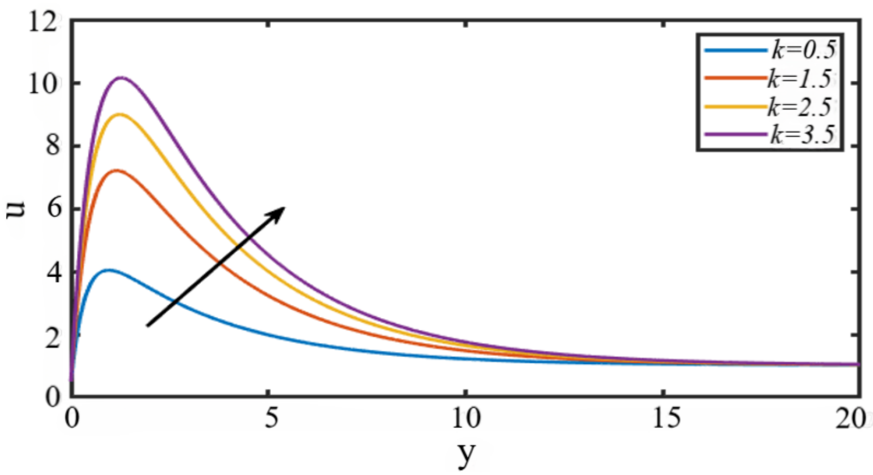

Figure 5. Influence of k on velocity profiles u

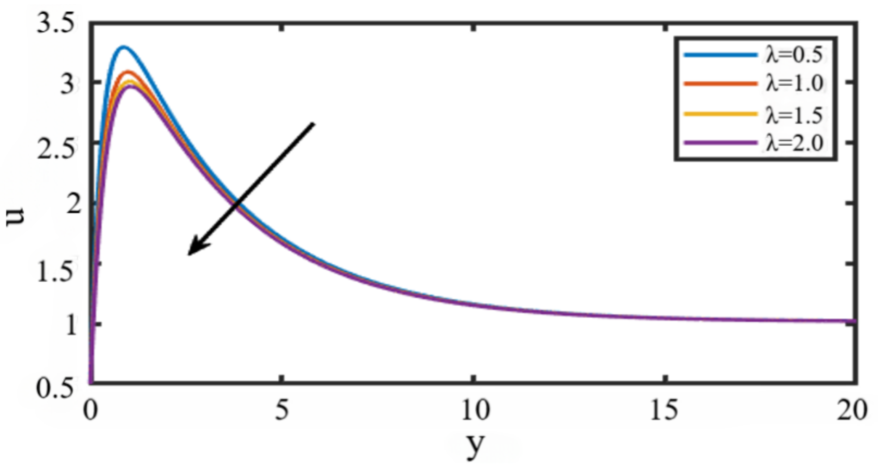

Figure 6. Influence of λ on velocity profiles u

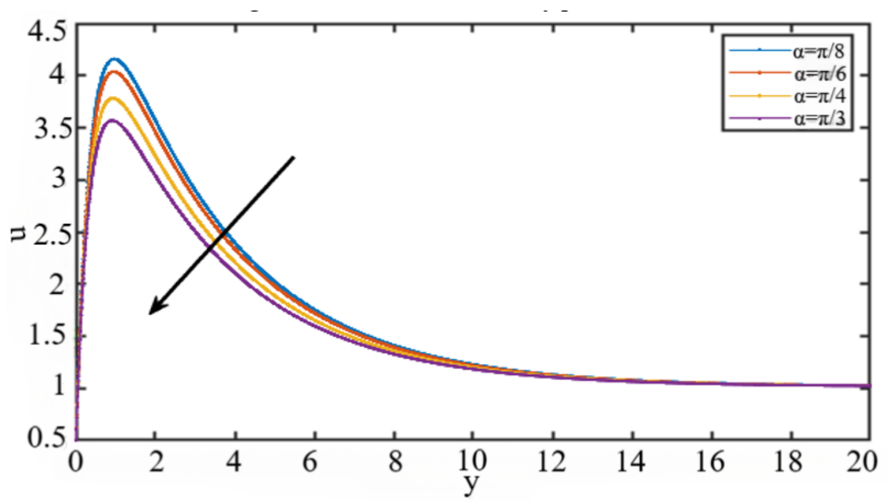

Figure 7. Influence of α on velocity profiles u

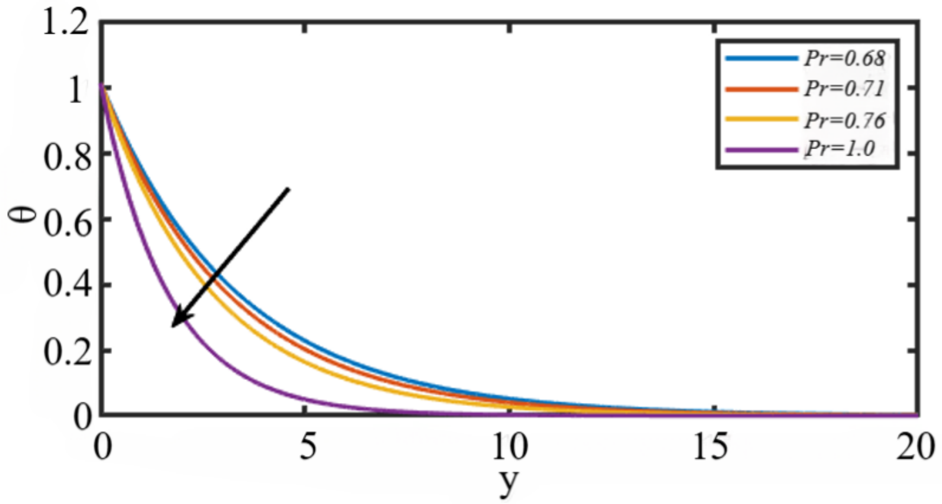

Figure 8. Influence of Pr on temperature profiles θ

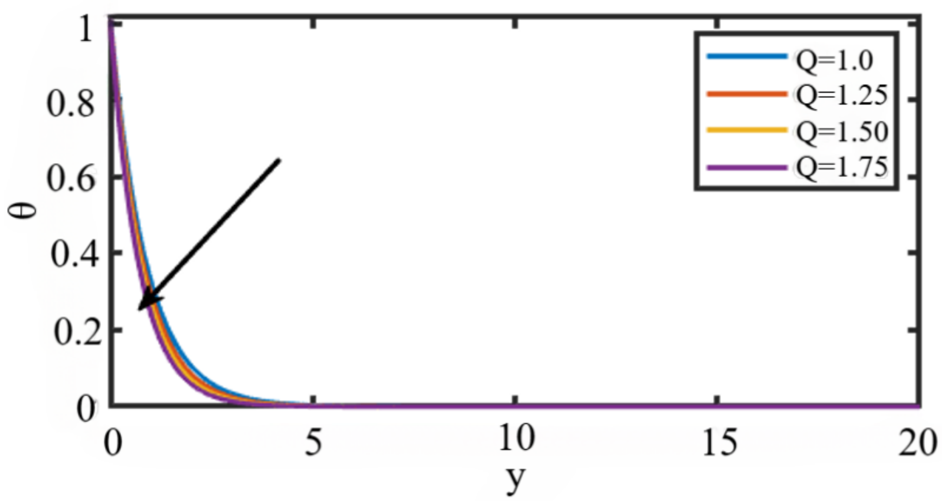

Figure 9. Influence of Q on temperature profiles θ

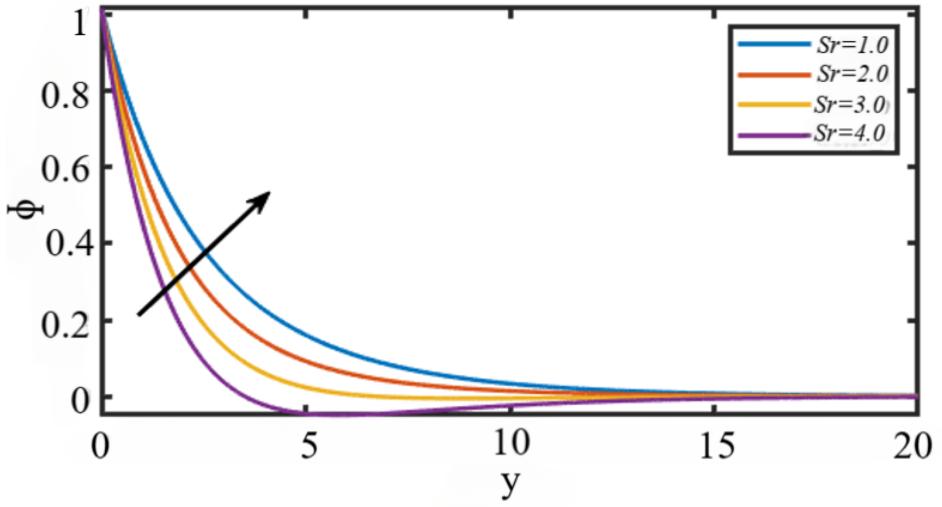

Figure 10. Without liquid cooling steady-state thermal

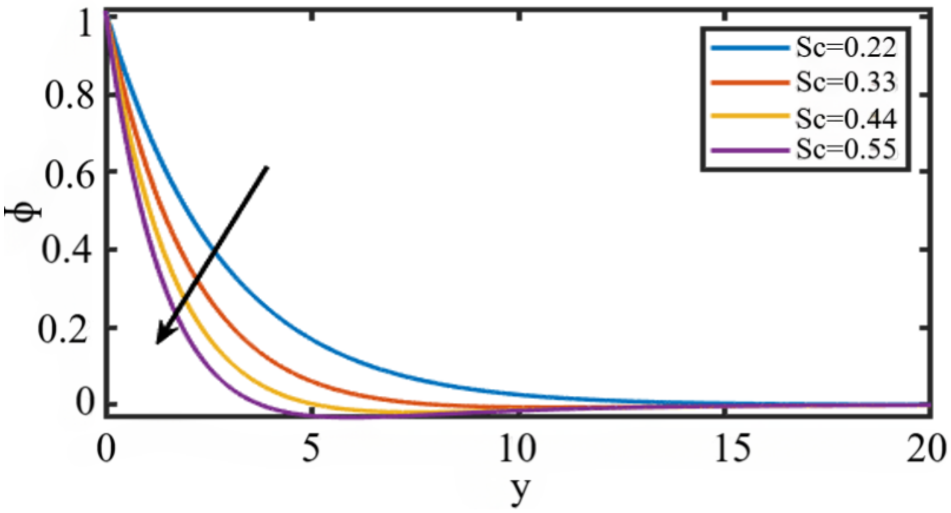

Figure 11. Influence of Sc on concentration profiles Φ

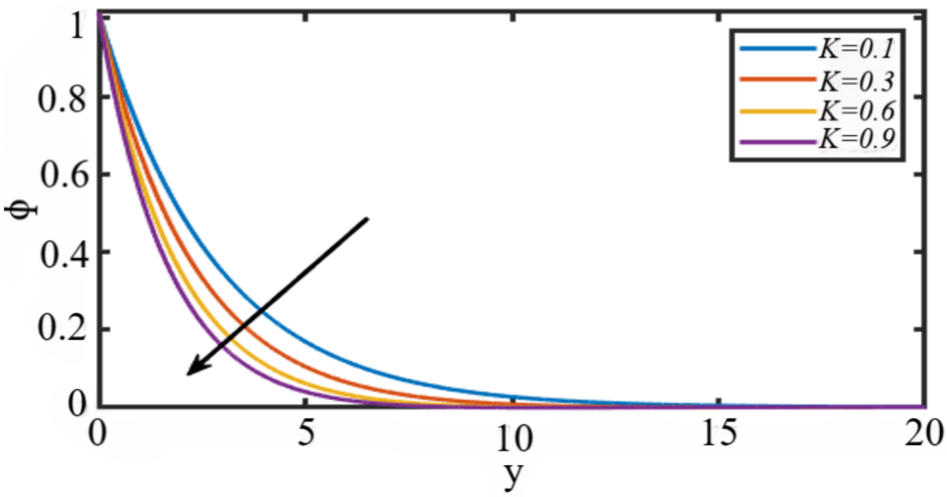

Figure 12. Influence of K on concentration profiles Φ

Figure 8 depicts the effect of Prandtl number on temperature profiles in presence of some selected fluids such as Hydrogen (Pr = 0.68), Air (Pr = 0.71), Carbon dioxide (Pr = 0.76) and Electrolytic solution (Pr = 1). From this figure, we observed that, an increase in the Prandlt number decreases the temperature of the flow field at all points. Due to the ratio of momentum diffusivity (kinematic viscosity) to thermal diffusivity. Figure 9 illustrate the influence of the heat absorption coefficient Q on the temperature profile. Physically speaking, the presence of heat absorption (thermal sink) effects has the tendency to reduce the fluid temperature.

Figure 10, Figure 11, Figure 12 demonstrate the variation of concentration field under the influence of the parameters Sc, Sr, and K. Figure 10 shows that the effect of Soret parameter on concentration. It is observed that the concentration is diminishes with increasing values of Soret number, this is consistent with the fact that an increase in Sc means a decrease of molecular diffusivity which results in a fall in the thickness of the concentration boundary layer as shown in Figure 11. The same behavior is observed in the case of Figure 12 it means the chemical reaction is diminished.

Table 1. Effect of various parameters on the skin friction

Gr	Gm	M	K	α	Skin Friction (τ)
5					6.2828
10					9.0189
15					11.7599
	6				6.9902
	12				11.1620
	15				13.1772
		1.5			4.9730
		2			4.0174
		2.5			3.3332
			1.5		5.6489
			2.5		4.7953
			3.5		4.2342
				$\pi/10$	6.8989
				$\pi/6$	6.2826
				$\pi/3$	3.6283

Table 2. Effects of various parameters on Nusselt number

Pr	Ec	M	Q	Nu
0.78				0.00150
0.79				4.5391
0.80				4.1840
	0.01			0.5888
	0.05			5.3022
	0.1			11.1940
		1.5		-0.5397
		2		-0.5668
		2.5		-0.5781
			0.1	-0.4717
			0.15	-0.5144
			0.20	-0.5366

Skin friction is an influential appearance that characterizes the frictional resistance at the dense exterior. From Table 1, the magnetic field, aligned magnetic field, inclined angle, thermal Grahosf Number, Casson Parameter, and chemical reaction all increase the skin-friction, while the mass Grahosf Nemours, heat sink parameter all decrease it. From Table 2, the rate of mass transference from the platter to the liquid area is slowed down by the Prandtl number; the heat source parameter and Time (t) are increased. From Table 3, it is noted that all the entries are positive. It is observed that Schmidt number (Sc), and chemical reaction parameter (K), increase the rate of mass transfer at the surface of the plate.

Table 3. Various values parameters on Sherwood Number

Sc	So	K	Sh
0.6			-0.6145
1.2			-1.1013
1.8			-2.5014
	0.5		-0.6145
	1.0		-0.4529
	1.5		-0.2608
		0.001	0.3581
		0.003	0.1430
		0.005	0.0052

5 Conclusion and Application

The present investigation leads to the following conclusions:

- The fluid velocity decreases with the increase in Aligned magnetic field, inclined angle, Casson parameter, chemical reaction and heat sink parameter whereas it is accelerated due to thermal diffusion.

- The fluid temperature drops with the increase in heat sink parameter and Prandtl number.

- The concentration level of the fluid rises with the increase in Soret number and falls due to Schmidt number and chemical reaction parameter.

- The skin-friction coefficient is increased under the effect of magnetic field, aligned magnetic field, inclined angle, thermal Grashof number, Casson parameter and chemical reaction parameter, whereas it is decreased with the increase in mass Grashof number, heat sink parameter.

- The rate of mass transfer is decreased from the plate to the fluid region due to Prandtl number, heat source parameter, where as it increases t.

- Sherwood Number increases with increasing values of Prandtl Number and Chemical Reaction Parameter.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflict of interest.

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